AKENTEN APPIAH-MENKA UNIVERSITY OF SKILLS TRAINING AND ENTREPRENEURIAL DEVELOPMENT

THE EFFECT OF THE USE OF ALGEBRA TILES MANIPULATIVES ON STUDENTS' MATHEMATICS ACHIEVEMENT IN ALGEBRA IN SOME SELECTED PUBLIC JUNIOR HIGH SCHOOLS IN BEREKUM WEST DISTRICT

PRINCE KUSI

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PRINCE KUSI

(8210620007)

A thesis in the Department of Mathematics Education, Faculty of Educational Studies, submitted to the School of Graduate Studies in partial fulfillment

of the requirements for the award of the degree of

Master of Philosophy

(Mathematics Education)

in the Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Kumasi.

MAY, 2023

DECLARATION

CANDIDATE'S DECLARATION

I, **Prince Kusi** declare that this thesis, except quotations and references contained in the published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in the part or whole, for another degree elsewhere.

Signature: ………………………… **Date**: ……………………………….

SUPERVISOR'S DECLARATION

We hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Kumasi.

Prof. Ebenezer Bonyah (Principal Supervisor)

Signature: …………………………… **Date**: …………………………………

Prof. Y. D. Arthur (Co-Supervisor)

Signature: .... **Date:** ....

DEDICATION

To my family, friends and all my loved ones.

ACKNOWLEDGEMENTS

I sincerely appreciate the efforts of everyone who made this work a success. They owe me a great deal. My supervisors at the Akenten-Appiah Menka University of Skills Training and Entrepreneurial Development (AAMUSTED), Kumasi, Professor Ebenezer Bonyah and Prof. Y. D. Arthur of the Department of Mathematics, deserve my deepest gratitude for helping me create this thesis with their helpful and constructive comments and suggestions. I would like to express my sincere gratitude to Dr. Eric Appiah-Twumasi of the AAMUSTED Mampong campus, Mr. Augustine Bediako of the Alfarouq College of Education, Mr. Emmanuel Teku, and Mr. Collins Asoma of the Berekum College of Education, all of whom provided me with consistent academic advice and supported me in various ways to accomplish this feat. For their moral support during the study, I would like to thank my lovely wife Juliana Agyeiwaa, and my children Giovanna, Anselma, Junior, Justice, and Justus. My mother Stella Amponsah, sister Joyce Kusi, and brothers Emmanuel Nana Kusi, Samuel Nana Kusi, and Joseph Nana Kusi deserve special recognition for their unfailing support.

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ABSTRACT

This study aimed to determine how the use of algebra tile manipulatives affected students' mathematics achievement in algebra in some selected public Junior High Schools in Berekum West District in the Bono Region of Ghana. Using the lottery method of simple random sampling, stratified and the purposive sampling techniques, 274 respondents who were all students in JHS form 2 were chosen to participate in the study. The research approach was mixed methods and quasi-experimental as its design. Descriptive statistics (mean, standard deviation, and coefficient of variation) were used to analyze the data, and the quantitative data were analyzed using the paired samples t-test. The qualitative data from the respondents underwent thematic data analysis. The findings of the study demonstrated that using algebra tiles to expand binomials and factorize trinomials in algebraic expressions increased student achievement. The study's findings also revealed that teachers limited was cardinal challenge of the perceived challenges of algebra tiles use. The results of the interviews supported the use of algebra tiles in the instruction of binomial expansion and trinomial factorization of algebraic expressions. Accordingly, the study recommended that the Berekum West District Directorate of Ghana Education Service (GES) focus on organizing frequent teacher collectives (professional development sessions) where mathematics teachers can gather to share ideas, solve problems, and discuss how to teach mathematics using manipulatives because teachers' lack of knowledge makes manipulatives classes difficult to run smoothly.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter of the research work introduces the whole research work which discusses the problem under investigation. The chapter contains the background to the study, a statement of the problem, The study's purpose, objectives of the study, research questions, hypotheses, significance of the study, delimitation, limitations of the study, the definition of terms, and organization of the rest of the study.

1.1 Background to the study

Students learn best through direct experience or interaction, which can be achieved by guiding them through specific activities or games to develop key ideas that will underpin their acquisition of specific mathematical skills and knowledge (Akcaoglu & Kale, 2016). The "discovery method," which relies on an experiential approach to teaching mathematics, is the philosophy of contemporary mathematics. Education has the potential to develop the mind's ability to critically analyze situations to solve problems that plague humanity. The ability of students to test hypotheses and reason logically is heavily dependent on their mathematical ability. Mathematics contributes significantly to society by improving students' critical, logical, and abstract thinking.

Mathematics has historically been a subject that many students struggle with and dislike as they progress through the grades due to students' negative attitudes toward the subject (Geist & King, 2008; Jameson, London, Sexton & Wenner, 2022). Most students dislike mathematics due to a number of elements, including instruction, learners' mental, due to a number of elements, notably instruction, learners' mental, affective, and psychomotor traits, subject matter, and the learning environment (Gafoor & Kurukkan, 2015). Algebra is an important topic in school mathematics. The focus of early algebra could be considered to 'compass algebraic reasoning and algebra-related instruction among early learners-from about 6 to 12 years of age' (Carraher & Schliemann, 2008, p. 670).

Early basic school should be used to introduce algebra and allow pupils to practice it to get ready for algebra in elementary and secondary schools. (National Council of Teachers of Mathematics [NCTM], 2000). As a mathematics educator, the solution is to teach toward relational understanding, or 'knowing both what to do and why.' Skemp (1976, p. 20). When learners are given the chance to use manipulatives in their classes, there is an interesting shift in how that subject is taught because it makes lessons more interactive, engaging, and student-driven. Evidence from empirical studies revealed that students grew more independent when given the choice or freedom to use manipulatives provided by their tutor (Moyer & Jones, 2004).

According to Ojose and Sexton (2009), having access to learning resources allowed students to develop their problem-solving techniques, encourage autonomous thought, and build confidence in learning mathematics, all of which led to greater achievement in mathematics. Learners must first be able to visualise their solutions to achieve this goal. To allow students to develop relational understanding, the solutions must give meaning to their experiences. The majority of educators believe that using concrete aids in teaching can help students understand and visualize concepts more clearly. According to research (Fennema, 1972), concrete aids aid in the development of links between the concept's concrete and symbolic representations. This helps the student understand how the concept was developed and gives meaning to the solution as it relates to a concrete experience.

In algebra, it is necessary to think in terms of sets of numbers rather than single numbers. As a result, algebra appears more abstract than arithmetic (Lave, 2021). Because algebra appears less concrete to students, they struggle with school mathematics and face significant challenges in the mathematics learning process (NCTM, 2000). Using variables and notations for variables is difficult (Chappell & Structchens, 2001). According to Edge and Kant (1992, as cited by Thornton, 2004), words represent something that can be touched or experienced, so learning a language is simple. As a result, when you see the word ‘orange’ or ‘television,’ you can envision it. They, however, claimed that it is challenging to study mathematics since it is frequently taught in a way that cannot be understood. If you do not understand the meanings of frameworks, you cannot imagine anything when you see $4x$ or x^2. In this instance, learning mathematics could be compared to learning to read without understanding the meanings of the words, as Edge and Kant discussed. Therefore, it is accurate to say that ‘conceptualizing variables and manipulating them are fundamental characteristics of algebraic learning.’ (Polites, Roberts & Thatcher, 2012). By offering a tangible foundation for learning, manipulative materials could be the means by which students' algebra learning becomes relevant and successful. Different definitions of manipulating materials can be found in the literature.

Explicit and physical representations of abstract mathematical concepts are achieved through manipulating materials, according to Moyer (2001). According to Sanderson (2021), manipulatives are ‘tangible models that combine mathematical ideas, appeal to numerous senses, and can be felt and moved around by learners’ (p. 11). Learners can move from thinking concretely to thinking abstractly by using manipulatives (Fennema, 1972). Making use of manipulatives allows students to transition from tangible to abstract thought. (Fennema, 1972). By giving pupils qualities they can see, hear, and touch, manipulative materials improve learning. This boosts student motivation and reduces the very few fascinating aspects of mathematics that students find to be intriguing. When pupils are introduced to new mathematical concepts, they are especially helpful (Kober, 1991).

Manipulative materials are concrete learning tools that, through their concretization, assist students in understanding abstract concepts (Boggan, Harper, & Whitmire, 2010; Cope, 2015; Ojose & Sexton, 2009; White, 2012), and thereby aiding students in connecting the tools that combine abstract mathematical ideas by giving them real-world experiences (Cates, Cass, Smith, & Jackson, 2003; Holmes, 2013). Students must be able to synthesize their knowledge and apply it to their own ideas in order to completely understand mathematical ideas (Boggan et al., 2010; Kelly, 2006); Additionally, using manipulatives allows pupils to express their mathematical ideas and enhance the degree of their understanding. (Ojose & Sexton, 2009). They also make learning fun by demanding active involvement from both learners and teachers (Boggan et al., 2010; Ojose & Sexton, 2009), which promotes equality of opportunity for all students and, as a result, results in long-term learning (Shin & Kang, 2015).

Kontas described manipulative materials as educational tools that improve teaching and learning and positively influence learners' conceptualization and interpretation processes in his 2016 research. One could wonder what exactly qualifies as a manipulator. According to Johnson, Hashtroudi and Linsay (1993), 'manipulative materials are objects used in instruction that have different dimensions and colours' (p. 10). Manipulative materials can ultimately be any objects that a creative teacher wishes to use to improve a learner's mathematical skills. According to Apondi (2015), manipulative materials have a place not only in basic school but also in higher grades.

By starting with basic objects like beans or buttons and working their way up to more complex tools like graphing calculators or computer software, learners can now discover a large amount of mathematics that was previously unfathomable. Any mathematical instruction should aim to aid learners in conceptual comprehension. Students can understand mathematics as an integration by linking techniques employed in one question to techniques used in an equivalent question. Functioning in today's fast-paced and changing environment requires having strong mathematical skills. We must be flexible in our approach to the usage of manipulatives because these are abilities that learners will employ in their daily lives.

Educators are continuously finding the best ways to employ manipulatives. They would do well to embrace the position that mathematics instruction should concentrate on helping learners understand mathematical ideas. Manipulative materials used by teachers have potential when used in a way that encourages students to reflect. According to Odum (2022), mathematical instruction should begin with experiences that are meaningful to learners and serve as symbols throughout their learning, and manipulative materials are vital tools in efforts to make mathematics more meaningful.

Manipulative materials not only help the learner's cognitive abilities, but they also help the learner's psychomotor skills develop (Cope, 2015) by taking into account the learner's senses of sight, touch, and hearing both inside and outside of the classroom. According to a study conducted in Nigeria (Dauda, Jambo & Umar, 2016), Success of learners is significantly influenced by the teacher's credentials, methods of teaching, and resources used in the classroom.

Teachers should always try to find ways to actively engage their students to not only understand concepts but also to create elements of fun and excitement to pique students' interest. Using manipulative materials has become one method of engaging students in fun learning that encourages student motivation. Manipulative materials have also helped make abstract ideas concrete for students, resulting in conceptual understanding.

However, as with any tool, they should be used with caution and care to achieve the best results. When used incorrectly, they can cause frustration or confusion, ultimately disrupting student motivation and overall learning opportunities. An empirical review of how well-suited manipulatives are for mathematics instruction and learning (Boggan, et al., 2010; Cope, 2015, Kontas, 2016) shows that when used in the mathematics classroom, manipulative materials improve students' understanding and performance. learners' attitudes, self-reported questionnaires, self-engagement, responsiveness, and confidence improve when they learn mathematics in a free environment where they are allowed to explore (Mukuka, Ndihokubwayo, Ukobizaba, Uwamahoro, 2021). As a result, educators need to be aware of the best ways to involve learners in the development of their interests, creativity, application, and discovery. (Ukobizaba, et al., 2021). Another possible method for enhancing instruction that will serve all students at all levels is to provide students the opportunity to learn about and consider the ramifications of their learning practices. (Stigler & Hiebert, 2004).

According to Mills and Mereku (2016), it was necessary to shift the focus of the classroom from the transmission of information to students' understanding of mathematical ideas. The efficacy of manipulative-based instruction strategies in math classes needs to be investigated. According to Moyer and Jones (2004), manipulatives are made to clearly and concretely represent intangible mathematical concepts, which are frequently challenging for learners to comprehend. As a result, they can be used as reliable resources in the classroom to help students understand complex concepts. Studies have also revealed that when manipulatives were used in lessons, 'learners seemed to be concerned, active, and involved,' viewing mathematics as a fun activity (Bolyard, Moyer & Spikell, 2002).

National and international reports show that Ghana and the rest of the world have suffered significantly as a result of students' poor mathematics performance. According to reports from the West African Examination Council (WAEC), mathematics was the subject in which more than 50% of all test-takers failed, with failure rates falling to around 40% from 2000 to 2005 and 40% from 2006 to 2010, respectively. The statistics on school placement once more show poor performance. For instance, only 210,282 candidates, or 62.16 per cent, qualified for admission to second-cycle institutions in 2008 out of the 338,292 candidates who participated in the BECE. A total of 395,649 students took the examination the following year, but only 198,642 (50.21) of them passed and were admitted to second-cycle institutions.

The percentage of BECE candidates performing poorly in Ghana has remained high, with 49.12 per cent in 2010 and 46.93per cent in 2016. (Iddi, 2016).

Anamuah-Mensah and Mereku (2005a) determined from their analysis of the

Trends in International Mathematics and Science Studies (TIMSS) report of 2003 that Ghana's performance in mathematics at grade 8, or Junior High School, form two, was poor.

Additionally, Anamuah-Mensah, Mereku, (2005b) found that Ghanaian students performed very poorly, with a mean score of 276 compared to the global average mean score of 467. Ghana placed 45th out of the 46 nations that took the 2003 TIMSS test. There were similar performances in 2007 and 2011.

The scale scores of 130 and 430 in 2007 were far below the average score of 500 and 800 (Anamuah-Mensah, Mereku, & Ghartey-Ampiah, 2008). The performance of students in Mathematics is generally assessed to be poor and therefore the suggestion to help students know or understand the relevance of mathematics in the country's educational progression to drive change in pupil's attitudes in Basic Education Certificate Examination (Chief Examiner's Report, 2011*)*.

These worrying trends are not different from what already exists in the researcher's area of study. In light of this, the study aims to investigate how algebra tiles manipulative materials affect students' mathematical achievement in the Junior High Schools in the Berekum West District of the Bono Region.

1.2 Statement of the problem

Due to the weak foundation of mathematics, students' motivation for the subject and their academic performance have declined in Ghana (Ampadu, 2012; Anamuah-Mensah, Mereku & Ampiah-Ghartey, 2008). The National Education Assessment (NEA, 2016) report also reveals that 40% of Junior High School students in Ghana do not meet the minimum competency requirement in mathematics, indicating that the country's students' overall performance in mathematics is still below acceptable levels.

This astonishing occurrence perfectly captured the situation of students' mathematics performance in the study area over the past five years as a major concern for all stakeholders and researchers looking for solutions. The statistics of the Basic Education Certificate Examination [BECE] over the last five years of the study area put the performance of the students in mathematics as follows: 2017 (55%), 2018 (60%), 2019 (65%), 2020 (63%), 2021 (68%) (data from Berekum West District Education Directorate, 2021 as cited by Asoma, 2022). The performance over the periods has not exceeded expectations, and it is not clear how the teachers' use of manipulatives in teaching mathematics in Junior High Schools is contributing to the problem.

A Literature review conducted by the researcher on the applications of manipulatives on students' understanding revealed that extensive studies have been conducted on the usage of manipulatives for teaching mathematics. (Erdogan, Menekse & Uludag, 2020; Liggett, 2017; Kontas, 2016; Ojose, 2008). However, few studies have been conducted on the application of manipulatives in the Ghanaian Junior High School context. Similarly, there seems to have been a lack of consensus on the effects of manipulative materials in the previous studies.

Because the various studies cited espousing the relevance of manipulative materials in mathematics were done in different jurisdictions whose jurisdiction is different from Ghana coupled with different respondents used has fortified the conceptualization of the study in Ghanaian context the to fill the gap.

1.3 Purpose of the study

The study aimed to investigate the effect of the use of algebra tiles as manipulative materials on the learners' achievement in algebra, particularly in teaching binomial expansions and trinomial factorizations in some selected public Junior High Schools in Berekum West District in Bono Region.

1.4 Objectives of the Study

The following were the specific objectives of the study:

1. to find out the Junior High School students' views on the usage of algebra tiles manipulative materials in mathematics lessons.
2. to determine the impact of the use of algebra tiles manipulative materials on students' achievement in algebra.
3. to find out the students' perceived challenges in the use of algebra tiles manipulative materials in teaching algebra in some selected Junior High Schools.
4. to explore how students' algebraic thinking differ for those who use algebra tiles manipulative materials and those who do not use them.

1.5 Research Questions

The following research questions served as a guide for the study:

1. What are the Junior High School students' opinions on the usage of algebra tiles manipulatives in mathematics lessons?

2. What is the impact of the use of algebra tiles manipulatives on the students' achievement in algebra?
3. What are the perceived challenges of the use of algebra tiles manipulative materials in teaching algebra in Junior High Schools?
4. How do students' abilities in algebraic thinking differ for those who use algebra tiles manipulative materials and those who do not use them?

1.6 Hypotheses

The research was guided by the following null and alternative hypotheses:

H0: There is no statistically significant difference in the mean performance of students taught using algebra tiles manipulative materials and those taught without the use of algebra tiles manipulative materials.

H1: There is a statistically significant impact between the mean performance of students taught using algebra tiles manipulative materials and those taught without the use of algebra tile manipulative materials.

1.6 Significance of the Study

The research findings will be beneficial because they are intended to help erase any fear and negative attitude in students that have developed toward the misconception that mathematics is difficult at the Junior High School level. The findings are expected to help students become more proficient in understanding and implementing new and simpler manipulative materials-based strategies for learning mathematics.

It will also be beneficial to the institutions where the study was be conducted since it will help improve the performance of the students in mathematics in those schools. Mathematics teachers would learn to focus their efforts on facilitating students' understanding and conceptualization rather than rote procedures, drills, and practice. By making this effort, manipulative materials could be used more easily in mathematics lessons. The attempt to incorporate the use of manipulatives would emphasis the thought process of students as they construct personal mathematics knowledge. Naturally, this will also assist students in making connections between actual events and mathematical symbolism used in their classrooms, enabling them to solve mathematical problems independently of their teachers' instructions.

Also, the findings will serve as the bases for District Education to organize professional development courses and in-service training programs for teachers in teaching mathematics through manipulatives. In this context, the study would provide vital information for Teacher Education Institutions for designing functional mathematics programs.

Again, the findings can guide curriculum developers in planning and designing an enriched mathematics curriculum for Ghanaian Basic schools in manipulatives.

Similarly, the study will enrich the existing literature on the usage of manipulatives for teaching algebra in mathematics both in global and local contexts.

1.7 Delimitation

Delimitations of a study are those features that arise from limitations in the scope of the study (Simon & Goes, 2013). Delimitation defines the boundaries, as do conscious exclusionary and inclusionary decisions made during the study plan's development.

Exclusionary and inclusionary decisions include the selection of objectives and research questions, variables of interest, theoretical perspectives to be used, methodology, and participants.

As a result, there are numerous approaches to studying student achievement in mathematics, but for the purpose and direction of this study, mixed method was used.

The investigation was limited to the impact of algebra tile manipulative materials on students' mathematical achievement.

Lastly, the study was restricted to Ghanaian Junior High School students in the Berekum West District in the Bono Region of Ghana.

1.8 Limitation

The study could not be expanded to include all of the students in the country. Due to time constraints and other study-related logistics, it was limited to some randomly selected public Junior High Schools in Berekum West District. The outcomes of this study will be a reflection of the country's targeted population in the educational sector. As a result, the conclusions drawn were based only on the schools selected and not generalized. However, the students used have characteristics in common with students from other parts of the country.

Another limitation of the study was that students had to draw out the tiles during the post-test (see Appendix A). On the post-test, the treatment group was asked to demonstrate their work by drawing out the tiles on their test sheet. This could have been a problem for the treatment group because it added an extra component to the problem-solving process. The use of an algebra tile-specific test ensures that students are assessed on their ability to comprehend algebraic expressions rather than their ability to organize tiles.

Purposive sampling was used, which limits the study's ability to be generalized to a larger population and may have some underlying bias. In future studies, a larger and more random sample size would be preferable.

1.9 Operational Definitions of Terms

School (Classroom) Environment: It is a place where Students and teachers come together to continue in a secure setting that will support their knowledge and learning.

Mathematics Achievement: It is the level of mathematical accomplishment and mastery attained by the student. That is an indicator of knowledge acquired through formal education, such as test scores and grades.

Manipulatives: They are physical or concrete objects that are used as teaching tools to engage students in the hands-on learning of mathematics.

Algebra Tiles: These are concrete aids (made of wood or cardboard) that are used to teach binomial expansions and trinomial factorisations of algebraic expressions.

Unit Tiles: These are tiny square tiles, which stand in for the number 1.

x - Tiles: These tiles are rectangular in shape and represent the variable 'x'.

x^2 −Tiles: These tiles are square-shaped in nature and represent the variable 'x' squared or x^2

Binomial expansion: This means multiplying two binomial expressions of the form

(x+ b) or (x – b) where 'b' is an integer to find the simplest form.

Trinomial factorization: This means writing an expression of the kind $x^2 + bx + c$, where a, b, and c are integers as a multiplication of two binomial expressions.

1.10 Organization of the Study

The study has been sectionalized into five parts.

The first chapter, which has already been thoroughly discussed, dealt primarily with the study's introduction. This chapter's main sections covered the study's background, problem statement, purpose, objectives, research questions, and hypothesis, as well as its significance, delimitation, limitations, operational definition of terms, and organizational structure.

The second chapter also includes a review of related literature. This is mainly about the books, articles, and materials read to find out what has been written and read about the topic. In Chapter three, which explains the procedures followed to conduct the study, methodology is also covered. This chapter's main focus is on research approach and design, population, sample, sampling methods, research tools, data collection data analysis and ethical consideration.

In Chapter 4, which is broken up into the result section and the discussion section, the analysis and discussion of the findings are covered.

The summary of the key research findings, the research's conclusions, and its recommendations are the main foci of the final chapter, which is also the concluding chapter.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

Reviewing theories, ideas, concepts and relevant empirical literature on manipulative materials is the focus of this chapter. It includes the viewpoints of a selection of researchers and disciplines discussed from a historical point of view as well as current thinking on the topic being researched.

2.1 Theoretical Framework

A theoretical literature review is an analysis of the body of knowledge on a specific subject. Key theories, concepts, and frameworks that have been used to study the subject are found there and examined. A theoretical examination's objectives are to give a thorough overview of the field's current state of knowledge, identify knowledge gaps, and create a theoretical framework for future research.

2.1.1 Constructivism

The study was guided by the constructivist philosophical viewpoint, which holds that learners play an important role in learning. Constructivism is a learning theory that views the learning environment as a "mini-society, a community of students involved in the activity, discourse, explanation, justification, and reflection" (Fosnot, 2013). Constructivist educational theory holds that each student builds their own knowledge, learning takes place in a social setting (the classroom), and some experiences are purposefully created by the teacher.

Constructivist teachers encourage appreciation and consideration of the opinions or points of view expressed by others, as well as mutual respect, allowing for the development of creative thinking and independence. Meaning, according to a constructivist, is the result of individuals (here, teachers) ‘establishing associations, reflecting on their activities, and modelling and constructing explanations’ (Fosnot, 2013, p. 280).

Modern-day theorists and investigators believe that real learning is about interaction, development, and growth rather than isolated student mastery of concepts (Fosnot, 2013).

Cognitive science asserts that learning occurs through the flexible organization and restructuring of knowledge experiences, and that in-depth conceptual learning necessitates structural changes in cognition without which disorder would follow. In other words, one of the central tenets of constructivist thought is that knowledge is actively constructed. This point of view contends that teachers should communicate and listen to their pupils in a way that encourages them to develop into readers and writers who think critically and profoundly (Fosnot, 2013).

Teachers who frequently collaborate with students gain a better understanding of how a specific student acquires knowledge and can thus become more responsive to the student’s needs. Lumadi and Monobe (2017) asserted that constructivist theorists’ preoccupation with subject matter content in most schools has resulted in a state in which emotional or relationship development is adversely inclined in their judgment, a prime focus of a constructivist-based instruction should be the development of a useful system of interpersonal interactions that will control the student’s school experience. They assert that ‘interpersonal relationships are the framework for the child’s construction of the self, of others, and subject-matter knowledge’.

The constructivist approach is supported by Bruner (1977), who claims that for schools to effectively serve as educational institutions, they must also actively contribute to student’s emotional and social growth (p. 9). Bruner developed four essential learning themes, one of which is ‘stimulating the desire to learn, creating interest in the subject being taught, the motive for learning and what he termed ‘intellectual excitement’ (p. 11). Bruner emphasizes the importance of researching the methods used by ‘successful’ teachers to determine effective practices.

Constructivism offers a natural frame and a core perspective for this study because constructivist researchers are interested in how people engage with one another in a given setting.

2.1.1.1 Bruner’s Theory of Representation

Bruner’s Theory of Representation asserts that learning is reinforced when it is designed to follow steps to facilitate understanding. Manipulative materials are frequently used in elementary school but are rarely used in upper-level mathematics classes, although they have benefits.

Bruner’s Theory of Representation has been used in the classroom to teach pre-algebra and algebra (Monobe & Lumadi, 2017). According to this theory, a child experiences three stages of representation when exposed to new information: enactive, iconic, and symbolic representation. The child needs to interact with materials in order to comprehend a notion throughout the enactive period. A youngster visualizes the items at the iconic level without actually manipulating them.

Last but not least, the child strictly manipulates symbols at the symbolic level and does not necessarily move the objects (Monobe & Lumadi, 2017). This theory can be used to explain how children develop and learn, as well as how students learn new material. The mathematical thought is then represented using pictures to stand in for the manipulative in the representational stage, and students should show that they can both visualize and communicate the concept at a pictorial level.

The concept is then expressed in an abstract form using symbols (such as numbers, operation signs, etc.). With the aid of manipulative materials, learners can have a concrete experience and grow in their ability to reason abstractly (Liggett, 2017). With the aid of manipulatives, students can strengthen their mathematical skills and draw connections between related concepts. Constructivist theory serves as the foundation for Bruner's theory. In the course of learning, students actively create new concepts by referencing their existing knowledge. (Holbrook, Rannikmäe & Soobard, 2020). The four features of Bruner’s (1977) theory of instruction are:

Redisposition to study, thus the encounters that inspire a person to appreciate studying generally or a specific subject. Two, framework for knowledge. It is feasible to organize knowledge so that a student can most easily understand it. The third one is modes of representation. The representation could take the form of words, images, or symbols. The last one is effective sequencing. Every learner follows the sequencing in accordance with their individual learning differences.

Constructivist theory has some bearing on schemas, which are cognitive frameworks that assist in organizing the world. As they learn a subject, people build their schemas for it and make new connections between what they have learnt. In mathematics, students must develop ‘well-connected schemas and sense-making skills. To accomplish this, one must first build ideas through a process that allows one to move from a concrete example to an abstract and symbolic idea rather than just using a formula.

In a study conducted by Palatnik and Koichu (2017), students examined an algebraic thinking issue involving the slicing of a pizza. They had to determine how many pieces could be produced by n straight cuts. First, the precise value of n was supplied to them. They practiced this example, in which they created images and tables. They followed procedures that enabled them to identify trends, extrapolate, and develop a formula for this question. They were delighted and eager to share their explanations for why it functions when they later discussed their findings. One of the students clarified that if all you have is a formula and you do not know what it means or can explain it, it is not fascinating (Palatnik & Koichu, 2017).

Steele and Johanning examined students who had ‘well-connected schemas’ and ‘partially constructed schemas’ as part of another study on a schematic-theoretic approach to mathematics problem-solving. Students with well-connected schemas were able to adapt what they did in the previous problem to the subsequent one to aid them when working through a sequence of tasks that, at first glance, do not appear to have much in common. They were eventually able to draw links between what they were doing and the first task, which asked for the number of squares in a border, after being provided a few specific examples. Students were able to use the formula created in the previous problem to solve the following one, and so on for subsequent problems.

They would not have been able to relate the formula to the subsequent problem if they were not aware of how they arrived at it or why it worked. Even if they had a formula, individuals who had partially developed schemas found it challenging to link to the subsequent problems because they did not build those connections from the beginning (Steele & Johanning, 2004). This study hinged on Brunner's theory of representation because the theory provided the needed context for students' learning using manipulative material.

2.1.1.2 Piaget's Theory of Cognitive Development

One of the founding proponents of constructivism was Jean Piaget. According to his theories, people exchange their ideas and experiences to produce new knowledge.

In the 1920s, developmental theorists Piaget and Bruner laid the theoretical groundwork for the practice of using manipulative tools in the classroom. The authors contend that children lack the capacity for abstract thought by nature. Instead, they interact with their surroundings to produce abstract ideas.

Children should thus have practical, hands-on encounters with manipulative materials to help them internalize new concepts (Fennema, 1972). Jean Piaget's constructivist theory is where the concept of manipulative material originates (Ojose, 2008). To help students develop abstract reasoning, concrete manipulative materials can be used to help students master their content (Carbonneau, Marley &Selig, 2013; Piaget, 1962). Allowing learners to manipulate objects gives them a context for something they may have previously only perceived as abstract. Their senses are aroused because of their potential for kinesthetic learning and tactile connections (Castro, 2017). Children should learn through their senses, according to Piaget's theory, and it is crucial for them to experience and manipulate the concepts they are learning as well as to experiment with and apply them. The learning process can be made more engaging for students, which can result in improved performance (Ross & Willson, 2012). The continued drill-and-practice method of teaching children, especially with higher-level mathematics, is no longer regarded as the best practice (Heddens 1986).

Mathematics is something that students can learn by doing on their own, but they cannot learn it by hearing a teacher lecture about it. Using manipulatives to include learners in their learning allows teachers to turn them from passive bystanders into active participants. Through their active participation and the use of manipulative material, students can learn without teacher guidance and instead through discovery (Carbonneau et al., 2013). Additionally, using manipulative material gives learners the chance to relate the mathematics they are learning to their meaning and experiences.

Additionally, it might assist in laying the groundwork required for students to access more abstract lessons (Ojose, 2008). Teachers have discovered that even the simplest manipulative material can help students learn more about beginning algebra (Allsopp 1999).

Dienes (1999), Piaget (1952), Skemp (1976), and Brownell’s (2016) ideas on learning suggest that children who acquire mathematics through encounters with manipulatives may be able to bridge the gap between concrete mathematics and abstract mathematics

(Jones & Tiller, 2017). To put it another way, learners who work with manipulatives are better equipped to understand and apply mathematical ideas to real life (Yildiz, 2016).

The sensorimotor stage, which lasts from birth to age 2, the preoperational stage, which lasts from age 2 to age 7, the concrete operational stage, which lasts from age 7 to age 11, and the formal operational stage are the four stages of a child's cognitive development that Piaget (1973) identified in his Theory of Cognitive Development (age 11 onwards). Children who are going through these stages build schemas first with actual activities, then with symbols. Children can only organize facts at the concrete operational stage if concrete things are offered. According to Piaget (1952), children need to interact with models and materials to comprehend abstract mathematics because they cannot do so through explanations and lectures.

Clements (2000) argues that before engaging in strictly abstract learning experiences, children must first engage in tangible, concrete-abstract, and pictorial-abstract learning experiences. The stages of cognitive development should therefore be taken into consideration when planning learning opportunities. According to cognitive development theory, concrete and symbolic models must be used in the classroom to benefit children with various levels of development (Fennema, 1972).

2.1.1.3 Dewey's Perspective on Constructivism

Progressive education uses mature standards, curriculum, and teaching strategies, according to Dewey (1938), who claimed that 'it is a consequence of unhappiness with traditional education.' He believed that traditional education could not serve young learners in the manner already mentioned. Young children should participate in learning activities that are socially engaging and age-appropriate as part of Dewey's progressive education theory (Dewey, 1938).

Dewey thought that the key to good teaching was social contact and that schools should be seen as social institutions (Flinders & Thornton, 2013). He contends that education should be viewed as a 'process of living rather than a preparation for future existence' (Flinders & Thornton, 2013, p. 355; Gutek, 2014). Dewey differed from philosophers who favoured conventional classroom settings because of this set of ideas. He believed that schools and classrooms should reflect real-life conditions, allowing children to engage in learning activities interchangeably and adaptably in a range of social settings as opposed to traditional classrooms (Dewey, 1938; Gutek, 2014). According to him, it was unethical for teachers to abruptly introduce a lot of academic material to children without taking into account their social context (Flinders & Thornton, 2013). Since this idea differs considerably from what is happening in classrooms with a strong emphasis on adopting the Common Core requirements, it would be a source of friction in the current educational system. Due to the emphasis on improving academic achievement through the adoption of Common Core standards in modern classrooms, Dewey's ideas are less common in schools (Haider-Markel & Theobald, 2009).

Learner-centered classrooms frequently employ the social learning theory and educational philosophies of John Dewey. He thought of the classroom as a place where children might interact socially while studying and working out issues collectively. Because they are valued as distinct persons in these classrooms, students construct their knowledge through personal meaning rather than through knowledge that is imposed by the teacher and activities that are guided by the teacher (Koerner, 2020). Children will be seen in these classes practicing new skills and using real-world problem-solving strategies as they learn. When lesson planning, teachers will incorporate the curriculum with a focus on project-based learning while taking into account the interests of the students. The educational process encompasses a child's overall intellectual, social, emotional, physical, and spiritual development in addition to academic growth (Koerner, 2020).

Even if Dewey's work seems to have fallen out of favour, at least in terms of current educational policy, it is still crucial in many departments of education (Haider-Markel & Theobald, 2009). Dewey's educational theories and contributions have been very helpful to educators over the years (Haider-Markel & Theobald, 2009). His belief that education should be centered on the needs of the students rather than the curriculum has had a significant impact on teachers who share his ideas and beliefs about education and how children learn best (Koerner, 2020).

2.2 Empirical Review

2.2.1 Impacts of Manipulative Use in Algebra and Algebra Tiles

'Mathematical manipulatives are physical objects that are made to clearly and visibly depict abstract mathematical concepts' (Moyer, 2001, p. 176). Manipulative items can be bought already made, made by teachers, or created by students with their teachers' help. Examples of manipulatives include base-ten blocks, Cuisenaire rods, tangrams, geoboards, pattern blocks, algebra tiles, and fraction strips (Furner & Worrell, 2017). They have the ability to 'introduce, practice, or correct' mathematical concepts (Boggan et al., 2010, p. 2). Learning mathematics requires more than just paying attention to the teacher. Through the use of manipulative objects, students move from being passive onlookers to active participants (Carbonneau, et al., 2013). With the use of manipulatives, students may spot patterns on their own and make generalizations (Kaiser, 2020).

Additionally, studies have shown that the use of manipulative materials by students to develop their mathematical understanding results in the most effective learning (Boggan et al., 2010). When students discover the mathematical concepts and relationships in this way, instruction will be learner-centred rather than teacher-centred with the teacher acting as the facilitator (Bishop, 2016). When teaching mathematics, it is important to encourage students to practice what they have learned and to use manipulative materials that support the growth of their cognitive, affective, and psychomotor domains (Okpube & Anugwo, 2016).

Again, manipulative materials encourage students to participate in the educational process and help them comprehend and visualize ideas more clearly (Bruins, 2014). So, instead of starting math lessons with representational models like pictures, diagrams, and figures, teachers should start with concrete manipulative materials. Students learn symbols and operations at an abstract level after the lessons, and eventually, they will not need manipulative materials (Furner & Worrell, 2017). All students, regardless of academic proficiency level, can benefit from using manipulative materials. Even for students who are adept at symbolic procedures, according to Cooper (2012), manipulative materials are helpful because they improve conceptual understanding by offering a fresh take on mathematics.

Furthermore, since kinesthetic learners learn best when they can touch or physically engage with the material they are studying, manipulative materials are useful tools for them in both elementary and secondary education (Çaylan, 2018). There are many advantages for students using manipulative materials. While using manipulative materials in math classes, there are a few issues that need to be addressed. There are various difficulties associated with the usage of manipulatives in the classroom. Instead of completing their assignments, students can use manipulative materials to play games. Again, there is a significant time waste associated with dispersing and gathering manipulative materials. For these reasons, before implementing manipulative materials in the classroom, the teacher should consider the amount of time and be aware of the possibility that students can use manipulative materials as toys (Magruder, 2012).

Second, the proper application of manipulative materials is also important. If manipulative materials are not used appropriately, they will not guarantee meaningful learning. Therefore, for it to be effective, manipulative tools must be used appropriately. (Furner & Worrell, 2017). To be used in this way, the manipulative must link casual and formal school mathematics, be appropriate for children's developmental stages, and match the mathematical proficiency of learners (Smith, 2009). (Boggan et al., 2010). Instead of just seeing the manipulative materials as toys, students need to comprehend the mathematical ideas behind them. Therefore, before beginning to teach the concepts, teachers should give students time to work with manipulative materials (Boggan et al., 2010).

Additionally, when choosing math manipulative materials, educators should take into account the curriculum's goals and objectives (Smith, 2009), and teachers should understand when, why, and how to use manipulative materials in the classroom (Kelly, 2006). They should assist pupils in drawing accurate connections between manipulatives and the ideas they represent (Ball, 1992). Students need to make a connection between concrete and abstract ideas while learning algebra. Manipulative materials can provide this connection (Bruins, 2014).

According to Piaget (1952), students should interact with models and materials because they cannot understand abstract mathematics through explanations and instructions alone. Similarly, to this, Bruner (1960) asserts that students' early exposure to and interactions with concrete objects serve as a foundation for their future learning of abstract concepts. With the help of manipulative objects, students can switch between literal and abstract interpretations of the concepts. (Fennema, 1972). According to research, students will understand the fundamental algebraic concepts of the equal sign, variable, and unknown more readily if manipulative materials are used first, before pictures and figures, then mathematical symbols (Akkaya, 2006, as cited in Çağdaşer, 2008). One of the materials available for the instruction of algebra is algebra tiles. Algebra tiles are a set of manipulatives used to illustrate mathematical processes requiring expressions containing variables and integers. (Larbi & Mavis, 2016). A set of algebra tiles is displayed in *Figure 2.1.*

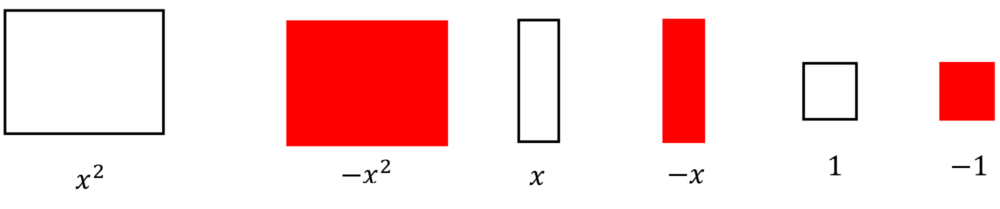


Figure 2.1 Algebra tiles manipulatives

A small square, a rectangle with an oblong shape, and a larger square are typically included in algebra tiles. The purposeful design of the tiles prevents the larger square's side length from being an integral multiple of the smaller square's side length. They could be utilized to conceptually and visually aid students in visualizing and understanding a variety of mathematical processes in algebra (Barba, 2022).

Additionally, algebra tiles can be used to teach the distributive property and factorisation of algebraic expressions, completing the square, and the operations of adding, subtracting, multiplying, and dividing integers (Leitze & Kitt, 2000). Because they allow students to interpret algebraic versions of mathematical problems, algebra tiles are useful manipulatives (Çaylan, 2018). Students can investigate algebraic expressions visually and practically by using algebra tiles. As a result, students can gain first-hand knowledge of the algebraic rules (Okpube & Anugwo, 2016). Also, using algebra tiles improves students' understanding of zero principles and prevents them from erroneously con fusing expressions like '$4x$' and '$4 + x$' (Picciotto & Wah, 1993). While simplifying algebraic expressions, students can construct a variety of pairs of zeros and generate various expressions without altering the values of the original expressions (Salifu, 2022a). Students typically utilize symbols like "x" and "y" to represent variables due to their widespread use, even though other symbols can also be employed. The arbitrary character of the variable idea can be understood by students using algebraic tiles.

However, there are some restrictions with algebra tiles. Algebraic tiles cannot be used to model polynomials beyond the first and second degrees (Salifu, 2022b). In addition, using algebra tiles to model complex examples is challenging. As a result, the symbolic form can be used to extend rules to complex examples. Fractions cannot be represented by algebra tiles. As a result, using algebra tiles to represent division equations is challenging (Magruder, 2012). Additionally, when modelling algebraic expressions with algebra tiles, the colour (red) of the rectangular algebra tile that represents $-x$ is used, though, in reality, an area cannot have a negative value (Tannous, 2020). As a result, this may cause students to have a misunderstanding. In several studies, algebra tiles have been employed. According to Caylan (2018) in the 7th, 8th, and 9th grades, the usage of algebra tiles dramatically increased students' comprehension of the algebraic concept of zero and operations with integers and polynomials. In an experimental study conducted by Larbi and Okyere, (2016), the usage of algebra tiles improved the treatment group's learners' comprehension of the mathematics learning process compared to the control group. According to Caylan (2018), students were more successful in solving formal linear equations with one variable when they used algebra tiles. When students connect geometry to factoring polynomials, it has also been discovered that using algebra tiles can help (Amidu, Nyarko & Salifu, 2020). Similarly, while demonstrating how to solve quadratic equations by filling in a square, students were able to connect geometric and algebraic ideas by using algebra tiles. (Zebracki, 2013). Though test results between students who used algebra tiles while factoring and those who did not differ, Larbi and Okyere (2014) discovered that junior high school students demonstrated meaningful and straightforward learning through the use of algebra tiles.

In a similar vein, students who used algebra tiles better conveyed how to multiply polynomials (Marsh, 2016). According to (Amidu et al., 2020), using algebra tiles helped teachers and students alike understand the concept of multiplying polynomials much better.

In Castro's (2017) pretest-posttest control group design experimental study, algebra tiles helped students with learning disabilities perform better in the treatment group.

When algebra tiles were utilized to increase Senior High school Students' conceptual knowledge of a system of two linear equations, Akpalu, Adaboh, and Boateng (2018) discovered that the experimental group's post-test results showed a statistically significant improvement.

Again, research on the usage of algebra tiles in teaching and learning algebra has generally focused on instruction and solving problems in linear equations in one variable to middle school students with algebra tiles (Magruder, 2012; Putri, Saraswati & Somakim, 2016), a system of two linear equations to senior high school learners by using algebra tiles (Akpalu et al., 2018), factoring algebraic expressions to high school students with algebra tiles (Larbi & Okyere, 2016; Malim, 2018) polynomial multiplication by using algebra tiles (Ünlüer & Kurtuluş, 2021; Wingett, 2019), distributive property to expand algebraic expressions (Larbi & Okyere, 2016); solving quadratic equations by completing a square (Vinogradova, 2007) and algebraic expressions to learners with learning disability through algebra tiles (Castro, 2017). According to these studies, middle school students used algebra tiles to solve linear equations and thought about their steps as they went (Magruder, 2012); high school students conceptually understood a pair of linear equations (Akpalu et al., 2018); they also learned factoring and distributive property meaningfully (Larbi & Okyere, 2016) and they could connect geometric

Concepts to fractions (Malim, 2018).

Additionally, high school students claimed that factoring was easy to learn, that they understood the concepts much better, and that they could explain polynomial multiplication better with the aid of algebra tiles (Goins, 2001). Besides, learners who previously struggled with algebra performed better when using algebra tiles (Ergene & Haser, 2019), and algebra tiles had a positive impact on the post-test results of students with learning disabilities (Castro, 2017).

In a manner similar to this, middle school pupils found algebraic expressions to be straightforward to model (Ünlüer & Kurtuluş, 2021), and algebra tiles assisted students in drawing connections between algebraic and geometric concepts (Vinogradova, 2007).

Even teachers' understanding of polynomial multiplication was aided by the use of algebra tiles (Amidu et al., 2020). Padmore (2017) examined the use of manipulative tools by Junior High School teachers in the Wa Municipality of Ghana, using a sample size of 94 teachers and 10 headmasters.

Regarding the benefits of using manipulation, he drew the following conclusions:

1. Learners' comprehension is increased and their ability to easily build their knowledge of the subject is aided by manipulative materials.

2. It frees up a lot of time for teachers, makes it simple for them to cover more material, inspires students, and aids in meeting their needs.

3. Enable students to connect mathematical symbolism to real-world contexts so they don't avoid math altogether.

4. Enabling students to solve problems, mathematical ideas, and concepts cooperatively.

5. Making mathematics enjoyable and simple for instructors to introduce concepts.

Moreover, his research's findings on the problems with manipulative use include;

1. Absence of suitable teaching manipulatives for teachers.

2. Absence of continual professional training on manipulative use.

3. An inadequate guide on how to use manipulatives for teachers.

4. Expensive preparation and acquisition of manipulatives.

5. Teachers are under too much stress.

6. Impact of large class size on the usage of manipulatives in mathematics tutoring.

To teach mathematics in three upper public primary schools, Salifu (2022) investigated the use of manipulatives. The sample for the study consisted of 200 students and nine (9) math teachers. The primary data collection tool was structured questionnaires. The study's findings indicated that using manipulatives helped teachers better explain mathematical concepts and made mathematics lessons for students very engaging and practical. His studies also reported, using manipulatives with students helped them perform better when solving mathematical problems. His research also revealed two other challenges that relate to instructors' use of manipulatives, including the inadequacy of some severalties designed specifically for mathematics and the challenge of creating or locating the appropriate sort of manipulatives to fit some complex topics. According to

Salifu's study, the use of manipulatives that are overly complex loses instructional time and that are difficult to distribute and control their use in big classes.

According to Amidu et al. (2020), the experimental group and control group showed statistically significant accomplishment differences between the pre- and post-test assessments. Additionally, the study found that the learners had fun using the algebra tiles. Algebra tiles, according to the study, also made it easier for students to comprehend algebraic ideas, making them prepared to use them in both on- and off-campus teaching situations (Aburime, 2017).

Caylan (2018) investigated how using algebra tiles affected sixth-grade students' proficiency with the subject, their ability to think algebraically, and their opinions on the subject. With 40 students from the sixth grade, they used a pretest-posttest control group design. The results showed that the algebra tiles encouraged a deeper understanding of the concepts and encouraged active participation from the sixth-grade students in the lessons. The study also showed that using algebra tiles made concepts easier to learn and remember, which contributed to the participant's enjoyment of the lesson. The sixth-grade students also noted that they can use algebra tiles to study the rules from their understanding and to switch between concrete and symbolic representations of concepts, investigate algebraic equations practically and graphically, and study the rules from their understanding. The participants concluded by saying that using algebra tiles improved their ability to visualize concepts and helped them to understand them conceptually (). Additionally, they claimed that using algebra tiles had made it easier for them to clarify muddled formulations and prevent blunders (Gningue, Menil & Fuchs, 2014).

Five algebra classes from high schools were used in a 1995 study by Sharp. The students were drawn from three suburban high school algebra classes (85% white, 10% African American, and 5% Hispanic) and two rural high school algebra classes (100% white). The students in the treatment group were taught how to factorize, add, subtract, multiply, and divide algebraic expressions using algebra tiles. The same units, but without these materials, were also taught to the control group.

Results showed no real distinction between the groups. However, the outcomes of the daily narrative data revealed that the majority of the learners who were taught algebra using the tiles said that the tiles added mental imagery that helped to learn. When they visualized the tiles, they claimed it was simple to think about algebraic operations.

2.2.2 Students Views on Algebraic Thinking

Algebra as a topic in mathematics appears to be very important because it helps students develop their critical thinking abilities. Algebra allows students to arrive at rational

solutions to real-word problems (Lehtonen).

Higher mathematics test scores, a better grasp of more complex mathematics, and a higher enrolment in high school are all results of succeeding in algebra throughout middle school (Wang & Goldschmidt, 2003). Thus, it is crucial to learn and comprehend algebra during the middle school years. 'The ability to represent quantitative situations in such a way that relationships among variables are clear' (Dewe et al., 2002). Algebraic thinking is similarly described by Kieran (2004) as 'the application of one or more representations that relate quantitative conditions' (p. 4). Instead of focusing on procedures, developing algebraic thinking helps students understand algebra in a meaningful way. Additionally, many students benefit from long-term learning when they begin to think algebraically at a young age (Booker & Windsor, 2010). Algebraic thinking can be facilitated when students have the chance to share their mathematical concepts and hypotheses in a collaborative learning environment that is appreciated and promoted in the classroom (Windsor & Booker, 2010). By changing the necessary teaching strategies rather than the mathematics curriculum, it is possible to maximize algebraic thinking. (Lawrence & Hennessy, 2002). Learners should be able to exercise their computational skills while modeling, exploring, discussing, forecasting, conjecturing, and evaluating their ideas in learning settings that are designed by teachers. (Blanton & Kaput, 2003, p. 75). Again, educators must design algebraic activities using resources that are already available. By requiring students to identify patterns, form hypotheses, and generalizations, and defend mathematical facts and relationships, they can adapt arithmetic exercises and word puzzles with just one solution to support learners' algebraic thinking (Blanton & Kaput, 2003).

Using algebraic symbols is a crucial component of algebraic thinking, according to Kieran (2004, as cited by Girit & Akyüz, 2016). Algebraic thinking does not, however, develop quickly. It first calls for knowledge of tangible objects, then of pictorial, tabular, graphic, and finally symbolic representations. To help elementary and middle school learners develop their algebraic thinking, situations with relationships in context and pictures should be presented to them. Middle school students are especially helped by using concrete models to create substantial relationships in their algebraic thinking (Lawrence & Hennessy, 2002).

Algebra requires the use of sets of numbers rather than just a few. Due to this, algebra appears to students to be more abstract than arithmetic (Kaput, 2017). Lack of understanding of the underlying logic is the main cause of students' struggles in algebra (McGregor,2020).

Students initially tend to calculate when learning algebra, much like when learning math. But before computation, algebra calls for the understanding, creation, and manipulation of algebraic expressions. Without conceptual understanding, students manipulating symbols will simply employ manipulation using mechanical means. (Kirshner & Awtry, 2004).

Early studies have found that certain students struggle with algebra. These exhibit a lack of comprehension of the equal sign (Kieran, 2004), misconceptions related to letters that represent variables (Kieran, 2004), and having disregarded that algebraic expression, such as 2b+5, is a solution to the problem. (Sfard & Linchevski, 1994) acknowledged that it can be difficult to solve one-variable equations in which the variable appears on both sides of the equals sign (Jupri & Drijvers, 2016).

Five categories of initial algebraic problems are addressed by Jupri, and Drijvers (2016). The application of mathematical operations, such as the addition or subtraction of like terms, to algebraic expressions is the first challenge. The second challenge comprehends how different letters can serve as placeholders, generalized numbers and variables in expressions. These four distinct methods of using letters in algebraic expressions were distinguished by Ely and Adams (2012). Unknown is a specific value (or a set of values) that can be identified using the available data.

As an example, x in ‘3+x = 12’ stands for a singular value, x and in ‘x^2- 3x = 6’ stands for two different values.

The term ‘variable’ is defined by Caylan (2018) as ‘the letter viewed as conveying a range of indeterminate values, and a systematic link is recognized to exist between two such ranges of values.’ Another way to say this is that a variable does not represent a single value or a limited collection of values. A group of values are represented by it. For instance, the variables in *‘*$y = \frac{1}{2}x + 6$*’* are x and y. A placeholder is a letter that substitutes for a number in a particular scenario or combination of circumstances. Other names for it include ‘given,’ ‘constant, parameter, and ‘coefficient.’ For instance, p, q, and r are placeholders in the equation ‘$px^2 + qx + r = 0$.’ (Particularly, coefficients). When all possible literal symbol replacements produce a true statement, as with identities, the term ‘generalized number’ is used (Miller, 2015). For example, the alphabets in ‘$p\ (q + r) = pq + pr$’ serve as generalized numbers. Thus, it can be said that among the crucial issues, instructors face should help learners understand the various uses of letters and how to tell them apart (Ely & Adams, 2012).

The third challenge is coming to terms with the absence of a resolution or the problem of the anticipated response (Tall & Thomas, 1991). Algebraic expressions (equations) without an equal sign or anything on the right side may be confusing to certain students, especially those who prefer to answer questions with precise numerical values. The fourth difficulty is recognizing how the equal sign can be used in different contexts. For example, while it indicates calculation in arithmetic, in algebra it indicates equivalence (Bush & Karp, 2013; Kieran, 2004). The fifth and final challenge is mathematization, which entails translating real-world issues into the language of mathematics, as well as the opposite (Jupri & Drijvers, 2016).

Additional study has been done in this area in Turkey such as (Akgün & Özdemir, 2006) that address middle school students' difficulties with variables and show results that are similar to those of the studies conducted abroad. According to research studies, it can be concluded that students generally struggle to comprehend letters' multiple meanings.

Additionally, research studies revealed a relationship between algebraic and arithmetic concepts. For instance, arithmetic misconceptions lead to errors when carrying out algebraic operations with integers (especially negative integers) and when oversimplifying the idea of cancelling (Norton & Irvin, 2007; Stacey & Chick, 2004). Students struggle to transfer their understanding of arithmetic to contexts involving algebra because they do not have a solid foundation in the subject. The majority of algebraic issues involve the concepts of fractions, decimals, negative numbers, comparison, ratios, percentages, or rates; therefore, in order to answer algebraic problems, students need to have a conceptual understanding of these concepts (Norton & Irvin, 2007; Stacey & Chick, 2004).

Regrettably, algebra expression is only treated in Junior High Schools as a set of rules to be applied and a set of procedures to be followed. Additionally, it is emphasized in instruction that it has little connection to mathematics, stands apart from other courses, and has little to do with real-world situations. (Kaput, 2017). As a result, a lot of students do not see algebra as a natural progression from arithmetic, and they have trouble connecting previously taught arithmetic principles with algebraic concepts.

The TIMSS results showed that whereas 58 per cent of eighth learners were able to understand that k + k + k + k is equivalent to 4k, just 47per cent of seventh graders were able to do so (Beaton, 1996). Şengül and Erdoğan (2014) discovered that 6th-grade learners in Turkey struggle to solve declarative, procedural, and conditional algebraic problems. The concept that a variable can represent any number is problematic for learners to comprehend, even though mathematics teachers believe that students can grasp it easily (Herscovics,2018).

Most students consider algebra to be ‘little more than numerous different kinds of guidelines about how to write and redraft strings of letters and numerals, instructions that must be remembered for the subsequent quiz or test,’ according to one survey (Kaput, 2017). Algebraic learning, however, involves more than just memorization of a rule set. It is necessary to comprehend the connotation of symbols, properties, and procedures for algebraic learning to be meaningful (Thomas et al., 2015).

The transition from arithmetic to algebra is difficult for students when there is an excessive reliance on textbooks with a procedural focus and teacher-centred instruction. As a result, they protest that they do not fully comprehend mathematical thoughts and that learning them is not worthwhile (Heyd-Metzuyanim, 2019). Algorithms are learned by heart by students, who do not understand why they are used. They struggle to see the answers. Because of this, students learn to apply rules without considering why they are appropriate (Skemp, 1976, as cited in Herscovics, 2018). Thus, it is crucial to give learners the chance to visualize their answers when educating students in algebra to help them comprehend algebraic ideas and develop justification-based approaches to algebra topics.

Though there are numerous studies on algebra learning and instruction, few studies have focused specifically on how teachers can teach algebra and what elements must be present in effective algebra learning situations (Kaya & Keşan & Izgiol, 2014). Hence, traditional methods are still used to teach algebra in schools (Doerr, 2004).

2.2.3 Manipulative Materials and the Teachers' Role

The methodical application of manipulative materials has the potential to have a substantial impact on the role that a teacher plays in the teaching-learning process. The most important adjustment that needs to be made is teachers' belief that they are the source of all information. The instructor who is engaged in active mathematics learning is no longer primarily concerned with the traditional notion of teaching, which includes lecturing, demonstrating, and other types of explicit exposition (Park & Choi, 2014).

The teacher should instead focus on organizing or encouraging proper interactions between the student(s) and the resources. This is not to claim that all telling behaviour has disappeared; rather, it is to say that it has generally decreased significantly. The previously discussed nature of learning can be directly linked to this newly defined role. It is assumed that appropriate experiences with materials will aid children in conceptual development because children learn best through active experiences (Fitzsimons, 2014).

This does not eliminate the role of the classroom teacher; they will always be needed to set up the right learning environments as well as to discuss, reflect on, and inspire further explorations by posing the right questions or providing the right guidance when it is most appropriate. In some ways, the teacher's role in using manipulative materials in a lab setting is more complex and demanding than the more conventional roles of telling and explaining. Small-group and individual projects will take precedence over class projects. Usually, a concurrent reduction in the emphasis on lecture and demonstration is used to achieve this (Suydam &Higgins, 1977).

A format like this enables the teacher to differentiate students' assignments more practically. The station approach appears to be the best option for this in terms of (a), (b), and (c). The station approach frees the teacher to interact with students individually and in small groups more extensively, addressing questions and concerns as they come up, and it calls for a new source of guidance in terms of the structuring of student activity. This is necessary because the teacher cannot give instructions to every group at once. An important part of this need can be met by task or assignment cards. The tasks assigned to students are clearly defined using these cards. The teacher only needs to choose those that are ideal for both individuals and/or groups (Beers, 2003). The teacher does not have to feel solely responsible for coming up with the assignments that the students are to complete because there are thousands of these individual assignment cards available commercially.

According to the National Council of Teachers of Mathematics (NCTM, 2000), educators must thoughtfully incorporate concrete manipulative materials into regular classroom instruction to help learners meet mathematics standards. The NCTM (2000) recommends using hands-on activities to help students understand and solve problems related to mathematical standards. Students can make abstract concepts physical by using this kind of practical instruction and the experiences it creates (Ojose, 2008).

Considering all students' needs, it is crucial to differentiate instruction using a range of modalities, including manipulative materials. Students can better understand abstract mathematical concepts and have more enjoyable mathematical experiences by using concrete, hands-on activities (Furner, Yahya, & Duffy, 2005). Utilizing manipulative materials can help students retain information and support the curriculum (Allsop, 1999). The use of these tangible objects can encourage students by making mathematics enjoyable and stimulating mathematical thought in them (Herbert, 1985).

Teachers can allow students to be creative and actively participate in their learning when they are having fun and are engaged. Teachers no longer have to rely simply on worksheets for student evaluation and exercises since manipulative tools allow them to include entertaining and engaging learning into their lesson plans (Furner et al., 2005). During assessments, teachers can determine whether or not students can put on their knowledge to practical positions by using manipulative materials. To solve problems and have fun during mathematics class, students can then use a variety of resources (Herbert, 1985; Kamina & Iyer, 2009). The idea of manipulating materials is based on Jean Piaget's constructivist theory (Ojose, 2008). The usage of concrete manipulating tools can promote subject mastery by assisting the growth of abstract reasoning (Carbonneau et al., 2013). When given the opportunity to employ manipulatives, students can give meaning to ideas that they may have previously only understood as abstract. Due to their capacity for kinesthetic learning and tactile connections, their senses are stimulated (Castro, 2017).

The philosophy of Piaget holds that children should learn through their senses and that they must experiment with and manipulate the ideas they are learning. Students may be more actively involved in the learning process, which may result in better performance

(Ross & Willson, 2012). Especially with higher-level mathematics, utilizing the same continuous drill and practice strategy to teach youngsters is no longer seen as best practice (Treffers, 2012). Students can only learn mathematics by actively engaging in their education; they cannot learn mathematics by passively absorbing a teacher's lecture. Teachers can get learners to become active participants in their education rather than merely spectators by involving them in it through the use of manipulative materials. By actively participating and using manipulative materials, students can learn through discovery as opposed to teacher-directed instruction (Carbonneau et al., 2013). Also, learners are better able to connect the mathematical concepts they are learning to their meaning and experiences when they use manipulative materials.

It might also assist in laying the groundwork necessary for learners to access further abstract teachings (Ojose, 2008). The use of even basic manipulative materials by students in beginning algebra has been found to increase learning, according to educators (Allsopp 1999). This is particularly accurate for disabled students who have a history of having difficulty understanding basic algebraic concepts (Lee et al., 2020). Teachers need to change from the status quo and start using various teaching philosophies to represent all students in light of the Common Core State Standards and the growing need to understand abstract mathematics. For students with disabilities to overcome the barriers of a challenging curriculum and achieve their goals, teachers must move toward instruction that does so.

When it comes to algebraic ideas like integer operations and evaluating equations and expressions, algebra tiles are beneficial for students (Chappell & Strutchens, 2001). Teachers must make sure that they are assisting their pupils in connecting the manipulative with the abstract concept it represents (Chappell & Strutchens, 2001). Students can mentally explain the ideas of expressions and integers by labeling each tile on the algebra tiles with its corresponding variable. Students who use algebra tiles can go further into topics they are already familiar with as they start to establish links between the tangible and abstract, leading to a higher comprehension (Salifu, 2022; Ojose, 2008). Using algebra tiles improved learning-disabled learners' accuracy while subtracting integers, according to Prosser and Bismarck (2023). Additionally, Strickland and Maccini (2013) found that enabling the use of algebra tiles during instruction increased students' understanding of algebraic concepts. Students with learning difficulties were able to demonstrate learning and improvement when it came to handling issues involving many linear expressions within an area, according to Strickland and Maccini (2013). Students with disabilities were only included in two of the fifty-five studies that used mathematics and manipulatives and were examined by (Carbonneau et al., 2013) in meta-analysis. Concerning the teaching of algebra to students with disabilities, there is a dearth of available research and literature. Kelly (2006) defined manipulative materials as items from the real world that kids use to interact and strengthen their conceptual understanding of mathematics. Mathematical manipulative materials help students form and understand concepts better. For instance, studies have shown that girls benefit most when both boys and girls are instructed to use a lot of manipulative materials, allowing them to perform almost on par with their male counterparts.

Yang and Chen (2010) conducted a study to determine how a digital pentominoes game affected gender differences and spatial ability, two crucial human factors, on students' performance. The pentominoes game significantly increased the students' spatial abilities, according to the results. The outcomes also showed that the difference in mathematics performance between males and females had been somewhat reduced by the online game.

To find out how the Dienes block approach affected learners' interest in number bases, Chiawa, Ibrahim and Kurumeh, (2010) conducted a study. The study used a quasi-experimental design with non-equivalent control groups. According to the study's outcomes, the experimental group performed better than the control group, which received instruction without the experimental approach, because they had a higher mean score. The results also showed that the mean scores of the participants in the experimental group, both male and female, did not differ significantly. The study's findings indicated that the use of manipulative materials in the tutorial room is successful in raising student achievement levels and delivering an engaging learning environment for all students, regardless of gender. Kurumeh, et al., (2010) conclusion is consistent with that of Bütüner and Baki (2020), who also discovered no differences in student performance based on gender when they were taught using manipulative materials.

Suydam and Higgins at the Mathematics and Science Information Reference Center (ERIC) at Ohio State University compiled a thorough review of the research on the usage of manipulative materials in 1976. Though the report stresses the need for more research, it generally concluded that manipulatives are effective in raising student achievement. When research studies were analyzed, manipulative treatments were favoured in 60% of the studies, while non-manipulative treatments were favoured in 10% of the studies. When compared to non-manipulative approaches, the use of manipulative materials produced student performance that was equal to or better in 90% of the studies reviewed, if studies in which no significant differences were found are interpreted as efforts that did not inhibit achievement. In light of this, Suydam and Higgins concluded that every grade level supports the value of using manipulative materials in teaching a variety of mathematical topics.

More studies support the use of written materials as well as visuals. The idea that manipulatives are most effective at lower grades is not well supported by the available evidence. All elementary school levels seem to benefit from the use of an activity-based approach that incorporates manipulative materials.

The findings in the Suydam and Higgins report's summary are comparable to those from earlier reviews of research on manipulatives and/or mathematical labs (Fennema, 1972; Fitzgerald, 1972; Kieren, 2004; Bauer, Reinsch & Wilkinson, 2013). According to the comment, 'their conclusion and some of ours are comparable in practically all circumstances' (Suydam & Higgins, 1976). Suydam and Higgins conclude by saying, 'We believe that classes utilizing manipulative materials will create more mathematical accomplishment than courses in which manipulative materials are not used, assuming the manipulative tools are used correctly.' The following ideas were made by Suydam and Higgins in 1976:

1. 'Manipulatives should be used frequently in a total mathematics program in a way consistent with the goals of the program'.

2. 'Manipulative should be used in combination with other aids, including pictures, diagrams, textbooks, films, and similar materials'.

3. ‘Manipulatives should be used in ways appropriate to mathematics content, and mathematics content should be adjusted to capitalize on manipulative approaches’.

4. ‘Manipulatives should be used with programs that encourage results to be recorded symbolically’.

2.2.4 The Role of School Environment, Motivation, Teacher Competence, and Student Behaviour

The school environment plays a major role in improving the disciplinary climate and self-efficacy of students contributing to the reduction of the achievement gap in students’ mathematics ability (Cheema & Kitsantas, 2014). The environment was seen a to have major the impact on learning outcome of students’ mathematics achievement. The authors observed that the following conditions moderate mathematics achievement.

1. Classroom Setup: The layout of the classroom can impact how easy it is to use algebra tiles. Teachers need to ensure that there is enough space for students to spread out and work collaboratively with the tiles.

2. Availability of Materials: Algebra tiles are a physical resource that requires an investment in materials. Schools need to ensure that teachers have access to enough tiles to effectively use them in their classrooms.

3. Teacher Training: Teachers need to be trained in how to use algebra tiles effectively in their instruction. Without proper training, teachers may not be able to fully capitalize on the benefits of the tiles.

4. Student Attitudes: Student attitudes towards mathematics and hands-on activities can affect how receptive they are to using algebra tiles. Teachers need to foster a

positive attitude towards math and provide students with opportunities to explore and manipulate the tiles in a supportive environment.

5. Curriculum Alignment: The school's curriculum and teaching objectives should be in line with how algebra tiles are used. Teachers need to ensure that the use of the tiles is integrated into the broader curriculum and supports the learning objectives of the course.

Similar to this, when people are properly motivated, they perform well and are more productive, which supports both national growth and development and citizen welfare (Clements-Croome, 2017). Therefore, it is impossible to overstate the influence of motivation on teachers' use of instructional resources. In the classroom, motivation supports and enhances the teaching and learning process. For instance, a motivated teacher will make every effort to do the task successfully and efficiently when assigned a class to teach. The teaching process does not consider discouragement, even when it fails. Additionally, classroom environments demonstrate how motivation affects teachers. Teachers' behavior is energized and roused for action by motivation. Not only the motivation energizes the behaviour, but it also sustains their interests and behaviour for a longer period of activity (Clements-Croome, 2017). A motivated state supports to increase proficiency and adequacy of behaviour.

A motivated teacher, for instance, approaches his classwork with zeal and excitement and also makes use of teaching aids to facilitate teaching and learning. Such teachers will always be present in the classroom and complete the required tasks. The performance and academic success of the pupils of the teachers could be considered as another effect of motivation on the teachers' usage of instructional resources (algebra tiles). The behavior is focused on a certain objective that the person has established for himself. In this case, the teacher's behavior or activity is not haphazard; rather, it is focused on helping the student attain a specific objective that he has set for himself. An example is when a teacher is determined to achieve efficiency in teaching and learning; such a person selects appropriate behaviour such as the use of instructional materials and other learning materials, to attain his set goals and the motive ends with the achievement of the goals (Bergsten, 2007).

Additionally, manipulatives motivate students to participate in the learning process and help them comprehend and visualize ideas more clearly (Bruins, 2014). To effectively teach mathematics, teachers should start their lessons with concrete manipulatives before moving on to representational models like pictures, diagrams, and figures.

Also, learning and teaching algebra concepts are frequent problems for both teachers and students alike (Garzon & Bautista, 2018). Students struggle to learn algebra and teachers struggle to identify the best way to teach algebra. The authors further argued that the issue of teachers' problems in using algebra tiles can be ameliorated through the following:

1. Training: Teachers need to receive proper training on how to use algebra tiles effectively in their instruction. This includes understanding the different types of

tiles, how to represent expressions and equations using the tiles, and how to use them to teach specific math concepts.

2. Practice: Teachers need to practice using algebra tiles themselves to become proficient with them. This can involve creating sample problems and exploring how different equations and expressions can be represented using the tiles.

3. Planning: Teachers need to plan how they will use algebra tiles in their instruction, including how they will introduce the tiles, what problems they will use them to solve, and how they will assess student learning using the tiles.

4. Flexibility: Teachers need to be flexible in how they use algebra tiles to meet the needs of their students. This may involve adjusting their approach based on student feedback or modifying the types of problems they use the tiles to solve.

5. Assessment: Teachers need to be able to assess student learning using algebra tiles. This includes understanding how to evaluate student work using the tiles, identifying areas where students are struggling, and providing feedback to help students improve their use of the tiles.

Again, when students use algebra tiles manipulatives in the classroom, their behaviour can vary depending on several factors (Chaurasia, 2019). Here are a few factors that can affect student behaviour:

1. Interest: If students are interested in using the algebra tiles, they are likely to be more engaged and motivated to learn. Teachers can help to generate interest by explaining the benefits of using the tiles and providing opportunities for students to explore and manipulate them.

2. Experience: Students who have prior experience using manipulatives may be more comfortable using algebra tiles. However, even students who are new to manipulatives can quickly become adept with the tiles with proper guidance and instruction.

3. Collaboration: Students who work collaboratively with the algebra tiles may exhibit more positive behaviour than those who work independently. Collaborative work allows students to share ideas, support one another, and engage in problem-solving together.

4. Frustration: Students who struggle with math concepts may become frustrated if they are unable to grasp the use of the algebra tiles. Teachers need to be aware of this and provide additional support and guidance to these students.

5. Creativity: Some students may exhibit more creativity when using the algebra tiles than others. Teachers can encourage creativity by providing open-ended problems that allow for multiple solutions and interpretations.

Overall, student behaviour when using algebra tiles can be positive and productive, especially when students are engaged, collaborative, and motivated to learn. Teachers can help to foster these behaviours by providing a supportive and encouraging classroom environment that emphasizes creativity, collaboration, and problem-solving.

2.2.5 Challenges in Using Manipulative Materials

The correct usage of manipulative materials is significant if manipulative materials are not used properly; using manipulative materials does not assure fruitful learning. Consequently, the appropriate use of manipulative materials is essential to ensure its efficiency. (Furner & Worrell, 2017). In order to be used in this way, the manipulative must link informal and formal school mathematics, be appropriate for the children's developmental stage, and match their mathematical proficiency (Smith et al., 2017). (Boggan et al., 2010). Instead of just seeing the manipulative materials as toys, students need to comprehend the mathematical ideas behind them. Therefore, before beginning to teach the concepts, teachers should give learners time to work with manipulative materials (Boggan et al., 2010). Researchers have documented the difficulties in many ways and from various angles while employing algebra tiles to solve linear equations in a single variable. The following difficulties with solving linear equations in one variable using algebra tiles were listed by Smith et al. (2017) and Magruder's (2012). According to Smith et al. (2017), modeling complex situations with algebra tiles is challenging and cannot be done for polynomials with degrees more than the first and second. Therefore, it is recommended that complex instances be given in symbolic form.

Similarly, because algebra tiles cannot represent fractions, it is difficult to divide equations with them (Magruder, 2012). Additionally, when modeling algebraic expressions with algebra tiles, the rectangle algebra tile's color indicates -x, even though the area in real life cannot have a negative value. Salifu (2022) identified a number of difficulties related to teachers' usage of manipulatives (such as algebra tiles), including; the inadequacies of a number of tools designed expressly for mathematics, as well as the challenge of creating or locating the appropriate manipulatives to go with some complex topics. The use of excessively complicated manipulatives wastes instructional time, and big class sizes make it harder to distribute and regulate the usage of particular manipulatives, according to Salifu's (2022) study.

Given the strength of the theoretical justifications for incorporating active learning opportunities into the mathematics classroom, why are manipulative materials not utilized more frequently? When asked this question, classroom teachers initially respond that a lack of funding is the main barrier to using manipulative materials more extensively. The actual causes are without doubt more complex than this.

According to a survey conducted by (Fey, 1979) at Ohio State University, manipulative materials use accounts for only 37% of kindergarten through grade 6 mathematics classes, and 9% of those classes never used them at all (Mckinney & Frazier, 2008). Implementing a program built around texts and workbooks is indeed simpler for the teacher to do than systematic material use. The process of teaching school-age children to take responsibility for their actions maintain self-control is difficult for many teachers to execute. Due to this, it is challenging for teachers to use lengthy intervals of uninterrupted on-task time, especially when that time seems to be more loosely structured than the methodical finishing of textbook pages. The inevitable transitional problems connected with classroom management and control may pose problems for teachers to give attention to approaches that make students enough opportunity to practice to develop the necessary level of self-control.

An accountability issue also stands in the way of widespread departures from the status quo. When achievement is gauged by the number of textbook pages ‘covered,’ using extra-textual activities seems incongruous and unhelpful. Furthermore, when students’ performance on a standardized test that assesses their capacity for symbolic computation makes up the main criterion for success in the program, the widespread use of manipulative materials appears to be almost counterproductive.

This situation is likely to remain unchanged until the general public understands that test scores cannot be used as a reliable indicator of true understanding and that the things, they are measuring may not ultimately be the most important results of the mathematics program. The lay public, classroom teachers, and university-level mathematics educators all have very different ideas about what basic mathematical abilities and knowledge are.

Shin et al., (2023) identified widespread false assumptions and subsequent abuses brought on by an overzealous acceptance of manipulative materials as the long-sought educational panacea. His list of false beliefs included the following:

1. Almost any manipulative aid may be used to teach any given concept.

2. Manipulative aids necessarily simplify the learning of mathematical concepts.

3. Good mathematics teaching always accompanies the use of manipulative materials.

4. The more manipulative aids used for a single concept, the better the concept is learned.

5. A single multipurpose teaching aid should be used to teach all or most mathematical concepts.

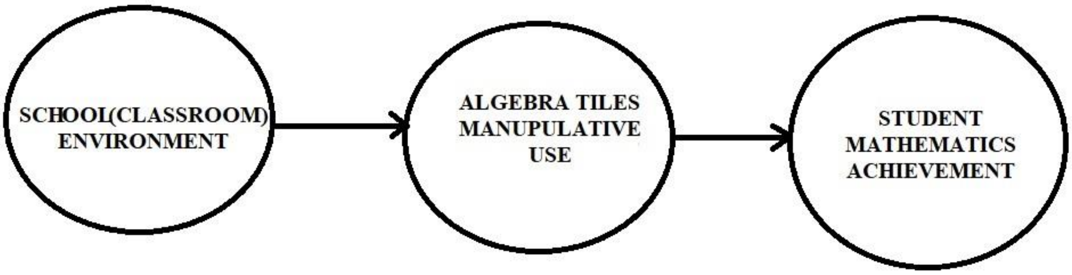
6. Manipulative aids are more useful in the primary grades than in the intermediate and secondary grades, and they are more useful with low-ability students than with high-ability students.

The decision of which manipulatives should be employed in the classroom for mathematics has many facets. In the regular classroom exercises, the questions around the usage of manipulatives are complicated and a little confusing. The things that most affect decisions have less to do with conceptual growth and math learning and more to do with the demands of daily survival. The problems are complicated, and redefining the main objectives of mathematics education will undoubtedly be necessary to resolve them.

2.3 Conceptual Framework

Moderating factors that explain the study's focus construct are addressed in the conceptual framework. The framework was based on the school setting, which constructivist theorists claim gives students a platform to build their knowledge through social interaction and environmental manipulation. The school setting may influence the use of manipulative materials (such as algebra tiles), which in turn may have an impact on students' mathematics achievement. For instance, when manipulative materials are not used frequently in the classroom, learners tend to have negative attitudes toward using them, and the opposite is also true (Larbi & Mavis, 2016). The learning environment which is a place where Students and teachers come together to continue in a secure setting that will support their knowledge and learning at school influences algebra tiles usage which in turn influences the performance of students in mathematics.

The use of algebra tiles in mathematics instruction is also based on the theory of constructivism, which suggests that students actively construct knowledge through their interaction with the environment. Students can examine, explore, and solve problems in a concrete and meaningful way by using manipulatives, which helps them make the connection between concrete manipulative experiences and abstract mathematical concepts. Students can once more explore and draw connections between various algebraic operations and ideas by manipulating the tiles. By giving algebraic concepts a tactile and visual representation, these tiles aid in students' understanding of them. They encourage participation, teamwork, and conceptual comprehension. The interactions of the variables explaining the construct are conceptually shown in *Figure 2.2*.



Researcher's Constructs, 2023.

Figure 2.1 Conceptual Framework Showing the Interaction between School Environment and Algebra Tiles Manipulative Use on Student Mathematics Achievement.

The school environment and manipulative (algebra tiles) manipulation are the predictor variables in the model that are used to determine how students' achievement in mathematics will be measured as the criterion or outcome variable. This suggests that the predictors can explain any variation in the outcome variable (student mathematics achievement) which is the degree of mathematical accomplishment and mastery attained by the student thus an indicator of knowledge acquired through formal education, such as test scores and grades. Mathematical Achievement goes beyond only being able to conduct computations and solve equations. Additionally, it requires the use of critical thinking, logical reasoning, pattern detection, spatial visualisation, and the capacity to apply mathematical ideas in practical contexts or situations. As students go through various mathematical themes and levels of difficulty, their mathematical achievement frequently builds upon their underlying knowledge and skills (Hill et al., 2005).

Characteristics like a school setting with a steady supply of manipulatives (algebra tiles) encourage teachers to use manipulative materials more, which unintentionally sparks teacher motivation and competence, which then influences students' behaviour and affects the students' achievement in mathematics. The school environment mediates manipulative use because some school environments provide incentives for manipulative use while others do not. In a situation where the school lacks the materials and no efforts are being made to procure them, teachers will decline interest, will have less competence in using them, and students will have poor attitudes, consequently affecting their achievement (Larbi & Okyere, 2014).

2.4 The Chapter Summary

The review of the literature found that manipulatives can be utilized to compensate for students' struggles with algebra learning. The reason why students use an algorithm and therefore learn with tiles is that they have trouble understanding the algebraic logic at work. Several studies found that using algebra tiles helped students learn through meaning. Some of the key difficulties raised in the literature to endanger students' algebra learning included their inability to visualize their solutions, difficulties comprehending multiple letter functions, and difficulty moving from arithmetic to algebra as a result of an overreliance on textbooks with a procedural focus and teacher-centred instruction. To overcome these challenges, manipulatives can be used in the algebra learning process, Students can changeover between concrete and emblematic images of the concepts, particularly with the aid of algebra tiles. With the use of algebra tiles, students can explore algebraic expressions visually and practical manner while learning the rules through their experiments. In addition, modelling with algebra tiles improves learners' picturing abilities and advances conceptual understanding. Learners can clear up their misunderstandings of different expressions and prevent mistakes by using algebra tiles. Internationally, the literature is disparate on the effectiveness of manipulatives, particularly algebra tiles, in solidifying students' mathematical abilities. This has made the study so important in the Ghanaian context because it uses variables prevalent in most of our Junior High Schools to assess the construct under study. The study hinged on constructivist theories, which assert that students construct their knowledge when allowed to interact with and manipulate their environment.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter provides an account of the study's methodology**.** It highlights the description of the study area and research paradigm, research approach, research design, population, sample and sampling techniques, intervention, data collection instruments used, data collection procedure, validity and reliability tests of instruments, data analysis as well as ethical consideration.

3.1 Study Area

This study was conducted in the Berekum West District in the Bono Region of Ghana. Berekum West is located in the North-Western corner of Bono with an area of 465.4 km^2. Based on a sample of 20 localities, the Berekum West District had 49,464 residents in 2020. Of these, 23,593 (47.7%) of them were males and 25,871 (52.3%) were females with a population density of106.3/km^2 (Ghana Statistical Service, 2020). For the year 2022, the district is currently expected to have 50,849 residents (Ghana Statistical Service annual population change of 2.8%, 2010 – 2021). The district also has a rural population of 33,422 (67.6%) and an urban population of 16,042(32.4%). The majority of the dialectal groups in the area are of the Akan ethnic group with a population of 43,845 representing 89.6%.

In terms of employment and income, agriculture dominates the economy. Plantain, cocoyam, cassava, vegetables, yam, and maize are the main crops grown, along with cashew, cocoa, citrus, palm kernel, pawpaw, and mango. Other exotic crops include cashew, cocoa, and palm kernel. Wawa, Odum, Sapele, Teak, Onyina, and Mahogany are a some of the larger trees that are found in the area.

There are a total number of 111 schools both privately and publicly owned in the district. Out of this number, 38 are pre-schools (KG), 49 are Primary Schools, 22 are Junior High Schools, and 2 are public Senior High Schools. The district has six (6) circuits, which consist of both rural and urban communities. The urban segment of this District is among the select few privileged areas in the region where access to fundamental social amenities is ensured. Numerous rural residents have been drawn to the availability of these amenities. As a result, these fundamental social amenities are under pressure. The school's classrooms, furniture, and other facilities, such as the curriculum materials, are among the over-utilized social amenities because they cannot accommodate the demands of the growing number of students in the district's populated urban and rural communities. The classrooms now have a large teacher-to-student ratio as a result of this. Besides this, the district was picked for the study because; It provided opportunities to look at how using algebra tiles affected learners' algebraic achievement.

3.2 Philosophical Underpinnings of the Research

How someone views the world through his or her lenses is the philosophical foundation of research. Research methodologies include qualitative, quantitative, and mixed methods, according to Anney (2014). However, each of these strategies has a paradigm that informs its research strategy and data analysis techniques (Creswell & Creswell, 2017). A fundamental set of presumptions that directs a researcher's methodology is known as a paradigm (Creswell & Creswell, 2017). The pragmatist paradigm served as the philosophical foundation for this study. The researcher uses 'what works' to find study answers when using the pragmatic approach (Creswell & Plano-Clark, 2014). The pragmatist asserts that research questions are essential for both subjectivists and objectivists to identify the true nature of the study (Creswell & Plano-Clark, 2014).

The pragmatist mixed-method approach was chosen as a result of the following criteria.; it provided a greater level of validity due to the correlation between quantitative and qualitative data, it allowed the researcher to create a comprehensive analysis that fully incorporated several important elements into the study and also helped in designing and validating the instruments.

to provide answers to different research inquiries and to create the quantitative phase, this provides a clear and thorough overview of the phenomenon under study (Bryman, 2016). Additionally, it aims to gather both quantitative and qualitative information to paint a vivid picture of the issue being studied.

3.3 Research Approach

The study employed a mixed-methods technique to collect information from the respondents to test the hypothesis and to examine the themes that arose from the interviewees.

Mixed-methods approach enables the researcher to gather qualitative and quantitative data on the phenomenon under study to allow for a proper understanding of the study's problem (Dawadi et al., 2021). Convergent parallel approach, exploratory design, embedded design, and explanatory design are the four categories of mixed methods designs that Creswell and Plano-Clark (2014) identified.

Based mostly on the various types of data acquired, the researcher used embedded design, which intends to perform a supporting and auxiliary role in a study. This strategy was chosen because the researcher combined the collection and analysis of both quantitative and qualitative data within a typical quantitative or qualitative research design to collect data from the frames of the respondents' viewpoints and, respectively, quantify their views.

3.4 Research Design

Research design, according to Van Wyk and Taole (2015), 'means all the problems involved in planning and carrying out a study project, from finding the problem to writing and publishing the findings.' Research design, according to Makrygiannakis and Jack (2018), places the researcher in the empirical setting and links the study's data and findings to the research questions. Additionally, research design, according to Van Wyk and Taole (2015), is 'a basic plan for a piece of research and includes four main ideas: the strategy, the conceptual framework, the question of what was studied, and the tools and procedures for collecting and analyzing empirical materials.' The researcher's plan of action dictates how the study's conceptual framework is presented.

The study's design was quasi-experimental. The term 'quasi-experiments' refers to studies in which the allocation of research subjects to two or more groups is not done at random (Dawadi, Ram & Sagun, 2021). Quasi-experimental is an empirical study used to appraise the causal effect of an intervention (treatment) on the target population from which participants are not assigned in groups (Creswell, 2014). The researcher controls some of the variables in this method, which incorporate components of both true experimental and quasi-experimental designs. Similarly, to this, Van Wyk and Taole (2015) proposed that ‘quasi-experiments’ are experiments in which research participants are not randomly assigned to two or more groups. The design enabled the researcher to study the cause and effect of integrating the algebra tiles manipulatives into teaching and learning to improve students’ performance’ in algebra (Creswell, 2014).

To gather data on the topic at the Junior High School level, the two groups took pre-and post-tests. The treatment group received an intervention using algebra tiles manipulative in the period between the pre-test and post-test. In contrast, the control group, was denied access to them, the experimental used them (algebra tiles) both during instruction and during practice. The control group continued to practice and drill abstract concepts using their text and worksheets.

This approach combines elements of both real experiments and designs quasi-experiments but have some factors that are controlled by the researcher. Because the researcher only intended to use intact groups rather than randomly assigning samples to the control and experimental groups, this design was chosen.

Besides, the interview was conducted for the qualitative phase. Interviewing particularly is an effective technique for collecting data about the lived experience of participants in a phenomenological study (Adhabi & Anozie, 2017). This enabled the participants to express their views and concerns freely and explicitly. The detailed nature of face-to-face interviews helped the researcher to obtain from interviewees, such information they were unable to give using a questionnaire.

The study relied on phenomenology as its design for the qualitative phase. Phenomenology is a design of inquiry coming from philosophy and psychology in which the researcher describes the lived experiences of individuals about a phenomenon as described by participants. Phenomenology is particularly suitable for research where the focus is on the uniqueness of a person's experiences, how experiences are made meaningful, and how these meanings manifest themselves within the context of the person both as an individual and in their many roles (Creswell, 2017). This description culminates in the essence of the experiences of several individuals who have all experienced the phenomenon (Creswell, 2017).

This design has strong philosophical underpinnings and typically involves conducting interviews (Creswell, 2017). Hermeneutic phenomenology, as Safadi (2021) notes, reduces a human subject's experiences with a phenomenon to a description of its 'essence,' usually written down. In light of this, a qualitative researcher will classify a phenomenon as a 'object' of human experience and assign it a value. The phenomenology design used in this study made it easier to comprehend the phenomenon of caring relationships in relation to teachers' conceptions and practices in mathematics teaching and learning. One fundamental premise of qualitative research is that people in the social world construct their perspective of fact through contact. In their normal social environments, the qualitative researcher interviews people and studies them (Safadi, 2021).

The guiding question behind phenomenology is this: What is the essence that all people experience about a certain phenomenon? Phenomenology's fundamental goal is to understand the very nature of a research topic by distilling human encounters with phenomena down to a description of its universal essence. What all participants share when they perceive the occurrence is described by phenomenologists (Shosha, 2012). Phenomenological research seeks to understand several individuals' common experiences of the phenomenon being studied, and analysis of the data gleaned is used to develop practice or policy, and to develop a deeper understanding of the features of the phenomenon (Creswell & Tashakkori,2007).

3.5 Population

'The parent group from which a sample is to be drawn is known as the population,' which is the total mass of observations. Population, however, refers to the characteristics of a particular group when discussing research methodology (Pandey & Pandey, 2021). Everyone in the population of all the Junior High School [JHS] students in Berekum West District was the target population. The total population of interest was 3711 JHS students and teachers totaling 610. (Berekum West Education Directorate, 2022). The accessible population consists of six Junior High Schools purposively selected with a population of eight hundred and seventy (870) to constitute the sampling frame.

3.6 Sample and Sampling Techniques

Sampling is the procedure of choosing a subset of the population to represent the whole population (Ssengendo, 2019). The study used a sample size of 274, [i.e n $=\frac{N}{1+N(0.05)^2} = \frac{870}{1+870(0.05)^2}$ = 274] out of 870 students which constitutes 31.49% of the students calculated at 95% confidence level (Yamane, 1973) as cited by Adam (2021). According to Appiah (2019), quantitative studies with a sample size of 5% to 20% of the population is sufficient for generalization.

To choose students for the sample size, the researcher employed a lottery method of simple random sampling. The lottery method of simple random sampling was used to select the respondents. In using the lottery method, a sampling frame made up of an alphabetical list of names of each student was used. The names indicated in the sampling frame were substituted with numbered cards such that each one corresponded to the name of the student. The cards were put in a box, mixed well enough, and randomly removed one by one without replacement. The number of any selected card was registered to match a student's name. This process was continued until the needed number of respondents was attained. The method was repeatedly used in all the selected schools to select 274 students who took part in the study. This was done to guarantee that every learner in every one of the chosen schools had a separate and equal chance of being chosen.

A stratified sampling technique was used to determine the proportions of students from each stratum of JHS levels to form the sample size. Table 3.1 shows the proportion of the stratum summing up to the sample size.

Table 3. 1 Sample Size of the Study

JHS	Students	Sample =($\frac{Stratum\ size}{Accessible\ pop.}\times \boldsymbol{Sample\ Size}$
Ayimom R/C	139	44
Jinijini St. Lucy	150	47
Nsapor D/A	141	44
Benkasa D/A	135	43
Botokrom R/C	147	46
Abi D/A	158	50
Total	**870**	**274**

(Berekum West Education Directorate, 2023).

The samples as seen in Table 3.1 from the six JHS purposively selected to constitute the sample size were made up of three schools (Ayimom, Jinijini and Nsapor) which have shown impressive performance in mathematics over the years as control group and three schools (Benkasa, Botokrom and Abi) whose performance have not been impressive over the years as the experimental group (Berekum West Education Directorate, 2022).

The students from the six Schools selected all speak Ashanti Twi fluently and fairly speak the English language. All the students have an average age of 15 and come from fairly the same socio-economic background, which is evident in their semblance of attitudes and behaviour. All the schools are situated in peri-urban settings with access to some appreciable level of social amenities.

According to Creswell and Plano-Clark, the researcher’s desire to choose participants who had experience with the primary phenomenon or qualities that were of interest to the study was the basis for the use of purposive sampling to choose the schools (2014).

Besides, six students from the schools were purposively selected and interviewed on the subject of study for the qualitative phase. According to Safadi (2021), for qualitative analysis, purposive sampling is most fitting. The researcher further explained that this method of sampling is grounded on the premise that the researcher needs to know and gain knowledge, so a sample from which the most can be learned must be chosen. With two to ten participants, according to Boyd (2001), research saturation can usually be achieved. Creswell (2017) advises ‘long interviews with up to 10 people for phenomenological studies. To achieve informational saturation, the study used six participants.

3.7 Data collection instrument

The items for the students' pre-test and post-test were made to be completely consistent. Their test results were used to determine their scores, which served as the study's foundation. Each question on the test was worth one point and was graded as correct or incorrect. The test consisted of ten questions and was based entirely on the student's textbook, Mathematics for Junior High Schools by Allotey (2012) and The Aki-Ola Series for JHS by Asiedu (2015).

Since students' understanding of algebraic expressions was evaluated, the study's pre-test and post-test periods lasted five weeks. Both the pre-test and the post-test assessed the students' general understanding of how to solve problems involving algebraic expressions. Combining similar terms, the distributive property (binomial expansion), and trinomial factorisation were all problems that were included in the individual tests. The students were interviewed using an interview guide to learn more about their opinions on the use of algebra tiles in mathematics lessons, the challenges they perceive with using manipulative materials to teach algebra in some selected Junior High Schools that were chosen, and how the use of algebra tiles affected how they approached the algebra achievement test.

The study used primary sources as well as other methods for gathering data. Primary sources are authentic sources where the researcher directly gathers data that has not been gathered before. Using a questionnaire about the variables in research objectives 1 and 3, primary data was gathered. The survey was divided into sections. Section A of the questionnaire consisted of the respondents' characteristics. Sections B and C which contained Likert-type items relating to manipulative materials used in teaching algebra in mathematics, the JHS students' opinions on the usage of algebra tiles in mathematics lessons, and the students' perceived challenges of using manipulative materials in the teaching of mathematics in the selected schools. The use of the questionnaire as a data collection tool allowed the researcher to quickly gather the opinions of a large number of respondents (Bryman, 2016).

Additionally, for the qualitative phase, interviews were conducted. In particular, conducting interviews is a useful method for gathering information about participants' actual lived experiences in phenomenological studies (Adhabi & Anozie, 2017). Semi-structured interviews were conducted within the literature's conceptual framework. According to Safadi (2021), semi-structured interviews are those in which 'the questions are more flexibly worded and consist of a mix of more or less structured questions', enabling the researcher to react to and be shaped by the situation as it unfolds and fully explore the emerging worldview of the respondent. These are described as 'one of the most powerful ways in which we try to understand our fellow human beings (Fontana & Frey, 2000). Interviews 'can offer insight into individuals' constructed words and the ways they present these constructions,' according to Wilson (2016). Interviews, both in focus groups and one-on-one, are necessary for examining personal perspectives because they give the study the contextual and anecdotal data that are essential to comprehend these interactions (Seidman, 2013). Participants in one-on-one interviews are typically those who do 'not hesitate to speak, who are articulate, and who can share ideas comfortably,' according to Creswell (2017). The researcher constructed the semi-structured interview guide under the guidance of research question 4.

Before the study, a pilot interview was performed to determine whether changes were appropriate to the planned interview procedure. The students chosen for the preliminary interview were not participants in the research. The semi-structured interview format of the study used an interview guide to help pilot the interview on the right path to remain close to the subject.

An introductory letter was collected from my department to enable the researcher to visit the schools. With the permission of the headmaster, the researcher interacted with the students and teachers. The researcher clearly indicated to the students what the study was going to be about and their right to withdraw if they wanted to. The interview section was done early in the morning, between the hours of 7 a.m. and 8 a.m., before classes began. Participants were reminded before each interview started that their involvement was purely voluntary, that they could cancel the interview at any time, and that they had the right to ask questions. Using the interview protocol, students were interviewed and audio recorded. The interview protocol consisted of questions developed from a range of associated research literature and back study questions.

3.8 Validity and reliability evidence

All items leading to latent variables in the questionnaire were rated from 1 to 5 to assess the respondent's opinions. A Likert-scale-type questionnaire was adopted in the sense that its' psychometric scale was devised to measure and quantify the subjective preferential thinking and feelings of a subject through social interactions (Taherdoost, 2016). The validity of the questionnaires was assessed through the supervisor's judgment and also by allowing experts in the mathematics department at Berekum College of Education to look at the items in the questionnaire for their validity. The Expert was employed to critically look at the content validity of the items to be convinced that the items are good to measure the construct under study. The reliability of the questionnaire items was established using a pre-test of respondents in another jurisdiction outside the study area. The Cronbach's alpha of all Likert-type questionnaires from the pre-test was computed, and items with a Cronbach’s alpha less than 0.7 were removed before commencing the study with the required questionnaires. Creswell (2014) opted that a Cronbach alpha coefficient of 0.70 is considered reliable and a good indicator of internal consistency.

3.9 Reliability Test

The researcher made use of two factors of 10 and 6 items to, respectively, find out the Junior High School students’ opinions about the use of algebra tiles as manipulatives in mathematics instructions and to find out the students’ perceived challenges in the usage of algebra tiles as manipulative materials in teaching algebra in mathematics in some selected Junior High Schools. (See Appendix A.) The reliability of Cronbach’s alpha value for all the test items concerning the constructs sought to be determined is shown in Table 3.2 after presenting the items to 30 respondents to assess the reliability of the questionnaires before embarking on the main study. The Cronbach’s alpha of all the items is greater than 0.7, indicating their internal consistency. According to Abel, Buff and Burr. (2016), a Cronbach alpha coefficient on a scale of above 0.7 is desirable to measure construction in a study. Table 3.2 shows the reliability of the questions and the constructs they seek to measure.

Table 3. 2 Reliability of Questionnaire Items Leading to Their Construct

Construct	No. of Items	Cronbach's alpha (α)
Students' views about the use of algebra tiles	10	0.72
Students perceived challenges with the use of algebra tiles	6	0.73

Source: Field Survey, 2023

3.10 Trustworthiness

To assess the reliability of the qualitative study as an alternative to more conventional quantitatively-oriented standards, Daniel (2019) suggested four criteria: credibility, transferability, dependability, and confirmability, which were adhered to to make the instrument for the qualitative data collected reliable and valid.

3.10.1 Credibility

Credibility means establishing that, from the perspective of the participants, the findings of the study are credible. The study's participants were the only ones qualified to assess the validity of the findings because their participation was the only way to determine the impact of using manipulative algebra tiles on students' algebra achievement. The most important method for building credibility is member checks, which involve displaying research materials to the subjects of the research (Daniel, 2019). Participants are allowed to express their agreement or disapproval of how the researcher has portrayed them. To do member checks, the researcher gave the students access to the transcripts of their interviews so they could make any edits or amendments they felt were necessary to fairly portray their opinions. The participants' agreement with the study's data outcomes was essential to this particular study.

3.10.2 Transferability

This refers to the degree to which it is possible to generalize the findings or transfer results to other settings. It describes the degree to which a study's outcomes can be applied to studies that share similar features (Safadi, 2021). Qualitative research does not intend to generalize its results; it is believed that other investigators stand a great chance of benefiting from the findings of qualitative research that shares related research questions in a similar context. Contrary to generalization, which means the findings of a given study can be generalized across all environments relevant to the context being studied, transferability deals with researchers connecting the findings in related settings and situations outside the scope of the preliminary study (Jensen, 2008).

3.10.3 Dependability

This discusses the stability of data over time. Dependability as a concept concerns itself with how a study's results will be replicable or repeatable with time (Trochim, 2006). In ensuring dependability, the researcher made sure that he followed strictly the standards in conducting research. All the respondents were assured of their anonymity. Again, the respondents were given the chance to pull out if they so wished. The interview guide and the document analysis guide were handed to the expert for validation. Again, the researcher collected enough related studies to strengthen the results and claims of this study. Finally, all authors cited have been duly referenced in the reference section.

3.10.4 Confirmability

The researcher's distinctive perspective could slant or skew the data collection or analysis. Interpretive validity (Elo et al., 2014) was beneficial in providing objective and consistent results, which was important if the study's findings needed to be verified by other researchers. The degree to which the researcher accurately interprets and reports the participants' points of view, thoughts, intentions, and experiences is known as their level of interpretive validity. To avoid biases or predispositions, the researcher actively engaged in critical self-reflection and adopted the reflexivity approach. Writing memos to keep track of personal presumptions allowed the researcher to limit and monitor biases. The techniques for verifying and reverifying the data were documented by the researcher throughout the investigation, creating a 'paper trail' of the researcher's actions.

Additionally, the researcher aggressively sought out and documented any bad occurrences that ran counter to earlier interviews. The supervisor's ongoing criticism was helpful for data analysis as well. To triangulate the data, each student's interview responses were analysed for similarities. To put it another way, the researcher examined whether the student's self-report (as obtained during the interview) was compatible with the nominators' descriptions of each student. The greatest emphasis was placed on the consistency between the students' self-reports about how learners' algebraic thinking differ for those who use algebra tiles and those who do not use algebra tiles.

3.11 Intervention

Algebra tiles, a form of manipulative tool that enhanced the pupils' development of algebraic skills, were presented to the class. According to Stiegelmeier and Moore (2019), manipulatives help pupils 'visualize' mathematical ideas and construct mathematical relationships. Algebra tiles were employed by the treatment group for the introduction, instruction, and application of algebraic expressions. The function of the tiles and their meaning were clearly explained to this group. There were boxes on the post-test for the treatment group's pupils to utilize algebra tiles because they had been using them throughout all of the work building up to it. The control group was denied access to algebra tiles and continued with normal drills and practice with the use of worksheets and teacher-led instruction.

3.12 Data collection procedure

A researcher was introduced to the schools where the data was to be collected for permission by way of an introductory letter from the department. A five-week experimental study was conducted on study participants. In weeks two through four, the intervention took place after the participants took a pre-test in week one. Since teachers were planning to teach the class for five weeks, this was done to avoid disrupting the regular schedule on the timetable.

During those weeks, the experimental group was taught algebraic expressions (binomial expansion and trinomial factorisation) using manipulative materials called algebra tiles, while the control group was not given access to these tools. During the fifth and final week, all participants took the post-test, which was different from the pre-test. Six students each were also drawn from the experimental and control groups and interviewed to address research question4.

3.13 Analysis of Data

Based on the results of the pre-test and post-test, data was analyzed. The group as a whole compared the test results of the learners. The Statistical Package for the Social Sciences (SPSS) version 25.0 was used to enter all the data. The statistical analysis did not include names or other personal data. All data were cleaned up before analysis to make sure there were no outliers (Dimitrov, 2012). Participants who missed a few days of the study were dropped from the data file. Paired samples t-tests was used to determine whether there was a statistically significant difference between the groups. A thematic analysis of the qualitative information from the interview was carried out to address research question 4.

According to the recommendations of Creswell and Creswell (2017), qualitative data were collected and then analyzed in accordance with the phenomenological principles. Phenomenological themes from the interviews were extracted for study, and they were then condensed into meaning units or larger themes. As these themes emerged, they were then transcribed and interpreted by examining the respondents' common experiences with the phenomenon of care.

This was further supported by what Safadi (2021) refers to as the constant comparative form of data analysis used in this study, which compared one part of the data with another to identify parallels and differences. Finding trends in the data that were arranged according to how they related to one another was the study's ultimate goal. To interpret the data gathered, several steps were taken in this analysis. Students' interviews were the first source of data in this report. Every interview was compared and contrasted with the preceding ones because the researcher used the same methodology for all of them. To record the researcher's initial observations and the participants' interpretations of the interview responses, preliminary notes and reflective memos were recorded during the interviews. The researcher also listened to the recorded interviews and made extra reflective memos in addition to the written notes that he had taken. The memos helped with subjects that needed clarity or further consideration and interpretation by translating oral words into written and understandable languages.

A manual transcription of interviews was used by the researcher for the Word document. This was accompanied by a study of electronic transcripts to equate them with digital records in order to guarantee accuracy. The researcher used a mobile phone recorder for interviews to help him connect personally to a computer. Each interview was handled separately, and distinct file names were given to each participant to make it easy to distinguish them. The transcripts of the interviews were sent to the interviewee through WhatsApp after they had been transcribed so that they could be reviewed and checked to see whether they accurately reflected what was meant to be referred to as the member's check.

Daniel (2019) proposed that members search for integrity and trustworthiness. To retain confidentiality, only the transcripts relating to each interviewee were received. Any information that was incorrect or that the participant did not want to provide was changed or omitted. After the necessary transcripts and data from the interviews were made, it was time to create the preliminary codes and highlight and encircle the information that stood out the most in the transcripts and Word documents. Once the themes arose from the relationship between the data, groupings were developed that summarized and gave significance to the transcriptions. Byrne, Cave and Raymer (2022) report that the assignment of codes will improve the study process and the general organisation of data, with each having a unique code that would allow themes to be generated and more easily identified. Byrne et al., (2022) also mention some of the required characteristics that investigators should engage in when coding.

In order to achieve this, it is necessary to be organized, deal with the ambiguity of the various responses and codes, be adaptable and creative, show the utmost respect for every participant and their responses, and maintain an open mind so that all themes and categories might emerge. A colour-coding technique was used to identify the codes, which contained words, sentences, and phrases that held the answers to the study questions.

To identify specific themes in the text, the codes were then put into a codebook. Each group of texts was likewise assigned a code, and the themes were discovered by carefully examining each line of text and looking for any methods, connections, acts, or effects of those activities. Codes and potential patterns have been used to establish groupings by organising and defining the main research-related associations. After concentrating on all of the transcripted interviews, the investigator compiled the data for additional research on the broad themes of the study's questions, including relationship trends, attitudes, behaviours, and obstacles.

3.14 Ethical Considerations

To guarantee the safety of the study's participants, the investigator considered all ethical considerations. These worries include protecting participants from any negative effects and maintaining their privacy and honour. All interviews were conducted with the interviewees' consent. The names of the interviewees were replaced with serial numbers to protect their privacy.

CHAPTER FOUR

ANALYSIS, RESULTS AND DISCUSSION

4.0 Overview

This chapter presents the findings and discusses them in light of the research questions on which the study was focused. The study's particular aims were as follows: to find out the Junior High School learners' opinions on the use of algebra tiles manipulatives in mathematics lessons, to determine the impact of the use of algebra tiles manipulative materials on students' achievement in algebra, to find out the students' perceived challenges in the use of algebra tiles manipulative materials in teaching algebra, and to explore how students' algebraic thinking differ for those who use algebra tiles and those who do not use them. A total of one hundred and thirty-nine (139) questionnaires were distributed to the respondents in the experimental group, and all 139 questionnaires were retrieved from the respondents, representing a response rate of 100%. The samples for the study comprised mainly form two students in some selected Junior High Schools in Berekum West District.

This chapter starts with the socio-demographic characteristics of the respondents, followed by the detailed analysis of the 'research questions of the study'. The findings were presented using frequencies, percentages, mean, standard deviation and coefficient of variation.

4.1 Socio-Demographic Background

This part addresses the socio-demographic background of the respondents, and the variables taken into consideration were gender and age. Table 4.1 displays the analysis of the outcome of the findings.

Table 4.1 Demographic Characteristics of the Respondents

Category	Variable	Frequency	Per cent
Gender	Male	131	47.8
	Female	143	52.2
	Total	**274**	**100**
Age	10 – 14 years	93	33.9
	15 – 19 years	177	64.6
	20 years and above	4	1.5
	Total	**274**	**100**

Source: Field survey, 2023

Table 4.1 shows that 143 respondents (52.2%) were female and 131 respondents (47.8%) were male, indicating that more female students participated in the study than their male counterparts. Regarding age, 93 respondents 33.9% of the total sample were between the ages of 10 and 14; 177 (64.6%) were between the ages of 15 and 19 and 4 (1.5%) were students aged 20 and up. From the findings, students between the ages of 15 and 19 made up the majority of the respondents.

Research Question 1: What are the Junior High School students' opinions on the use of algebra tiles manipulative materials in mathematics lessons?

To find out what the JHS students thought on the usage of algebra tiles as manipulative materials in lessons in mathematics, the respondents were given a 5-point Likert scale to rate various items, 1 being the least and 5 being the highest. The means and standard deviations of respondents' responses were calculated for analysis purposes. With mean ranks, the responses were analyzed. The analysis of the respondents' views is shown in Table 4.2.

Table 4.2 Students' views on the use of Algebra tiles manipulative materials in Mathematics Lessons

Variable	Mean	Stds.
Algebra tiles enabled me to remember the concept easily.	4.626	.605
I enjoy the use of algebra tiles in activities while learning algebraic concepts.	4.453	.818
I can use the algebraic rules from my own experiences with the help of algebra tiles	4.245	.815
With the help of algebra tiles, I understood and comprehended the concept better.	4.237	1.047
Algebra tiles clarified the concept and helped me learn the concept better.	4.223	1.083
Using algebra tiles in group work may provide an idea-rich environment for learning the basic algebra concepts	4.209	.928
Algebra tiles have the potential to help me to internalize algebraic ideas.	4.180	.773
The transition from arithmetic to algebra is difficult for students when there is an excessive reliance on textbooks	4.166	1.094
Algebra tiles enabled me to perform complicated operations easily	4.137	.987
When I saw the operation on the board before the use of algebra tiles, I did not understand what to do.	3.496	1.218
Grand mean and Standard Deviation	**4.197**	**.937**

Source: Field Survey, 2023

The results as depicted in Table 4.2 revealed that algebra tiles enable me to remember concepts easily is the most influential measure of the learners' views on the use of algebra tile manipulatives in mathematics lessons. It attained a mean of 4.626, with a standard deviation of .605 showing the homogeneity of views expressed by the respondents. I enjoy the use of algebra tiles in activities while learning algebraic concepts was the second most influential measure. It gained a mean score of 4.453 and a standard deviation value of .818 indicating common views expressed by the respondents. I can use the algebraic rules from my own experiences with the help of algebra tiles was rated third as a measure. It secured a mean of 4.245 and a standard deviation of .815 which signified the respondents were of high measure to students' views on the use of algebra tiles manipulatives materials in mathematics lessons. Additionally, it highlights the respondents' consistent responses, demonstrating how well the items assess this sub-construct. With the help of algebra tiles, I understood and comprehended the concept better was the next rated response by the respondents. It secured a mean score of 4.237 and a standard deviation value of 1.047, showing a common opinion shared by the respondents. Again, algebra tiles clarified the concept and helped me learn the concept better was another measure, which secured a mean score of 4.223 Standard 1.083. Also, using algebra tiles in group work may provide an idea-rich environment for learning basic algebra concepts was the next rated measure, it recorded a mean of 4.209 and a standard deviation of .928 revealing the homogeneity of views expressed by the respondents. Algebra tiles have the potential to help me to internalize algebraic ideas was rated seventh which obtained a mean of 4.180 and a standard deviation of .773 which also shows the homogeneity of views expressed by the respondents. The transition from arithmetic to algebra is difficult for students when there is an excessive reliance on textbooks was the eighth variable measured obtaining a mean score of 4.166 and a standard deviation of 1.094 indicating a similar opinion shared by the respondents. The ninth rated variable, Algebra tiles enabled me to perform complicated operations easily, obtaining a mean score of 4.137 and a standard deviation of .987 indicating the expression of similar sentiments shared by the respondents. When I saw the operation on the board before the use of algebra tiles, I did not understand what to do was the last rated measure. It recorded a mean score of 3.496, a standard deviation value of 1.218 indicating heterogeneity sentiments shared by the respondents.

In all, students' ratings on views on the usage of algebra tiles manipulative materials in mathematics lessons yielded a grand mean of 4.197, a standard deviation of .937 and a coefficient of variation of 22.32% showing a strong homogeneity of views expressed.

Research Question 2: What is the impact of the use of algebra tiles manipulative materials on the students' achievement in algebra in mathematics?

In an attempt to answer this research question, a quasi-experiment design was used with some schools selected as intact groups to constitute the experimental and control group respectively to examine the impact of algebra tiles on students' mathematical ability and achievement. The two groups, Control and Experimental, underwent pre- and post-testing to compare their baseline knowledge and achievement respectively to determine the efficacy of the intervention. The hypothesis: *There is a statistically significant impact between the mean performance of students taught using algebra tiles manipulative materials and those taught without the use of algebra tile manipulative materials* was formulated.

A paired-samples t-test was conducted to evaluate the impact. The paired samples t-test was used after the data was checked and satisfied the assumption of linearity, normality and absence of outliners (see Appendix B). The results in Tables 4.3 and 4.4 showed a significant increase in scores of the students from before the intervention (M = 2.60, SD = 1.16) to after the intervention (M = 8.32, SD = 1.41), t (138) = 44.61, p = .000 (two tail). With a 95% confidence interval of 5.46 to 5.96, the mean increase in test scores was 5.71. The large effect size is indicated by the eta square test statistic (.94).

Table 4.3 Combined Paired Samples Statistics for the Control and Experimental Groups

Test	Control			Experimental		
	N	Mean	SD	N	Mean	SD
Pre-test	135	2.7926	.9068	139	2.6043	1.1648
Post-test	135	5.0963	1.0429	139	8.3165	1.4092

Table 4.4: Paired Samples Test for the Experimental Group

	Mean	Std. Deviation	Std. Error Mean	95% CI Lower	95% CI Upper	t	df	Sig (2-tailed)	Eta square
Post-test – Pre-test	5.71223	1.50961	.12804	5.45905	5.96541	44.612	138	.000	.94

Calculating the effect size for the paired sampled t-test

The researcher can confidently say that the interventions significantly changed the students' achievement based on the results in Tables 4.3 and 4.4. The results, however, do not reveal anything about how much of an impact the interventions had. To accomplish this, one method is to compute an effect size test statistic. One of the most widely used effect size statistics, eta squared, is calculated and interpreted as:

Eta squared = $\frac{t^2}{t^2+(N-1)}$, where ***t*** refers to t- statistic shown in Table 4.4, ***N***, refers to the number of respondents provided in the Table 4.3

Eta squared = $\frac{(44.612)^2}{(44.612)^2+(139-1)}$ = .94

The Guidelines for interpreting this value are as follows:.01 = small effect,.06 = moderate effect, and.7 = large effect (Cohen 1988, pp. 284–27). The researcher can infer from the calculated eta square’s value of .94 that there was a significant effect because there was a significant difference between the test scores obtained before and after the intervention.

Research Question 3: How do students perceive the challenges of the usage of algebra tiles manipulatives in teaching algebra in Junior High Schools?

The respondents were asked to rate 5-point Likert scale items measuring students’ assessments on the usage of algebra tiles as manipulatives in mathematics lessons, with 1 representing the least rating and 5 representing a strong rating (see Appendix A), to examine students’ perceived challenges of using algebra tiles as manipulative materials in teaching algebra in junior high schools. The mean, standard deviation and coefficient of variation of the respondents’ responses were calculated for analysis purposes. The mean ranks were used to analyse the responses. In Table 4.5, the findings of the analysis are displayed.

Table 4.5: Students' perceived challenges of the use of algebra tiles manipulative materials in teaching algebra in Junior High Schools

Variable	Mean	Stds.
Limited knowledge by teachers hinders smooth operation of algebra tiles class	4.554	.554
Large class sizes make distribution and control of the use of algebra tiles difficult.	4.439	.553
The use of algebra tiles wastes instructional time	4.410	.657
The use of algebra tiles if not properly monitored in class can cause chaos	4.209	.913
Modelling complicated examples with algebra tiles is difficult to understand	4.079	.893
Algebra tiles cannot denote fractions hence it is difficult to carry out the division of equations by using algebra tiles	3.849	1.245
Grand mean and std deviation	**4.257**	**.803**

Source: Field Survey, 2023

The results in Table 4.5 indicated that Limited knowledge by teachers hinders the smooth operation of the algebra tiles class is rated as the most influential measure of the variables. It got a mean score of 4.554 and a standard deviation value, of .554 revealing the homogeneity of views expressed by the respondents. Large class size makes distribution and control of the use of algebra tiles difficult and was rated second as a measure of students perceive challenges of the use of algebra tiles. It recorded a mean of 4.439 and a standard deviation of .553 indicating similar responses expressed by the respondents that the item strongly measures the subconstruct. The third rated measure was the use of algebra tiles waste instructional time which also attained a mean of score of 4.410 and a standard deviation value of .657 which signified respondents' views were homogeneous. The use of algebra tiles if not properly monitored in class can cause chaos was the next measure It recorded a mean of 4.209 standard deviation of .913 which also reveals that respondents shared similar opinions**.** The next rated variable was modeling complicated examples with algebra tiles is difficult to understand with a mean of 4.079 and a standard deviation of .893, showing that the views expressed by respondents were homogeneous. Algebra tiles cannot denote fractions hence it is difficult to carry out the division of equations by using algebra tiles was rated last obtaining a mean of 3.849 and a standard deviation of 1.245 indicating a little heterogeneity of respondents. The grand mean, 4.257, standard deviation, .803 and CV, 18.86%, show that the respondents' shared strong and similar views and a homogeneous opinion that students have perceived challenges in the use of algebra tiles as manipulative materials in teaching algebra in Junior High Schools.

Research Question 4: How do students' abilities in algebraic thinking differ for those who use algebra tiles and those who do not use them?

In answering this research question, Semi-structured, open-ended interviews were conducted using a qualitative research paradigm approach. To give the participants' actual lived experiences while using the algebra tiles, this was done utilizing a phenomenology study approach. A deeper understanding of the problem is provided through phenomenology. The population for this phase consisted of six Junior High School learners, who were purposefully chosen among 274 research participants from the six schools (one from each school), to share their algebraic experiences with the use of manipulatives such algebra tiles and the teaching of algebraic expressions. In qualitative studies, Byrne et al. (2022) claim that a sample size of no more than 20 is sufficient to obtain saturation. Purposive and theory-or concept-based sampling methods were employed in this investigation. Because the study's participants had broad backgrounds in knowledge and had prior experience studying mathematics with algebra tiles, a concept-or theory-based approach was adopted. The Purposive sampling was also adopted because the researcher targeted some students who learn using materials and others who do not. The instrument used was a semi-structured, open-ended interview developed by the researcher to collect data on students' views on algebraic thinking and the manipulatives used. The semi-structured, open-ended interview was designed with the study's aims in mind. The semi-structured interview guide was pretested on students at Jinijini Methodist JHS, which was not part of the study. Prior to the primary study, the pretesting was carried out to identify any potential problems that needed to be fixed. Fortunately, no problems were discovered during the pretesting, thus no change was necessary. In order to address ethical concerns, the students' consent was sought through personal contact, and they were assured that they could stop the study if they felt uncomfortable doing so. Based on the availability and comfort of each student, dates and precise hours for the interviews were decided. Informing participants of the study's goal, the researcher was forthright and honest with them. The researcher set up a one-on-one interview session with the learners at their schools.

Experts in mathematics education were asked to evaluate and provide feedback on the questionnaires in the scope of clarity, ambiguity, relevance, and generality as part of a semi-structured interview process to ensure validity and reliability. The researcher also made sure that high standards for secrecy, credibility, transferability, reliability, conformability, and voluntary involvement were upheld throughout the data collection procedure. The researcher made sure that the data collection method took place in a real-world setting. The respondents were asked the identical questions throughout the semi-structured, open-ended interview, and in a same sequence. Combining participant reviews with verbatim descriptions from participants increased the validity of the data collected. Participants were requested to check the researcher's synthesis of the interviews to ensure that the information was accurately represented. To make sure that the information recorded accurately reflected the participants' true positions or opinions, the participants were required to read the transcript of their conversation. For reliability checks and to serve as verbatim reports of what happened during the interview sessions, audio recordings of the interviews were made.

The researcher immediately typed in writing to manually transcribe the audio recordings after the interview. This was done in order to eliminate any statements that overlapped and make it easier to get verbatim accounts from the participants. According to comparable patterns based on the study's themes, the replies were classified, described, and categorized. (See Appendix B). Additionally, the data had links and interrelationships, which helped the narrative discussions that supported the study's main themes. The verbatim presentation was done in a way that ensured anonymity by recording the date and the identity or code of the informant.

The semi-structured, open-ended interview data were analyzed using the theme analysis approach. All of the semi-structured, open-ended interview replies were first transcribed by the researcher, who then organized them into pertinent themes that matched the study's objectives. By taking into account the patterns and relationships, the comments were categorized and paraphrased. The topics were supported by the verbatim writing of some responses.

The research question posed in this study guided the presentation and discussion of the study's findings. Discussions are used to compare the findings to previous literature.

The research participants were asked to share their views on the algebraic thought associated with Algebra Tiles' manipulative use, and students from the experimental group identified the following thoughts about Algebra Tiles, which were captured as the themes:

Algebra tiles are interesting, good, and sometimes complex and confusing; convenient, exciting, practical, enjoying, amazing, activity-based, understanding concepts but slow and time-consuming; and, finally, very simple, effective, motivating, confident, and improving performance.

On the theme of algebra lessons being interesting, good, and sometimes complex and confusing the results of the semi-structured interviews established that the respondents hold the view that algebra lessons are interesting but complex and confusing. Some of them confirmed that when lessons are taught using algebra tiles, they feel happy, while others express the contrary. For instance, when the respondents were asked how they see algebra lessons, the following responses were given:

'***Interesting since you are em supposed to combine numbers and also alphabets in solving the algebra so I feel a little bit simple and sometimes confusing and comple***x ***but the tiles have reduced it a little'***.

'***Algebra lessons have variables, numbers so solving variables and numbers are good in mathematics because when you get to high levels you will learn all those numbers, the variables, and the numbers so learning algebra is good'.***

'Sometimes there is confusion in the lesson because the question they might give you involve variables and numbers but using algebra tiles helped us to get the answer easily'

Similarly, on the theme of algebra tiles *being* convenient, exciting, practical, enjoying, amazing, and activity-based, aiding in the understanding of concepts but slow and time-consuming, the findings of the semi-structured interviews revealed that the respondents believe that the use of algebra tiles has made them enjoy algebra lessons. Some of them acknowledged that they feel content and happy when algebra tiles are used in the lessons, but that it is slow and time-consuming. When prompted, the respondents sharedwith the researcher their feelings and how the lesson was done when algebra tiles were used in algebra lessons? The following answers were provided:

'Okay when you are using the algebra tiles it is quite convenient. For me using the algebra tiles did not only created fun, but helped me in comprehending the topic. It was so interesting enjoyable, amazing feeling, the lesson was very practical and we had fun but it wastes time'.

'Algebra tiles are amazing sir, because that was my first time of using the algebra tiles to solve such a question so I was so excited it has increased my love for mathematics most especially algebraic expression which most of us find it difficult to understand. I did not know that there was such an excellent manipulative. I was surprised that there is manipulative that can help me do expansion and factorization. It was so interesting, enjoying, amazing feeling, very practical lesson and we enjoyed ourselves'.

'Okay sir I felt excited because for me learning with the algebra tiles was slower and time consuming. I still enjoyed the lessons by working with my colleagues in groups. Group members were actively involved, very happy and the lesson was full of activities and practical'.

Lastly, regarding the theme of algebraic tiles being very simple, effective, motivating, making learners confident, and improving performance, the findings of the semi-structured interviews established that the respondents hold the view that algebra tiles have motivated them and are very good and effective. They went further to say that they feel comfortable and confident when using the tiles, which has greatly improved their performance in algebraic expressions. The respondents said they will tell their headmasters to buy them so that their mathematics teachers can use them in algebra lessons. The following answers were given by the respondents when asked, 'Will you advocate for the inclusion of the use of algebra tiles in the teaching of algebraic expressions, and how useful and effective will it be'?

'Sir I have already started that because it is very effective erh sometimes you see not everybody is sharp so you combining erh may be alphabets and numbers maybe it can just confuse you simple so when you are using the algebra tiles you know that may be this colour resembles may be negative or positive, this is for negative square numbers, this is for positive square numbers so if you use it is very, very simple and effective and you get the answers very, very correct. The use of algebra tiles has improved my understanding and performance in algebraic expressions'.

'I will do that. We shall tell the headmaster to buy the algebra tiles so that the mathematics teacher can use them to teach us. The use of the tiles in algebraic

expression was effective in a way that it makes you feel confident and comfortable when using it because when they give you erm an algebra erm question using the tiles may help you find the right answer because you have the negative and the positive so when you are given a positive question about algebra you may be able to solve it so the use of algebra tiles has motivated me and has improved my performance in algebra'.

'Sir I will advocate paa. It was good but it takes much time to get the answer so when you are using it, it takes much time to get the answer so but using it is very good. I suggest that my headmaster buys all these things (algebra tiles) for our maths teachers because sometimes it is not all the students in our class understand such a question so when the maths master is using it, it go na help all of us to understand it perfectly and improve our performance in expansion and factorisation in the algebraic expression'.

However, students in the control group also identified the following algebraic thoughts from the tiles, that were recorded as the themes:

Very confusing, difficult, and hard; the application of rules and procedures is abstract, boring, confusing, not easy, and not interesting; I have not heard of algebra tiles, but algebra tiles will be effective, useful, and improve performance.

On the theme of algebra lessons being very confusing, difficult, and hard, the results of the semi-structured interviews deduced that the respondents hold the belief that algebra lessons are hard and confusing. Again, when the respondents were asked how they see algebra lessons, the following responses were given by some of them:

'Oh, when they are teaching us, we find it difficult but when the teacher finishes teaching us, we can do some examples and get some correct'

'Algebra lessons in class are err hard and difficult'.

'Very confusion and it's hard'.

Additionally, on the theme of algebra lessons involving more application of rules and procedures and being abstract, boring, confusing, not easy, and not interesting, the results of the semi-structured interviews showed that the respondents hold the belief that algebra lessons are not interesting. Some said that the lesson was abstract and that they felt bored because the rules were many and they were confused at some point.

When the participants were questioned to share with the researcher their feelings and how the lesson was done after the algebra lessons, the following were some of the answers they provided:

'The teacher wrote the topic and examples on the board and discuss with us how to apply the rules and procedures in solving examples. I felt bored because the rules were many and I was confused at a point'.

'Yes, sir we have been taught algebraic expressions. The teacher explained to us the rules and steps used to do expansion and factorization of algebraic expressions and gave us some examples to work them using the but sir it was not easy it was abstract. On the feelings sir Hmm it was not interesting'

'Sir you taught us. You told us to follow some steps, techniques and rules, on the feelings, hmm the lesson was abstract'.

Finally, on the theme of I have not heard of algebra tiles, but algebra tiles will be effective, useful and improve performance, the findings of the semi-structured interviews showed that the respondents indeed have not heard the name algebra tiles before. Respondents hold the view that algebra will be very useful, effective and can improve their performance in algebra. When the respondents were asked whether they have heard of algebra tiles manipulative and advocated for the inclusion of the usage of algebra tiles in the teaching of algebraic expressions and how useful and effective will they will be, the following were some of the responses they provided.

'Sir, I have not heard of algebra tiles before, nobody has ever mentioned it to me so we sometimes find it difficult to understand algebra lessons. Yes, sir, I will say so because when they start teaching us, we find it more difficult, we will be asking our friends what is this. what is this? Sir, I will say that when they use these algebra tiles to teach us it will be very effective and we will be able to understand it well and fast

'I have not sir', I have not heard the name before. Sir, I will do the advocate, I have not used it before but it can help me and the individuals in the class to understand the lesson fast. Sir the way you taught us, if you had used the tiles, I would have performed better than I did I trust that the material will be very effective and more useful in maths lessons.'

'No sir I have not. I know that when algebra tiles are used it is going to increase our vim in learning maths, it will help us to improve our lesson and performance in mathematics, some of us don't like maths may be because of how they teach us so I think that the use of algebra tiles will be effective, useful and will make the lesson practical for us to like maths and this may help us get better scores in maths class test.

When I was in JHS1 in my previous School our maths teacher used to teach us using teaching and learning materials but ever since I came to this school, the interest that I have in mathematics has gone down because our maths teacher does not use any teaching learning materials in teaching us'.

Based on these excerpts, the researcher can conclude that using algebra tiles was very useful and effective because it made lessons very practical, made students very active, improved students' performance, particularly that of the experimental group, and improved learners' understanding of algebraic concepts.

4.2 Discussion of Findings

The study assessed the impact of the usage of algebra tiles manipulatives on students' mathematics achievement in algebra in some selected public Junior High Schools in the Berekum West District. The specific objectives of the study were to ; find out the Junior High School students' views on the use of manipulative algebra tiles in mathematics lessons, determine the impact of the use of manipulative materials on students' achievement in algebra, find out the students' perceived challenges in the usage of algebra tiles manipulatives in teaching algebra in mathematics in some selected Junior High Schools and explore how students' algebraic thinking differ for those who use algebra tiles and those who do not use them. It is believed that the use of algebra tiles as manipulatives may influence the students' performance and achievement in algebra. The findings of the quantitative information have been presented, followed by the findings of the qualitative data.

The purpose of the first research question was to examine what Junior High School students thought about the use of manipulative algebra tiles in mathematics lessons. To get the students’ opinions on the usage of algebra tiles, the construct was measured using 10 variables. The survey’s results revealed that JHS students in a few particular public schools had more consistent opinions about how using algebra tiles helped them learned the subject. Their views measured on the construct obtained a grand mean of 4.197, standard a deviation of .937 and a coefficient of variation of 22.32% showing homogeneity of opinions expressed by the interviewees. The results of the study showed that the usage of algebra tiles helped in the student’s comprehension and understanding of concepts as reflected in the views expressed by the respondents.

This finding corroborates the theoretical standpoint of Bruner’s (1960) and Piaget’ (1926) Theories of Representation and Cognitive Development respectively which posited that the use of manipulatives moderate positively on the achievement outcome of learners (Gningue, Menil, & Fuchs, 2014).

Besides, the present study also lends support to the research done by (Larbi & Okyere, 2016; Malim, 2018) on the use of manipulatives in teaching algebra which yielded results aligning with this study that manipulatives reinforce students’ achievement in mathematics.

Additionally, the use of manipulative materials and how it affects students’ performance in algebra was the focus of the second research objective. The sample was analysed using paired samples t-test to compare the means and the standard deviations of both pre-test and post-test results for the students before the intervention and after the intervention for both control and the experimental groups. An examination of the group means in the pre-test indicates that the control group (M = 2.793, SD = .907) was higher than the experimental group ($M = 2.604$, $SD = 1.165$). This might be because of the student's educational background and algebraic prior knowledge.

The mean score of the students in the pre-test, however, was 2.604, whereas the post-test mean score was 8.317, an increase of 5.713. This shows that on the post-test, every student in the experimental group scored higher. This increase in scores may be attributable to the use of the algebra tiles in the teaching of factorisation and expansion of algebraic expressions. The statistical significance of the mean score difference between the experimental group's post-test and pre-test ($M = 5.713$, $SD = 1.509$) was also investigated using the paired sample t-test. This was carried out to assess how algebra tiles affected students' algebraic achievement. From the results in Table 4.4, the student's performance improved statistically from the pre-test ($M = 2.604$, $SD = 1.165$) to the post-test ($M = 8.317$, $SD = 1.409$), $t(138) = 44.612$, $p < 0.05$. The large effect size is indicated by the eta-squared statistics (.94).

This demonstrates that the usage of algebra tiles accounts for 94% of the variance in the scores, meaning that the experimental group's pre-test and post-test were clarified by the instructional strategy (the use of algebra tiles) used to teach the subject. The findings also show that the intervention had a significant impact on the students, indicating that they significantly improved their comprehension and mastery of the algebraic concept. Thus, using Algebra tiles as a teaching tool or aid improved the students' algebraic achievement.

The pre-test and post-test results for the experimental group showed a significant difference with a large effect size, indicating that the algebra tiles significantly improved the student's performance.

This result is also consistent with studies conducted by (Amidu et al., 2020; Akpalu, et al, 2018; Aburime, 2007), which found that using algebra tiles improved students' performance. Ergene and Haser (2021) as cited by Salifu (2022a) study nonetheless revealed a contrary finding that there was no statistically significant difference between the groups in terms of procedural algebra achievement and attitudes to mathematics.

Also, the third research objective was to determine how students felt about the challenges associated with using manipulative algebra tiles in some Junior High Schools in Berekum West District. The perceived challenges with using algebra tiles were assessed once more using a questionnaire with six variables. The findings in Table 4.5, with a grand mean of 4.257, a standard deviation of .803, and a coefficient of variation of 18.86% demonstrate that students expressed strong and similar opinions about the perceived challenges of the usage of algebra tiles manipulatives in teaching algebra in Junior High Schools. The survey results showed that there were challenges, notably limited knowledge by teachers hinder the smooth operation of the use of algebra tiles in teaching algebra in class. The findings of the study are not different from those discovered by (Magruder, 2012; Salifu, 2022b; Furner & Worrell, 2017; Smith et al., 2017) that manipulatives use in the classroom is beset with some teething challenges which include a waste of instructional time, making instructional activities complex and rendering teaching ineffective when class size is large. Further along, the authors argued that modelling complicated examples with algebra tiles is difficult and hinders its effective use.

Lastly, the fourth objective sought to explore how students' algebraic thinking differ for those who use algebra tiles and those who do not use them.

From the interview conducted, the researcher deduced that students from the experimental group stated that they understood the lesson very well, had fun, and adored the usage of the algebra tiles in algebra lessons. Similarly, the respondents indicated that algebra tiles have greatly improved their scores (marks) in algebra lessons, have also helped them understand the concepts of algebra, and that they can now solve some complicated examples in expansion and factorisation using algebra tiles. Although it was time-consuming, the respondents claimed that using algebra tiles had boosted their spirits, confidence, and interest and dispelled any misconceptions they had about mathematics. The students have since decided to pursue higher levels of study in the subject.

However, students who were randomly chosen from the control group claimed that they were unaware of any manipulatives known as algebra tiles. The respondents expressed hope that after learning how to use and apply algebra tiles, it will be easier for them to comprehend algebraic concepts, which will improve their performance (scores) in algebra.

The interview results supported the usage of algebra tiles in teaching binomial expansion and trinomial factorisation. The learners who used algebra tiles appeared to have multiple embodiments of the same concept, providing a better understanding. The views shared by students align with the findings of Cooper (2012), who indicated that 'manipulative materials are helpful because they improve conceptual understanding by offering a fresh take on mathematics. Salifu (2022a) also asserted that manipulatives waste instructional time and that huge class sizes make supply and control of the use of some manipulatives difficult. The results of this study are also consistent with the qualitative study conducted by Yıldız (2016). According to the study's findings, the majority of middle school students expressed a desire to learn mathematics by using manipulatives, and they claimed that doing so would allow them to have fun while learning. Students also asserted that using manipulatives helped them develop positive attitudes to mathematics and improved their understanding of the subject. The results of this study also support Dewey's Social Learning Theory, which contends that learning occurs in a social environment and that knowledge is independently generated by the learner. [the classroom] (Flinders & Thornton, 2013).

4.3 Chapter Summary

The findings from the analysis were deliberated in this chapter, along with the results. The discussion of the response rate was followed by a presentation of the respondents' demographic background data, an examination of the study questions, and the results are finally discussed.

CHAPTER FIVE

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

5.0 Overview

The summary of the results, the inferences made, and the recommendations made in light of this study's findings are presented in this chapter. It also makes recommendations for additional study.

5.1 Summary of the Study

The current study aimed to determine how using algebra tiles manipulative materials affected students' algebraic achievement, particularly when teaching binomial expansions and trinomial factorisations in a few designated public Junior High Schools in the Berekum West District of the Bono Region. The following research objectives gave focus to the study: to determine the Junior High School students' opinions on the usage of algebra tiles manipulative in mathematics instructions; to determine the impact of the usage of algebra tiles manipulative materials on students' achievement in algebra; to find out the students' perceived challenges in the use of algebra tiles manipulative materials in teaching algebra and to explore how students' algebraic thinking differ for those who use algebra tiles and those who do not use them.

The study's sample consisted of 274 JHS 2 students from a few chosen public schools in the Berekum West District of the Bono Region. Semi-structured interviews; structured questionnaires, and tests were used as data collection tools for the qualitative and quantitative respectively. The research approach was mixed methods and quasi-experimental as its design. Descriptive statistics and the paired samples t-test were used to analyse the quantitative data. The transcripts of the interview data were coded and synthesized to thread the themes.

5.2 Major findings

The study revealed intriguing findings worthy of consideration which include the following:

One, the outcome of the study demonstrated that algebra tiles usage helps in the learners' comprehension and understanding of concepts as reflected in the views expressed by the survey respondents.

Two, the outcome of the research revealed that manipulatives use significantly improves learners' mathematics achievement in algebraic concepts. This was evidenced by the high eta square statistics (.94) proving the efficacy of the intervention.

Three, the survey results also show that there were challenges notably, limited knowledge by teachers hinders the smooth operation of the usage of algebra tiles in teaching algebra in class.

Four, the interview results supported the usage of algebra tiles in teaching binomial expansion and trinomial factorization. The learners who used algebra tiles appeared to have multiple embodiments of the same concept, providing a better understanding.

5.3 Conclusion

The research's findings showed that learners had positive thoughts about the use of manipulative as their rating of the construct on the views was high. This suggests that algebra tiles manipulatives use was popular among the students in the study areas.

Similarly, algebra tiles manipulatives have a positive impact on the students' achievement as the results of the post-test revealed marked improvement in student achievement. The students who used the algebra tiles consequently had a better understanding of what expanding binomials are. The use of algebra tiles in the trinomial factorisation produced a greater understanding of the concept, as shown by both the statistical data and the student interviews, which were overwhelmingly supportive of this claim. Additionally, the experiment's post-test results showed a significant improvement with a large effect size.

Again, it also came to light that students were fraught with some teething challenges in the use of the algebra tiles manipulative. The study's findings revealed that teachers' limited knowledge was the cardinal challenge of the perceived challenges of algebra tiles use.

From the results of the interviews, it was evident that there was a distinct, observable difference between the mental images that the students who used algebraic tiles had of the concept and those who did not. Furthermore, this imagery had a connection to prior mathematical understanding specifically, the area of a rectangle. Students who were taught using algebra tiles had access to manipulating both algebra tiles and diagrams in addition to symbols, as opposed to students who were taught without them. Again, the interview revealed that the student's opinions on the use of algebra tiles were overwhelmingly positive.

5.4 Recommendation

Based on the results of the study, the following recommendations are being proposed. One, students are excited, motivated, happy, confident and have fun when they use algebra tiles in algebra lessons so it is recommended that Education Directorate in Berekum West District should supply algebra tiles to all Schools in the District.

Second, because of the positive impact that algebra tiles have on students' achievement, it is recommended that teachers in Ghana Education Service in Berekum West District should be motivated to use teaching learning resources such as algebra tiles in their lessons so that the performance of their students will be improved.

Additionally, the District Directorate of Education in Berekum West should concentrate on holding regular teacher collectives and Professional Development Sessions where mathematics teachers can come together to exchange ideas, work out problems, and talk about how to teach mathematics using manipulatives because limited knowledge by teachers hinders the smooth operation of manipulatives (algebra tiles) class.

Finally, since teaching mathematics through the use of algebra tiles has been identified as time-consuming and as an approach requiring teacher resourcefulness, the District Directorate of Education in Berekum West should again offer incentives and general improvements to the condition of service of teachers to influence them to do their best.

5.5 Suggestions for Further Studies

This research centred on using algebra tiles to teach students about trinomial factorization and binomial expansions.

Research could examine the impact of algebra tiles on students' comprehension of a range of other ideas, including grouping related terms, expanding and factoring straightforward algebraic expressions, comprehending the meaning of 'variable', resolving linear equations, and resolving quadratic equations. These studies might offer more proof in

favour of using algebraic tiles.

The effects of algebra tiles on concept retention could be the subject of further study. A larger sample of interviews in a study on concept retention might offer more information about how students think, how their mental images form, and how their mental images affect concept retention. Finally, further research can be conducted on teachers' opinions on the algebra tiles usage in algebra instructions since the current study is limited to only students.

REFERENCES

Abel, J. P., Buff, C. L., & Burr, S. A. (2016). Social media and the fear of missing out: Scale development and assessment. *Journal of Business & Economics Research (JBER)*, *14*(1), 33-44.

Aburime, F. (2007). 'How Manipulatives Affect the Mathematics Achievement of Students in Nigerian Schools.' *Educational Research Quarterly* 31.1:3-15.

Adam, A. M. (2021). A study on sample size determination in survey research. *New Ideas Concerning Science and Technology Vol. 4*, 125-134.

Adhabi, E., & Anozie, C. B. (2017). Literature review for the type of interview in qualitative research. *International Journal of Education*, *9*(3), 86-97.

Akcaoglu, M., & Kale, U. (2016). Teaching to teach (with) game design: Game design and learning workshops for preservice teachers. *Contemporary Issues in Technology and Teacher Education*, *16*(1), 60-81.

Akgün, L., & Özdemir, M. E. (2006). Students' understanding of the variable as the general number and unknown: A case study. *The teaching of mathematics*, (16), 45-51.

Akpalu, R., Adaboh, S., & Boateng, S. S. (2018). Towards improving senior high school students' conceptual understanding of the system of two linear equations. *International Journal of Educational Research Review*, *3*(1), 28-40.

Allotey, G. A. (2012). Mathematics for Ghanaian Junior High Schools

Allsopp, D. H. (1999). Using modelling, manipulatives, and mnemonics with eighth-grade math students. *Teaching Exceptional Children*, *32*(2), 74.

Ampadu, E. (2012). Students' Perceptions of their Teachers' Teaching of Mathematics: The Case of Ghana. *International online Journal of Educational sciences*, *4*(2).

Amidu, B., Salifu, A. S., & Nyarko, S. (2020). The effect of Algebra tiles manipulative on preservice teacher's Mathematics knowledge in teaching basic Algebra. *International Journal of Mathematics and Statistics Studies*, *8*(2), 26-39.

Anamuah-Mensah, J., & Mereku, D. K. (2005). Ghanaian JSS 2 students' abysmal mathematics achievement in TIMSS 2003: A consequence of the basic school mathematics. *Mathematics connection*, *5*(1), 1-13.

Anamuah-Mensah, J., Mereku, D. K., & Ghartey-Ampiah, J. (2008). TIMSS Ghana Report 2007: Findings from IEA's Trends in International Mathematics and Science Study at the Eighth Grade.

Anney, V. N. (2014). Ensuring the quality of the findings of qualitative research: Looking at trustworthiness criteria.

Apondi, J. A. (2015). *Impact of instructional materials on academic achievement in mathematics in public primary schools in Siaya County, Kenya* (Doctoral dissertation, University of Nairobi).

Appiah, J. (2019). *Financial literacy and saving behaviour among Teaching and Non-Teaching Staff of Swedru School of Business* (Doctoral dissertation, University of Cape Coast).

Asiedu, P. (2015). Mathematics for Junior High Schools in Ghana. *Aki-Ola Series Gibrime Publishing, Accra.*

Asoma, C., Ali, C. A., Adzifome, N. S., & Eric, A. K. (2022). Mathematics Teachers' Problem-Solving Knowledge, Practices and Engagement among Public Junior High Schools in Berekum West, Ghana. *East African Journal of Education and Social Sciences (EAJESS)*, *3*(1), 29-37.

Ball, D. L. (1992). Magical Hopes: Manipulatives and the Reform of math education. *American Educator: the professional journal of the American Federation of Teachers*, *16*(2).

Barba, K. M. (2022). Mathematical Identity and the Role of the Educator. *Journal of Mathematics Education at Teachers College*, *13*(1), 7-13.

Beaton, A. E. (1996). *Mathematics Achievement in the Middle School Years. IEA's Third International Mathematics and Science Study (TIMSS)*. Boston College, Center for the Study of Testing, Evaluation, and Educational Policy, Campion Hall 323, Chestnut Hill, MA 02167.

Beers, K. (2003). When kids can't read*: What teachers can do* (Vol. 20, No. 3). Portsmouth, NH: Heinemann.

Bergsten, C. (2007). Investigating Quality of Undergraduate Mathematics Lectures. *Mathematics Education Research Journal*, *19*(3), 48-72.

Bishop, T. J. (2016). *Subsequent course pass rates in modularized developmental mathematics courses in select community colleges in North Carolina: The differences between teacher-centred, student-centred, and computer-centred delivery* (Doctoral dissertation).

Blanton, M. L., & Kaput, J. J. (2003). Developing elementary teachers' algebra eyes and ears. *Teaching children mathematics*, *10*(2), 70-77.

Boggan, M., Harper, S., & Whitmire, A. (2010). Using Manipulatives to Teach Elementary Mathematics. *Journal of Instructional Pedagogies, 3*.

Booker, G., & Windsor, W. (2010). Developing algebraic thinking: Using problem-solving to build from numbers and geometry in primary school to the ideas that underpin algebra in high school and beyond. *Procedia-Social and Behavioral Sciences*, *8*, 411-419.

Boyd, J. P. (2001). *Chebyshev and Fourier spectral methods*. Courier Corporation.

Brownell, C. A. (2016). Early Social Development Research Lab. Prosocial behavior in infancy: The role of socialization. *Child Development Perspectives, 10*(4), 222-227.

Bruins, B. E. (2014). The effectiveness of manipulatives in a high school algebra II class (Unpublished master's thesis). Eastern Kentucky University, Richmond.

Bruner, J. S. (1960). On learning mathematics1. *The Mathematics Teacher*, *53*(8), 610-619.

Bruner, J. S. (1977). Early social interaction and language acquisition. *Studies in mother-infant interaction*.

Bryman, A. (2016). *Social research methods*. London: Oxford University Press.

Bush, S. B., & Karp, K. S. (2013). Prerequisite algebra skills and associated misconceptions of middle grade students: A review. *The Journal of Mathematical Behavior, 32*(3), 613-632.

Bütüner, S. Ö., & Baki, A. (2020). The Use of History of Mathematics in the Mathematics Classroom: An Action Study. *International Journal of Education in Mathematics, Science and Technology*, *8*(2), 92-117.

Byrne, Z. S., Cave, K. A., & Raymer, S. D. (2022). Using a generalizable photo-coding methodology for assessing organizational culture artifacts. *Journal of Business and Psychology*, 1-15.

Çağdaşer, B. T. (2008). *Cebir öğrenme alanının yapılandırmacı yaklaşımla öğretiminin 6. sınıf öğrencilerinin cebirsel düşünme düzeyleri üzerindeki etkisi* (Doctoral dissertation, Bursa Uludag University (Turkey).

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of educational psychology*, *105*(2), 380.

Carraher, D. W., Martinez, M. V., & Schliemann, A. D. (2008). Early algebra and mathematical generalization. *ZDM*, *40*, 3-22.

Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction on solving area and perimeter problems by students with learning disabilities. *Learning disabilities research & practice*, *18*(2), 112-120.

Castro, S. (2017). Algebra Tiles Effect on Mathematical Achievement of Students with Learning Disabilities Capstone Projects and Master's Theses.

Çaylan, B. (2018). *The effects of using Algebra tiles on sixth grade students' Algebra achievement, algebraic thinking, and views about using Algebra tiles* (Master's thesis, Middle East Technical University).

Chaurasia, P. (2019). Using Algebraic Tiles from Secondary Mathematics Kit. *the NCERT and no matter may be reproduced in any form without the prior permission of the NCERT.*, *45*(1), 19.

Chappell, M. F., & Strutchens, M. E. (2001). Creating connections: Promoting algebraic thinking with concrete models. *Mathematics teaching in the Middle School*, *7*(1), 20-25.

Cheema, J. R., & Kitsantas, A. (2014). Influences of disciplinary classroom climate on high school student self-efficacy and mathematics achievement: A look at gender and racial–ethnic differences. *International Journal of Science and Mathematics Education*, *12*, 1261-1279.

Clements, D. H. (2000). 'Concrete' manipulatives, concrete ideas. *Contemporary issues in early childhood*, *1*(1), 45-60.

Clements-Croome, D. (2017). Effects of the built environment on health and well–being. In *Creating the Productive Workplace* (pp. 3-40). Routledge.

Cooper, T. E. (2012). Using Virtual Manipulatives with Pre-service Mathematics Teachers to Create Representational Models. *International Journal for Technology in Mathematics Education*, *19*(3).

Cope, L. (2015). Math manipulatives: Making the abstract tangible. *Delta Journal of Education, 5*(1), 10-19.

Creswell, J. W. (2017). Research design: Qualitative, quantitative, and mixed methods approach. London: Sage publications.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approach*. London: Sage Publications.

Creswell, J. W., & Plano Clark, V. L. (2014). *Designing and conducting mixed methods research.* (2nd ed.). Thousand Oaks, CA: Sage.

Creswell, J. W. (2014). *A concise introduction to mixed methods research*. Sage publications.

Creswell, J. W. (2007). Differing perspectives on mixed methods research. *Journal of mixed methods research*, *1*(4), 303-308.

Daniel, B. K. (2019). What constitutes a good qualitative research study? Fundamental dimensions and indicators of rigour in qualitative research: The TACT framework. In *Proceedings of the European conference of research methods for business & management studies* (pp. 101-108).

Dauda, B., Jambo, H. E., & Umar, M. A. (2016). Students' Perception of Factors Influencing Teaching and Learning of Mathematics in Senior Secondary Schools in Maiduguri Metropolis, Borno State, Nigeria. *Journal of Education and Practice, 7*(20), 114-122.

Dawadi, S., Sagun, S., & Ram, A. G. (2021). 'Mixed-methods research: A discussion on its types, challenges, and criticisms.' *Journal of Practical Studies in Education* 2, no. 2: 25-36.

Dewe, P., & O'Driscoll, M. (2002). Stress management interventions: what do managers actually do? *Personnel Review*, *31*(2), 143-165.

Dewey, J. (1938). The philosophy of the arts. *John Dewey: The Later Works*, *13*, 357-368.

Dienes, Z., & Perner, J. (1999). A theory of implicit and explicit knowledge. *Behavioral and brain sciences*, *22*(5), 735-808.

Dimitrov, D. S. (2012). Therapeutic proteins. *Therapeutic Proteins: Methods and Protocols*, 1-26.

Doerr, H. M. (2004). Teachers' knowledge and the teaching of algebra. *The Future of the Teaching and Learning of Algebra The 12th ICMI Study*, 265-290.

Elo, S., Kääriäinen, M., Kanste, O., Pölkki, T., Utriainen, K., & Kyngäs, H. (2014). Qualitative content analysis: A focus on trustworthiness. *SAGE open*, *4*(1), 2158244014522633.

Ely, R., & Adams, A. E. (2012). Unknown, placeholder, or variable: what is x? *Mathematics Education Research Journal*, *24*(1), 19-38.

Erdogan, F., & Sengul, S. (2014). A study on the elementary school students' mathematics self-concept. *Procedia-Social and Behavioral Sciences*, *152*, 596-601.

Ergene, B. Ç., & Haser, Ç. (2021). Students' algebra achievement, algebraic thinking and views in the case of using algebra tiles in groups. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi*, *15*(2), 254-281.

Fennema, E. H. (1972). Models and mathematics. *The Arithmetic Teacher*, *19*(8), 635-640.

Fey, J. T. (1979). Mathematics teaching today: Perspectives from three national surveys. *The Mathematics Teacher*, *72*(7), 490-504.

Fidele, U., Kizito, N., Angel, M., & Jean, U. (2019). Insights of teachers and students on mathematics teaching and learning in selected Rwandan secondary schools. *African Journal of Educational Studies in Mathematics and Sciences*, *15*(2), 93-106

Fitzsimons, M. (2014). Engaging students' learning through active learning. *Irish Journal of Academic Practice*, *3*(1), 13.

Fitzgerald, W. M. (1972). About Mathematics Laboratories.

Flinders, D. J., & Thornton, S. J. (2013). Introduction to part two. *The curriculum studies*.

Fontana, A., & Frey, J. H. (2000). The interview: From structured questions to negotiated text. *Handbook of qualitative research*, *2*(6), 645-672.

Fosnot, C. T. (2013). *Constructivism: Theory, perspectives, and practice*. Teachers College Press.

Furner, J. M., & Worrell, N. L. (2017). The importance of using manipulatives in teaching math today. *Transformations*, *3*(1), 2.

Furner, J. M., Yahya, N., & Duffy, M. L. (2005). Teach mathematics: Strategies to reach all students. *Intervention in school and clinic*, *41*(1), 16-23.

Gafoor, K. A., & Kurukkan, A. (2015). Why High School Students Feel Mathematics Difficult? An Exploration of Affective Beliefs. *Online Submission*.

Garzón, J., & Bautista, J. (2018). Virtual Algebra Tiles: A pedagogical tool to teach and learn algebra through geometry. *Journal of computer assisted learning*, *34*(6), 876-883.

Geist, E. A., & King, M. (2008). Different, not better: gender differences in mathematics learning and achievement. *Journal of Instructional Psychology*, *35*(1).

Ghana. Statistical Service. (2022). 2020 Population and housing census*: Summary report of final results*. Ghana Statistical Service.

Girit, D., & Akyüz, D. (2016). Algebraic thinking in middle school students at different Grades: Conceptions about the generalization of patterns. *Education Journal of Science and Mathematics Education*, *10*(2), 243-272.

Gningue, S. M., Menil, V. C., & Fuchs, E. (2014). Applying Bruner's Theory of Representation to Teach Pre-Algebra and Algebra Concepts to Community College Students Using Virtual Manipulatives. *Electronic Journal of Mathematics & Technology*, *8*(3).

Goins, K. B. (2001). *Comparing the effects of visual and algebra tile manipulative methods on student skill and understanding of polynomial multiplication*. University of South Carolina.

Gutek, G. L. (2014). Maria Montessori: Contributions to educational psychology. In *Educational psychology: A century of contributions* (pp. 171-186). Routledge.

Heddens, J. W. (1986). Bridging the gap between the concrete and the abstract. *The Arithmetic Teacher*, *33*(6), 14-17.

Herbert, E. (1985). One Point of View: 'Manipulatives Are Good Mathematics'. *The Arithmetic Teacher*, *32*(6), 4-4.

Herscovics, N. (2018). Cognitive obstacles encountered in the learning of algebra. In *Research issues in the learning and teaching of algebra* (pp. 60-86). Routledge.

Heyd-Metzuyanim, E. (2019). Changing teaching practices towards explorative mathematics instruction–The interweaving of teacher identity and pedagogical discourse. *Teaching and teacher education*, *86*, 102862.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American educational research journal*, *42*(2), 371-406.

Holmes, A. B. (2013). Effects of manipulative use on PK-12 mathematics achievement: A meta-analysis. Poster presented at the meeting of Society for Research in Educational Effectiveness, Washington, DC.

Iddi, A. (2016). *A comparative assessment of the academic performance among public and private Junior High Schools in the Tamale Metropolis of Ghana* (Doctoral dissertation).

Jensen, N. M. (2008). Nation-states and the multinational corporation. In *Nation-States and the Multinational Corporation*. Princeton University Press.

Johnson, M. K., Hashtroudi, S., & Lindsay, D. S. (1993). Source monitoring. *Psychological bulletin*, *114*(1), 3.

Jones, J. P., & Tiller, M. (2017). Using concrete manipulatives in mathematical instruction. *Dimensions of Early Childhood*, *45*(1), 18-23.

Jupri, A., & Drijvers, P. (2016). Student difficulties in mathematizing word problems in algebra. *Eurasia Journal of Mathematics, Science and Technology Education*, *12*(9), 2481-2502.

Kaiser, G. (2020). Mathematical modelling and applications in education. *Encyclopedia of mathematics education*, 553-561.

Kamina, P., & Iyer, N. N. (2009). From concrete to abstract: Teaching for transfer of learning when using manipulatives.

Kaput, J. J. (2017). What Is Algebra? What is Algebraic Reasoning? In *Algebra in the early grades* (pp. 5-18). Routledge.

Kaya, D., İzgiol, D., & Keşan, C. (2014). The investigation of elementary mathematics teacher candidates' problem-solving skills according to various variables.

Kelly, C. A. (2006). Using manipulatives in mathematical problem solving: A performance-based analysis. *The mathematics enthusiast*, *3*(2), 184-193.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it. *The mathematics educator*, *8*(1), 139-151.

Kirshner, D., & Awtry, T. (2004). Visual salience of algebraic transformations. *Journal for research in mathematics Education*, *35*(4), 224-257.

Kober, N. (1991). What We Know about Mathematics Teaching and Learning. EDTALK.

Koerner, M. (2020). At the Heart of a Teacher: Schooling in a Pandemic. *Journal of Interdisciplinary Studies in Education*, *9*(1), 172-174.

Kontas, H. (2016). The Effect of Manipulatives on Mathematics Achievement and Attitudes of Secondary School Students. *Journal of Education and Learning*, *5*(3), 10-20.

Kurumeh, M. S., Chiawa, M. A., & Ibrahim, M. O. (2010). Dienes Multibase Blocks' Approach an Effective Strategy for Improving Students' Interest in Number Bases among Secondary School Students in Mathematics. *Research Journal of Mathematics and Statistics*, *2*(3), 101-104.

Larbi, E., & Okyere, M. (2014). Algebra tiles manipulative and gender differences in learning and achievement in mathematics: A case of Sunyani West Municipality.

Larbi, E., & Mavis, O. (2016). The use of manipulatives in mathematics education. *Journal of Education and practice*, *7*(36), 53-61.

Lave, J. (2021). The Culture of Acquisition and the Practice of Understanding 1. In *Situated cognition* (pp. 17-35). Routledge.

Lawrence, A., & Hennessy, C. (2002). *Lessons for algebraic thinking: Grades 6-8*. Math Solutions.

Lee, J., Bryant, D. P., Ok, M. W., & Shin, M. (2020). A systematic review of interventions for algebraic concepts and skills of secondary students with learning disabilities. *Learning Disabilities Research & Practice*, *35*(2), 89-99.

Lehtonen, D. (2022). 'Now I Get It': Developing a Real-World Design Solution for Understanding Equation-Solving Concepts.

Leitze, A., & Kitt, N. A. (2000). Algebra for all: Using homemade algebra tiles to develop algebra and pre-algebra concepts. *The Mathematics Teacher*, *93*(6), 462-520.

Liggett, R. S. (2017). The Impact of Use of Manipulatives on the Math Scores of Grades 2 Students. *Brock Education: A Journal of Educational Research and Practice*, *26*(2), 87-101.

Lumadi, R. I., & Monobe, R. J. (2017). The management of instructional media in the general education and training band in schools in the Limpopo Province. *Journal of Educational Studies*, *16*(2), 82-97.

Magruder, R. L. (2012). *Solving linear equations: A comparison of concrete and virtual manipulatives in middle school mathematics*. University of Kentucky.

Makrygiannakis, G., & Jack, L. (2018). Designing a conceptual methodology for structuration research. *Meditari Accountancy Research*.

Malim, T. (2018). Development of Manipulative Module of Algebraic Tiles for Quadratic Expressions and Equations. *Advances in Natural and Applied Sciences*, *12*(8), 33-35.

Marsh, D. L. (2016). Using Manipulatives to Investigate ESOL Students' Achievement and Dispositions in Algebra.

McGregor, K. K. (2020). How we fail children with developmental language disorder. *Language, speech, and hearing services in schools*, *51*(4), 981-992.

McKinney, S., & Frazier, W. (2008). Embracing the principles and standards for school mathematics: An inquiry into the pedagogical and instructional practices of mathematics teachers in high-poverty middle schools. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 81*(5), 201-210.

Menekşe, B. O. Z., Uludağ, G., & Erdoğan, S. (2020). The Effect of The Manipulative Materials on The Early Mathematical Skills. *Bartın University Journal of Faculty of Education*, *9*(3), 492-500.

Miller, J. C. (2015). *Class numbers of totally real number fields*. Doctoral dissertation, Rutgers University-Graduate School-New Brunswick.

Mills, E. D., & Mereku, D. K. (2016). Students' performance on the Ghanaian junior high school mathematics national minimum standards in the Efutu Municipality. *African Journal of Educational Studies in Mathematics and Sciences, 12,* 25-34.

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in mathematics*, *47*(2), 175-197.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching children mathematics*, *8*(6), 372-377.

Moyer, P. S., & Jones, M. G. (2004). Controlling choice: Teachers, students, and manipulatives in mathematics classrooms. *School Science and Mathematics*, *104*(1), 16-31

National Council of Teachers of Mathematics (NCTM) (2000). Principles and Standards for School Mathematics. Reston, VA: NCTM.

National Educational Assessment. (2016). Finding report Accra: Assessment Service Unit

Norton, S., & Irvin, J. (2007). A concrete approach to teaching symbolic algebra. In *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia* (Vol. 2, pp. 551-560).

Odum, J. K. (2022). Manipulative Materials in Teaching Mathematics among Junior High School Teachers: A Literature Review. *Scholars Journal of Science and Technology*, *3*(4), 787-795.

Ojose, B. (2008). Applying Piaget's theory of cognitive development to mathematics instruction. *The mathematics educator*, *18*(1).

Ojose, B., & Sexton, L. (2009). The effect of manipulative materials on mathematics achievement of first grade students. *The mathematics educator*, *12*(1), 3-14.

Okpube, N. M., & Anugwo, M. N. (2016). Card Games and Algebra Tic Tacmatics on Achievement of Junior Secondary II Students in Algebraic Expressions. *International Journal of Evaluation and Research in Education*, *5*(2), 93-100.

Padmore, E. A. (2017). *Examining the use of manipulative materials in teaching Mathematics among junior high school teachers in the Wa municipality* (Doctoral dissertation, University of Education, Winneba).

Palatnik, A., & Koichu, B. (2017). Sense making in the context of algebraic activities. *Educational studies in Mathematics*, *95*(3), 245-262.

Pandey, P., & Pandey, M. M. (2021). *Research methodology tools and techniques*. Bridge Center.

Park, E. L., & Choi, B. K. (2014). Transformation of classroom spaces: Traditional versus active learning classroom in colleges. *Higher Education*, *68*, 749-771.

Piaget, J. (1952). The Child's Conception of Number (C. Gattegno & FM Hodgson, Trans.) New York.".

Piaget, J. (1962). The relation of affectivity to intelligence in the mental development of the child. *Bulletin of the Menninger clinic*, *26*(3), 129.

Piaget, J. (1973). *The child and reality: Problems of genetic psychology. (Trans. Arnold Rosin)*. Grossman.

Picciotto, H., & Wah, A. (1993). A new algebra: Tools, themes and Concepts. *Journal of Mathematical Behavior*, *12*(1), 19-42.

Polites, G. L., Roberts, N., & Thatcher, J. (2012). Conceptualizing models using multidimensional constructs: A review and guidelines for their use. *European Journal of Information Systems*, *21*, 22-48.

Prosser, S. K., & Bismarck, S. F. (2023). Concrete–Representational–Abstract (CRA) Instructional Approach in an Algebra I Inclusion Class: Knowledge Retention Versus Students' Perception. *Education Sciences*, *13*(10), 1061.

Rannikmäe, M., Holbrook, J., & Soobard, R. (2020). Social Constructivism—Jerome Bruner. *Science Education in Theory and Practice: An Introductory Guide to Learning Theory*, 259-275.

Ross, A., & Willson, V. (2012). The effects of representations, constructivist approaches, and engagement on middle school students' algebraic procedure and conceptual understanding. *School Science and Mathematics*, *112*(2), 117-128.

Safadi, S. (2021). *A Hermeneutic Phenomenological Study of the Experience of Using Appreciative Inquiry in Jordan* (Doctoral dissertation, Leeds Beckett University).

Salifu, A. S. (2022a). Mathematics tutors' views on benefits and challenges of using Algebra tiles in teaching linear equations in one variable. *Mediterranean Journal of Education*, *2*(1), 81-93.

Salifu, A. S. (2022b). The effects of balance model and algebra tiles manipulative in solving linear equations in one variable. *Contemporary mathematics and science education*, *3*(2).

Sanderson, D. (2021). The Role of Mathematic Manipulatives in the Preschool Classroom. In *Official Conference* (p. 17).

Saraswati, S., Putri, R. I. I., & Somakim, S. (2016). Supporting Students' Understanding of Linear Equations with one Variable using Algebra Tiles. *Journal on Mathematics Education*, *7*(1), 19-30.

Seidman, G. (2013). Self-presentation and belonging on Facebook: How personality influences social media use and motivations. *Personality and individual differences*, *54*(3), 402-407.

Sexton, J., London, D., Jameson, M. M., & Wenner, J. M. (2022). Thriving, Persisting, or Agonizing: Integrated math anxiety experiences of university students in introductory geoscience classes. *Education sciences*, *12*(9), 577.

Sfard, A., & Linchevski, L. (1994). The gains and the pitfalls of reification—the case of algebra. *Educational studies in mathematics*, *26*, 191-228.

Shin, W. S., & Kang, M. (2015). The use of a mobile learning management system at an online university and its effect on learning satisfaction and achievement. *International Review of Research in Open and Distributed Learning*, *16*(3), 110-130.

Shin, M., Simmons, M., Meador, A., Goode, F. J., Deal, A., & Jackson, T. (2023). Mathematics Instruction for Students with Learning Disabilities: Applied Examples Using Virtual Manipulatives. *Intervention in School and Clinic*, *58*(3), 198-204.

Shosha, G. A. (2012). Employment of Colaizzi's strategy in descriptive phenomenology: A reflection of a researcher. *European Scientific Journal*, *8*(27).

Simon, M. K., & Goes, J. (2013). Assumptions, limitations, delimitations, and scope of the study.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching*, *77*(1), 20-26.

Smith, G. (2009). *Democratic innovations: Designing institutions for citizen participation*. Cambridge University Press.

Smith, C. P., Tinkler, A., DeMink-Carthew, J., & Tinker, B. (2017). Pre-Service Mathematics Teachers' Experiences with Proficiency-Based Learning. *Mathematics Teacher Education and Development*, *19*(1), 107-120.

Sengendo, F. (2019). *Teachers'Instructional Effectiveness and Learners' Enjoyment of Secondary School Mathematics in Kira Municipality-Wakiso District* (Doctoral dissertation, Makerere University).

Stacey, K., & Chick, H. (2004). Solving the problem with algebra. *The Future of the Teaching and Learning of Algebra the 12th ICMI Stud.*

Steele, D. F., & Johanning, D. I. (2004). A schematic–theoretic view of problem solving and development of algebraic thinking. *Educational Studies in Mathematics*, *57*, 65-90.

Stiegelmeier, T., & Moore, J. D. (2019). Effective mathematics instruction for native American elementary students: Strategies for using manipulatives. *Journal of Teacher Action Research*, *5*(3).

Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational leadership*, *61*(5), 12-17.

Strickland, T. K. (2016). Using the CRA-I strategy to develop conceptual and procedural knowledge of quadratic expressions. *Teaching Exceptional Children*, *49*(2), 115-125.

Strickland, T. K., & Maccini, P. (2013). The effects of the concrete–representational–abstract integration strategy on the ability of students with learning disabilities to multiply linear expressions within area problems. *Remedial and Special Education, 34(3), 142-153.*

Suydam, M. N., & Higgins, J. L. (1976). Review and synthesis of studies of activity-based approaches to mathematics teaching. *Recommendations from research, Columbus, OHIO: ERIC Clearinghouse for Science, Mathematics and Environmental Education*.

Suydam, M. N., & Higgins, J. L. (1977). Activity-based learning in elementary school mathematics: Recommendations from research.

Taherdoost, H. (2016). Validity and reliability of the research instrument; how to test the validation of a questionnaire/survey in research. *How to test the validation of a questionnaire/survey in research.*

Tall, D., & Thomas, M. (1991). Encouraging versatile thinking in algebra using the computer. *Educational studies in mathematics*, *22*(2), 125-147.

Tannous, C. H. (2020). Influencing Student Achievement: Teaching, Learning and Teachers' Attributes.

Theobald, N. A., & Haider-Markel, D. P. (2009). Race, bureaucracy, and symbolic representation: Interactions between citizens and police. *Journal of Public Administration Research and Theory*, *19*(2), 409-426.

Thomas, C. N., Van Garderen, D., Scheuermann, A., & Lee, E. J. (2015). Applying a universal design for learning framework to mediate the language demands of mathematics. *Reading & Writing Quarterly*, *31*(3), 207-234.

Thornton, S. (2004). Standards for excellence in teaching mathematics. *Canadian Mathematics Education Study Group Groupe Canadien D'étude En Didactique Des Mathématiques*, 119.

Tran, C., Smith, B., & Buschkuehl, M. (2017). Support of mathematical thinking through embodied cognition: Nondigital and digital approaches. *Cognitive Research: Principles and Implications*, *2*, 1-18.

Treffers, A. (2012). *Three dimensions: A model of goal and theory description in mathematics instruction—The Wiskobas Project* (Vol. 3). Springer Science & Business Media.

Trochim, W. M. (2006). *The* Jossey *research methods knowledge base, (2nd ed.).* Retrieved from: http://www.socialresearchmethods.net/kb/

Trochim, W. M., Cabrera, D. A., Milstein, B., Gallagher, R. S., & Leischow, S. J. (2006). Practical challenges of systems thinking and modeling in public health. *American Journal of public health*, *96*(3), 538-546.

Ukobizaba, F., Ndihokubwayo, K., Mukuka, A., & Uwamahoro, J. (2021). From what makes students dislike mathematics to its effective teaching practices. *Bolema: Boletim de Educação Matemática*, *35*, 1200-1216.

Ünlüer, İ., & Kurtuluş, A. (2021). The examination of conceptual and procedural understanding processes of eighth grade students in the subjects of identities and factoring. *Turkish Journal of Computer and Mathematics Education*, *12*(1), 22-70.

Van de Walle, S. (2016). When public services fail: A research agenda on public service failure. *Journal of Service Management*.

Van Wyk, M. M., & Taole, M. (2015). Research design. *Educafional research: An African approach, Oxford University Press, Cape Town*.

Vinogradova, N. (2007). Teacher to teacher: Solving quadratic equations by completing squares. *Mathematics teaching in the middle school*, *12*(7), 403-405.

Wang, J., & Goldschmidt, P. (2003). Importance of middle school mathematics on high school students' mathematics achievement. *The Journal of Educational Research*, *97*(1), 3-17.

West Africa Examination Council (2011). Basic Education Certificate Examination, Chief Examiner's Report, WAEC: Accra.

White, K. M. (2012). *The effect of an instructional model utilizing hands-on learning and manipulatives on math achievement of middle school students in Georgia*. Liberty University.

Wilkinson, J. H., Bauer, F. L., & Reinsch, C. (2013). *Linear algebra* (Vol. 2). Springer.

Wilson, B. B. (2016). *Caring, creativity, and collaboration: how teachers perceive, understand, and enact high expectations* (Doctoral dissertation, Queensland University of Technology).

Yang, J. C., & Chen, S. Y. (2010). Effects of gender differences and spatial abilities within a digital pentominoes game. *Computers & Education*, *55*(3), 1220-1233.

Yildiz, A. (2016). The Geometric Construction Abilities of Gifted Students in Solving Real-World Problems: A Case from Turkey. *Malaysian Online Journal of Educational Technology*, *4*(4), 53-67.

Zebracki, J. E. (2013). *A study of students' preference towards different methods of finding solutions to Quadratic Equations* (Doctoral dissertation).

APPENDICES

APPENDIX A

AKENTEN APPIAH-MENSAH UNIVERSITY OF SKILLS TRAINING AND ENTREPRENEURIAL DEVELOPMENT

QUESTIONNAIRE

The study examines the effect of the use of algebra tiles manipulatives on students' mathematics achievement in algebra in some selected public Junior High Schools in Berekum West District. It is a research project being carried out as a partial fulfillment of the requirements for the awarding of a master's degree in philosophy in mathematics education.

Your kind cooperation will be highly appreciated by participating and provide accurate responses to this survey. The information you shall provide will be confidential and anonymous and will be solely used for academic purposes. The findings of this study will be communicated to you through your head teacher to guide and provide useful information for your effectiveness of learning.

Section A: Demographic Background of Participants

Please tick (√) under the space provided that corresponds to what you think regarding to the statement.

Sex Male [ ] Female [ ]

Age 10 - 14 years [ ] 15 - 19 years [ ] Above 20 years [ ]

Section B: Students' views about the use of algebra tiles in mathematics lessons.

Please rate the following items to determine your views on the use of algebra tiles in mathematics class. The rating scale is as follows 1-Strongly Disagree, 2-Disagree, 3-Not sure, 4-Agree, and 5-Strongly Agree.

	Students' views about the use of algebra tiles	**Responses**				
S/N		**1**	**2**	**3**	**4**	**5**
1	I can use the algebraic rules from my own experiences with the help of algebra tiles					
2	I enjoy the use of algebra tiles in activities while learning algebraic concepts.					
3	Algebra tiles have the potential to help me to internalize algebraic ideas.					
4	Using algebra tiles in group work may provide idea-rich environment for learning the basic algebra concepts					
5	Algebra tiles clarified the concept better and helped me learn the concept better.					
6	When I saw the operation on the board before the use of algebra tiles, I did not understand what to do.					
7	With the help of algebra tiles, I understood and comprehended the concept better.					
8	Algebra tiles enabled me to remember the concept easily.					
9	Algebra tiles enabled me to perform complicated operation easily					
10	Transition from arithmetic to algebra is difficult for students when there is an excessive reliance on textbooks					

Section C: Students' perceived challenges in the use of algebra tiles manipulative

Please rate the following items to determine your perceived challenges in the use of algebra tiles in mathematics class. The rating scale is as follows 1-Strongly Disagree, 2-Disagree, 3-Not sure, 4-Agree, and 5-Strongly Agree.

	Perceive challenges of the use of algebra tiles	**Responses**				
S/N		**1**	**2**	**3**	**4**	**5**
1	Modeling complicated examples with algebra tiles is difficult to understand					
2	The use of algebra tiles waste instructional time					
3	Algebra tiles cannot denote fractions hence it is difficult to carry out the division of equations by using algebra tiles					
4	Large class sizes make distribution and control of the use of algebra tiles difficult.					
5	Limited knowledge by teachers hinders smooth operation of algebra tiles class					
6	The use of algebra tiles if not properly monitored in class can cause chaos					

AKENTEN APPIAH-MENSAH UNIVERSITY OF SKILLS TRAINING AND ENTREPRENEURIAL DEVELOPMENT

INTERVIEW GUIDE

The study examines the effect of the usage of algebra tiles manipulatives on students' mathematics achievement in algebra in some selected public Junior High Schools in Berekum West District. It is a research project being carried out as a partial fulfillment of the requirements for the awarding of a master's degree in philosophy in mathematics education.

Your kind cooperation will be highly appreciated by participating and provide accurate responses to this interview. The information you shall provide will be confidential and anonymous and will be solely used for academic purposes. The findings of this study will be communicated to you through your head teacher to guide and provide useful information for your effectiveness of learning.

Section A: Demographic Background of Participants

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Sex Male [ ] Female [ ]

Age 10 - 14 years [ ] 15 - 19 years [ ] Above 20 years [ ]

SECTION B: Interview Guide to Explore How Students' Algebraic Thinking Differ for Those who use Algebra Tiles and Those Who do not use them.

1. How do you see algebra lessons in class?

2. You have been taught Expansion and factorization of algebraic expression please share with me your feelings and how was the lesson done when algebra tiles are used in algebra lessons?

3. Will you advocate for the inclusion of the use of algebra tiles in the teaching of algebraic expressions and how useful and effective will it be?

(Experimental Group)

1. How do you see algebra lessons in class?

2. You have been taught Expansion and factorization of algebraic expression please share with me your feelings and how was the lesson done?

3. Have you heard of algebra tiles manipulatives?

Researcher (explain algebra tiles manipulatives and its usage to students)

follow up

Will you advocate for the inclusion of the use of algebra tiles in the teaching of algebraic expressions and how useful and effective will it be?

(Control Group)

SECTION D1: Pre-Test Questions and Answers Administered to JHS 2 Students.

1. $x + 3x + 4x - 2x + 3$

$= \mathbf{6x + 3}$

2. $2x - 6x + 2 + x - 4$

$= \mathbf{-3x - 2}$

3. $2y + 6 - 2y + 6 - 2 - 4 = \mathbf{6}$

4. $2x + 4y - x + y + 4$

$= \mathbf{3x + 5y + 4}$

5. $3x + 4 + 2y - 1 + 2x - 2y$

$= \mathbf{5x + 3}$

6. $2x^2 + 3x - x + 5 - 2x$

$= \mathbf{2x^2 + 5}$

7. $3x^2 - 5x + 2x^2 + x + 2$

$= \mathbf{5x^2 - 4x + 2}$

8. $x^2 + 2x + 4x^2 - 2x + 1$

$= \mathbf{5x^2 + 1}$

9. $4x^2 - 4 + 3x - 4x^2 + 4 - 3x = \mathbf{0}$

10. $3x - 2x^2 + 2 + 4x^2 - x - 2$

$= \mathbf{2x^2 + 2x}$

SECTION D2: To find out the Impact of the use of Algebra Tiles Manipulative Material on Students' Achievement in Algebra in Mathematics.

Mathematics Achievement Test

Post-Test Sample Questions Administered to JHS 2 Students.

Expand and simplify the following algebraic expressions. (Using algebra tiles)

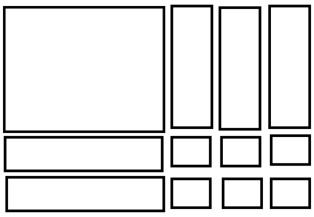
1. $(x + 3)(x + 2)$
2. $(x + 1)(x + 2)$
3. $(x - 1)(x + 2)$
4. $(2x + 1)(x + 4)$
5. $(x + 3)(x - 2)$

Factorize the following expressions (using algebra tiles).

1. $x^2 + 2x + 1$
2. $2x^2 + 2x - 4$
3. $2x^2 + 4x + 2$
4. $x^2 + 3x + 2$
5. $x^2 + 5x + 6$

Solutions to Post Test Questions –Expansion

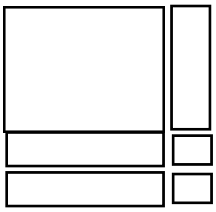
1. $(x + 3)(x + 2)$



$(x + 3)(x + 2)$

$=x^2 + 5x + 6$

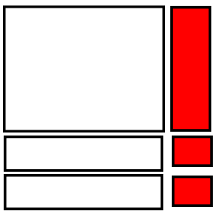
2. $(x + 1)(x + 2)$



$(x + 1)(x + 2)$

$= x^2 + 5x + 6$

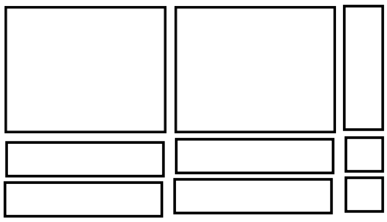
3. $(x - 1)(x + 2)$



$(x - 1)(x + 2)$

$= x^2 + x - 2$

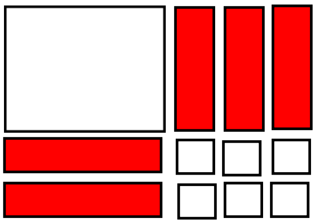
4. $(2x + 1)(x + 4)$



$(2x + 1)(x + 4)$

$= 2x^2 + 5x + 2$

5. $(x - 3)(x - 2)$

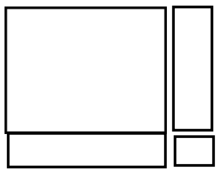


$(x - 3)(x - 2)$

$= x^2 - 5x + 6$

Solutions to Post Test Questions –Factorization

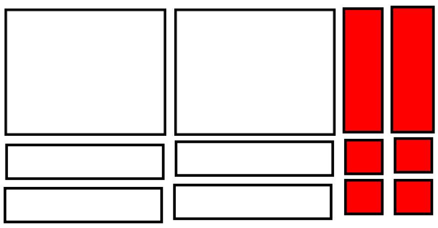
1. $x^2 + 2x + 1$



$x^2 + 2x + 1$

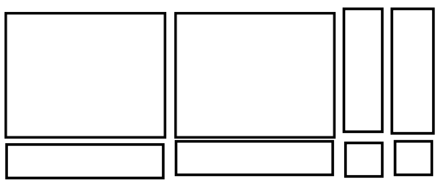
$= (x + 1)(x + 1)$

2. $2x^2 + 2x - 4$



$2x^2 + 2x - 4 = (2x - 2)(x + 2)$

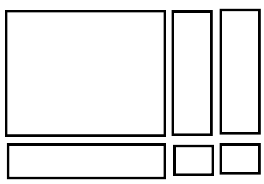
3. $2x^2 + 4x + 2$



$2x^2 + 4x + 2$

$= (2x + 2)(x + 1)$

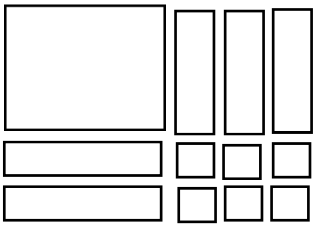
4. $x^2 + 3x + 2$



$x^2 + 3x + 2$

= $(x + 2)(x + 1)$

5. $x^2 + 3x + 2$



$x^2 + 5x + 6$

$= (x + 3)(x + 2)$

APPENDIX B

Gender of the Respondents

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	MALE	131	47.8	47.8	47.8
	FEMALE	143	52.2	52.2	100.0
	Total	274	100.0	100.0	

Age of the Respondents

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	10-14	93	33.9	33.9	33.9
	15-19	177	64.6	64.6	98.5
	ABOVE 20	4	1.5	1.5	100.0
	Total	274	100.0	100.0	

Reliability

Case Processing Summary

		N	%
Cases	Valid	30	100.0
	Excluded[a]	0	.0
	Total	30	100.0

Reliability Statistics on Students' Views on the use of algebra tiles in mathematics lessons.

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.720	.690	10

Reliability Statistics on Students' perceived challenges in the use of algebra tiles manipulative

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.726	.738	6

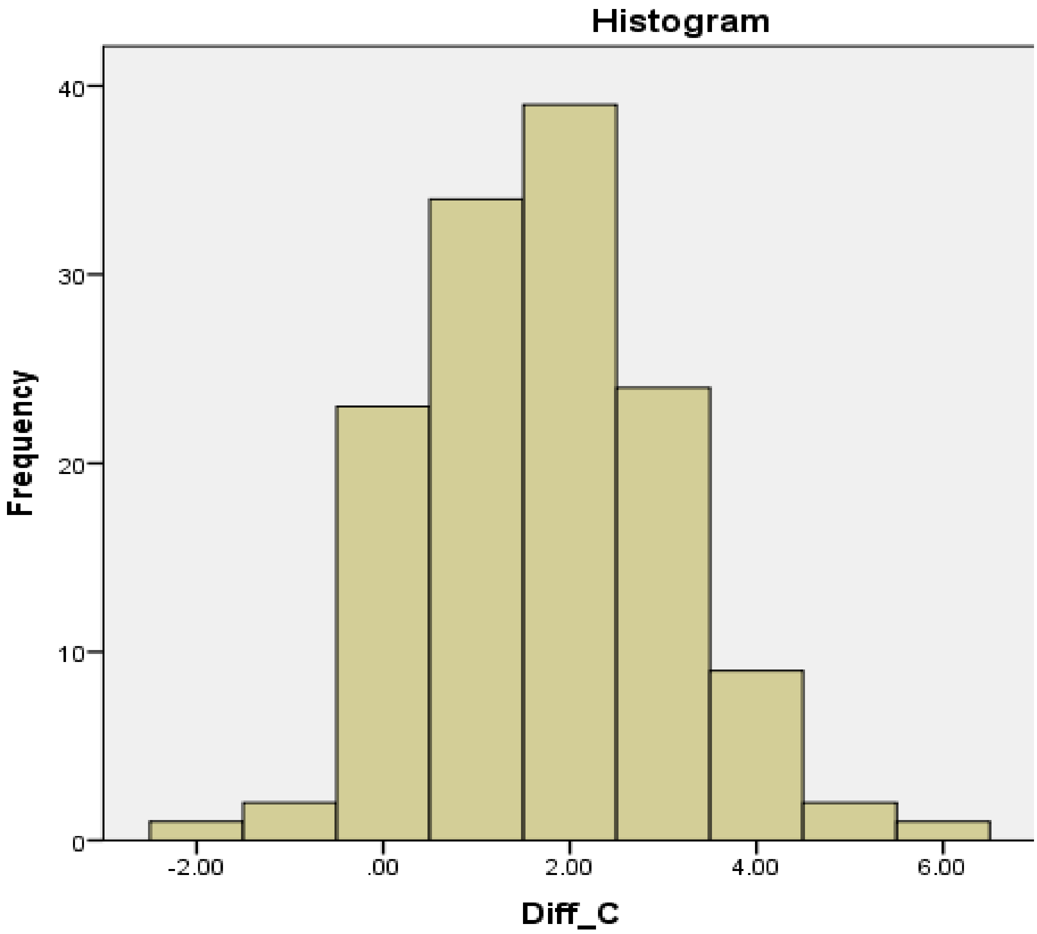
Descriptive Statistics on Students' views about the use of algebra tiles in mathematics lessons.

	N	Minimum	Maximum	Mean	Std. Deviation
Algebra tiles enabled me to remember the concept easily	139	1.00	5.00	4.6259	.60521
I enjoy the use of algebra tiles in activities while learning algebraic concepts	139	1.00	5.00	4.4532	.81847
I can use the algebraic rules from my own experiences with the help of algebra tiles	139	1.00	5.00	4.2446	.81509
With the help of algebra tiles, I understood and comprehended the concept better	139	1.00	5.00	4.2374	1.04661
Algebra tiles clarified the concept and helped me learn the concept better	139	1.00	5.00	4.2230	1.08376
Using algebra tiles in group work may provide idea-rich environment for learning the basic algebra concepts	139	2.00	5.00	4.2086	.92841
Algebra tiles have the potential to help me to internalize algebraic ideas	139	2.00	5.00	4.1799	.77321
Transition from arithmetic to algebra is difficult for students when there is an excessive reliance on textbooks	139	1.00	5.00	4.1655	1.09410
Algebra tiles enabled me to perform complicated operation easily	139	1.00	5.00	4.1367	.98688
When I saw the operation on the board before the use of algebra tiles, I did not understand what to do	139	1.00	5.00	3.4964	1.21807
Valid N (listwise)	139				

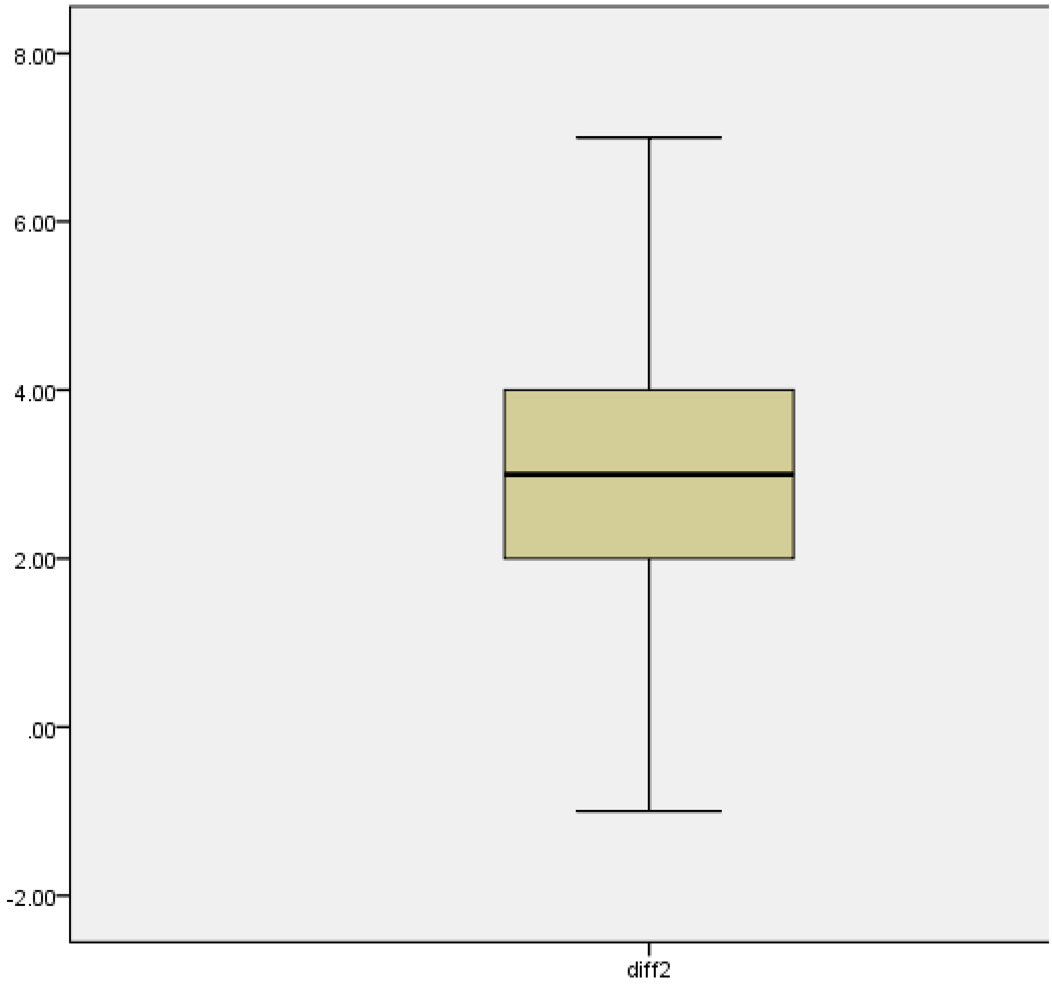
Descriptive Statistics on Students' perceived challenges in the use of algebra tiles manipulative

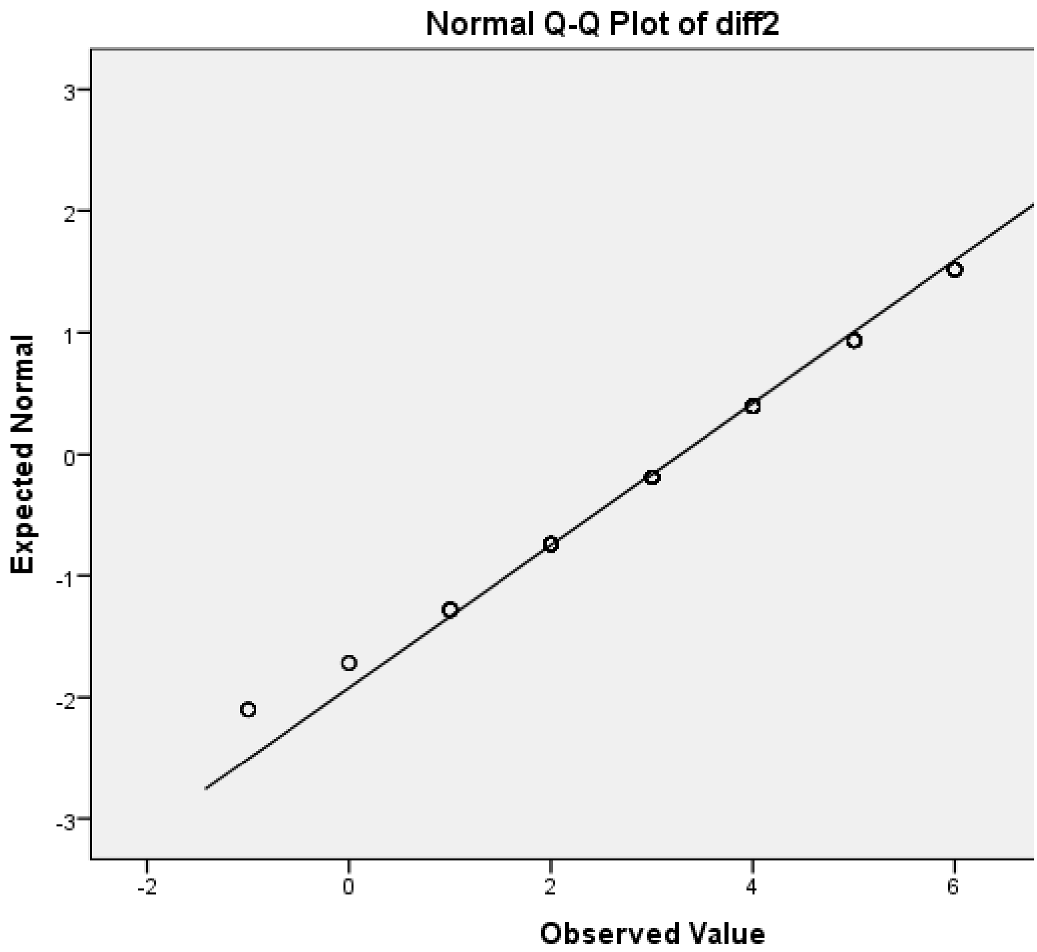
	N	Minimum	Maximum	Mean	Std. Deviation
Limited knowledge by teachers hinders smooth operation of algebra tiles class	139	2.00	5.00	4.5540	.55394
Large class sizes make distribution and control of the use of algebra tiles difficult	139	3.00	5.00	4.4388	.55319
The use of algebra tiles waste instructional time	139	1.00	5.00	4.4101	.65732
The use of algebra tiles if not properly monitored in class can cause chaos	139	1.00	5.00	4.2086	.91267
Modelling complicated examples with algebra tiles is difficult to understand	139	1.00	5.00	4.0791	.89333
Algebra tiles cannot denote fractions hence it is difficult to carry out the division of equations by using algebra tiles	139	1.00	5.00	3.8489	1.24478
Valid N (listwise)	139				

Tests for Normality on Students Mathematics Achievement



BOX AND WHISKER PLOT




Normal Q-Q Plot of diff2
Expected Normal
Observed Value
3
2
1
0
-1
-2
-3
-2
0
2
4
6

Paired Samples Statistics for Control Group

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	PRE_TEST_C1	2.7926	135	.90683	.07805
	POST_TEST_C1	5.0963	135	1.04292	.08976

Paired Samples Statistics for experimental Group

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	PRE_TEST_SCORES	2.6043	139	1.16481	.09880
	POST-TEST_-SCORES	8.3165	139	1.40923	.11953

Results of the Paired Samples t-test on pre and post tests on the Achievement of students in maths.

		Paired Differences					t	df	Sig. (2-tailed)
		Mean diff	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	POST_TEST_SCORES - PRE_TEST_SCORES	5.71223	1.50961	.12804	5.45905	5.96541	44.612	138	.000

Interview Transcriptions on how Students' Algebraic Thinking Differ for those who used Algebra Tiles and those who did not use them.

INTERVIEW TRANSCRIPTIONS

Student A (Experimental Group)

INTERVIEWER	**INTERVIEWEE**	**CODE**
How do you see algebra lessons?	***'Interesting since you are em supposed to combine numbers and also alphabets in solving the algebra so I feel a little bit simple and sometimes confusing and comple***x but the tiles have reduced it a little'.	Interesting but sometimes confusion
You talked about complexity, what makes it complex?	'***The complex is em you em having to do with numbers and also alphabets and***	Complex and confusion

	sometimes exponents so you having to add, subtract like everything sometimes you may be asked to find a variable which is sometimes complex and confusion'.	
You have been taught Expansion and factorization of algebraic expression please share with me your feelings and how was the lesson done when algebra tiles are used in algebra lessons?	***'Okay when you are using the algebra tiles it's quite convenient. For me using the algebra tiles not only created a fun but assisted me in comprehending the topic. Algebra tiles helped me discover more tricks, tactics, and approaches and remove my misunderstanding and perception***	Convenient, interesting, enjoying, making fun, practical lesson and helped in understanding and remembering of concepts.

	about maths'. 'I learnt well with the introduction of the algebra tiles since complex operations were made easier for me during the intervention period, because I remember concepts easily'. ***The teacher distributed algebra tiles to us We were shown x^2 tiles, x tiles and 1 tile and their negatives and demonstrated to us how it is used to expand and do trinomials factorisation. The teacher use tiles to perform some***	

	examples after which we were asked to use the materials to practice some examples and it worked perfectly. It was so interesting enjoying, amazing feeling, very practical lesson and we had fun'.	
'Will you advocate for the inclusion of the use of algebra tiles in the teaching of algebraic expressions and how useful and effective will it be'?	***'Sir I have already started that because it is very effective erh sometimes you see not everybody is sharp so you combining erh may be alphabets and numbers may be it can just confuse you simple so when you are using the algebra tiles you know that may be this***	very simple, effective and have improved performance

	colour resembles may be negative or positive, this is for negative square numbers, this is for positive square numbers so if you use it is very, very simple and effective and you get the answers very, very correct. The use of algebra tiles has improved my understanding and performance in algebraic expressions'.	

INTERVIEW TRANSCRIPTIONS

Student B (Experimental Group)

INTERVIEWER	INTERVIEWEE	CODE
How do you see algebra lessons?	‘***Algebra lessons have variables, numbers so solving variables and numbers are good in maths because when you get to high levels you will learn all those numbers, the variables and the numbers so learning algebra is good’.***	Good
You have been taught Expansion and factorization of algebraic expression please share with me your feelings and how was the lesson done when algebra tiles are used in algebra lessons?	***‘Okay sir’ I felt excited because for me learning with the algebra tiles made the lesson very practical and interesting although the lesson delivery was slower***	Slower and time consuming but exciting, practical, activity based, active involvement of learners, understanding of concept

	and time consuming. 'I still enjoyed the lessons by working with my classmates in groups'. 'The usage of the tiles aided me and my group to comprehend the concepts and making algebra tiles for teaching is not difficult'. 'The lessons were practical and every group participated actively and so the concepts were simply understood by all of us'. The teacher put us into groups and shared	

	the algebra tiles materials to the groups. He explained to us the tile that represent x^2 tiles, x tiles and one tiles some coloured tiles that represent the negative tiles. He used some of the tiles to do some examples and asked to us to do the same. Group members were actively involved, very happy and the lesson was full of activities and practicals'.	

'Will you advocate for the inclusion of the use of algebra tiles in the teaching of algebraic expressions and how useful and effective will it be'?	***'I will do that. We shall tell the headmaster to buy the algebra tiles so that the maths teacher can use it to teach us. The use of the tiles in algebraic expression was effective in a way that it makes you feel confident and comfortable when using it because when they give you erm an algebra erm question using the tiles may help you find the right answer because you have the negative and the positive so when you are given a positive question about algebra you may be able to solve it so the use of algebra tiles has motivated me and has***	Confident, comfortable, motivation and has improved performance.

	improved my performance in algebra'.	

INTERVIEW TRANSCRIPTIONS

Student C (Experimental Group)

INTERVIEWER	INTERVIEWEE	CODE
How do you see algebra lessons in class?	***'Sometimes there are confusion in the lesson because the question they might give you involve variables and numbers but using algebra tiles helped us to get the answer easily'***	confusion
You have been taught Expansion and factorization of algebraic expression please share with me your feelings and how was the lesson done when algebra tiles are used in algebra lessons?	***'Algebra tiles are amazing sir, because that was my time using the algebra tiles to solve such a question so I was so excited it has increased my love for mathematics especially algebraic***	Amazing, exciting, excellent manipulative, practical and enjoyable.

	expression which most of us find it difficult to understand. 'I didn't know that there was such an excellent manipulative'. 'Hmmm, I was surprised that there is a manipulative that can help me do expansion and factorization'. 'If all instructors were using concrete materials like this one then maths would have been seen as a hands-on subject than intangible'. 'The algebra	

	intervention starter inspired me to participate actively during the sessions. 'The teacher distributed algebra tiles to the class'. 'He then shows x^2 tiles, x tiles and 1 tile and their negatives to us and draws them on the chalkboard'. 'He then took us through how they are used to expand and do trinomials factorisation'. 'The teacher use tiles and did some examples after which we were asked to use the	

	materials to practice some examples and it worked correctly'. 'It was so interesting enjoyable, amazing feeling, a very practical lesson and we enjoyed ourselves'.	
Will you advocate for the inclusion of the use of algebra tiles in the teaching of algebraic expressions and how useful and effective will it be?	***'Sir I will advocate paa'. 'It was good but it takes much time to get the answer so when you are using it, it takes much time to get the answer so but using it is very good. I suggest that my headmaster buys all these things (algebra tiles) for***	Very Good, perfect understanding and improve performance but it takes much time.

	our maths teachers because sometimes it is not all the students in our class understand such a question so when the maths master is using it, it go na help all of us to understand it perfectly and improve our performance in expansion and factorisation in algebraic expression'.	

INTERVIEW TRANSCRIPTIONS

Student D (Control Group)

INTERVIEWER	INTERVIEWEE	CODE
How do you see algebra lessons in class?	***'Oh, when they are teaching us, we find it difficult but when the teacher finishes teaching us, we can do some examples and get some correct'***	Difficult
You have been taught Expansion and factorization of algebraic expression please, share with me your feelings and how was the lesson done?	***'The teacher wrote the topic and examples on the board and discuss with us how to apply the rules and procedures in solving examples'. For instance, in trinomial factorisation, you pick the constant term and multiply by the coefficient of say x^2 to get the two factors which are used to replace the middle term so that the***	Application of rules, felt bored because the rules were many

	factorisation can be done easily. He illustrated how to solve some examples on the board. I felt bored because the rules were many and I was confused at a point.	
Have you heard of algebra tiles manipulatives? Researcher (explain algebra tiles manipulatives and their usage to students) ***Follow up*** 'Will you advocate for the inclusion of the use of algebra tiles in the teaching of algebraic expressions and how useful and effective will it be'?	***'Sir, I have not heard of algebra tiles before, nobody has ever mention it to me so we sometimes find it difficult to understand algebra lessons.*** ***'Yes sir, I will say so because when they start teaching us, we find it more difficult, we will be asking our friends what is this? what is this? So, I think when they use these algebra tiles, we will be***	Have not heard of algebra tiles before but algebra tiles will be very effective and useful.

	able to get the concept right and understand it very well and improve our performance than not using it. Sir, I will say that when they use these algebra tiles to teach us it will be very effective and we will be able to understand it well and fast and our score in class exercises will go up so I think it will be very effective and useful also'.	

INTERVIEW TRANSCRIPTIONS

Student E (Control Group)

INTERVIEWER	INTERVIEWEE	CODE
How do you see algebra lessons in class?	***'Algebra lessons in class is err hard and difficult'***	Hard and difficult
You have been taught Expansion and factorization of algebraic expression please share with me your feelings and how was the lesson done?	***'Yes, sir we have been taught algebraic expressions. The teacher explained to us the rules and steps used to do expansion and factorization of algebraic expressions and gave us some examples to work them using the but sir it was not easy it was abstract. For the feelings sir Hmm it was not interesting,***	The lesson was abstract not easy and not interesting
Have you heard of algebra tiles manipulatives?	***'I have not sir', I have not heard the name before'.***	Have not heard of and used algebra tiles before but I trust they will be

Researcher (explain algebra tiles manipulatives and their usage to students) ***Follow up*** 'Will you advocate for the inclusion of the use of algebra tiles in the teaching of algebraic expressions and how useful and effective will it be'?	***'Sir I will do the advocate; I have not used it before but it can help me and the individuals in the class to understand the lesson fast. Sir the way you taught us, if you had used the tiles, I would have performed better than I did I trust that the material will be very effective and more useful in maths lesson'.***	effective, useful, and improve my performance.

INTERVIEW TRANSCRIPTIONS

Student F (Control Group)

INTERVIEWER	INTERVIEWEE	CODE
How do you see algebra lessons in class?	***'Very confusion and it's hard'.***	Very confusing and hard.
You have been taught Expansion and factorization of algebraic expression please share with me your feelings and how was the lesson done?	***'Sir you taught us. You told us to follow some steps, techniques, and rules to do binomial expansion and trinomial factorisation. The teacher together with the whole class solved some examples of both expansion and factorisation on the board. The teacher then wrote some examples on the board and asked some pupils to come to the board and apply the rules***	'Abstract, memorisation application of rules and procedures in solving expansion and factorisation of algebraic expressions.'

	and the techniques to solve them. ***For the feelings, hmm the lesson was abstract our maths teacher does not use any teaching learning materials in his teaching I have memorised the rules and apply the rules and some tactics to solve some examples.***	
Have you heard of algebra tiles manipulatives? Researcher (explain algebra tiles manipulatives and their usage to students) 'Will you advocate for the inclusion of the use of algebra tiles in the teaching of algebraic expressions and how useful and effective will it be'?	***'No sir I have not*** ***'Yes, sir I will advocate for the use of algebra tiles. The way you taught us and the illustrations that were done on the board, I know that when algebra tiles are used it is going to increase our vim in***	Have not heard of algebra tiles, don't like mathematics but the use of algebra tiles will make the lesson practical and improve our score and performance in mathematics.

	learning maths, it will help us to improve our lesson and performance in mathematics, some of us don't like maths may be because of how they teach us so I think that the usage of algebra tiles will be effective, suitable and will make the lesson practical for us to like maths and this may help us get better scores in maths class test'. When I was in JHS1 in my previous School our maths teacher used to teach us using teaching and learning materials but ever since I came to this school, the interest that I have in mathematics has gone down because our	

	maths teacher does not use any teaching learning materials in teaching us.	