AKENTEN APPIAH-MENKA UNIVERSITY OF SKILLS TRAINING AND ENTREPRENEURIAL DEVELOPMENT

RELATIVE VIABILITY OF THE METHOD OF CONJUGATE LINEAR EQUATIONS IN SOLVING QUADRATIC EQUATIONS: A COMPARISON WITH THE CONVENTIONAL FACTORIZATION METHOD

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A thesis in the Department of Mathematics Education, Faculty of Applied Sciences and Mathematics Education, submitted to the School of Graduate Studies in partial fulfilment
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DECLARATION

CANDIDATES' DECLARATION

I, **Samuel Alhassan**, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature………………… Date……………………….

SUPERVISORS' DECLARATION

We hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis/dissertation/project as laid down by the Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development.

Prof. Ebenezer Bonyah (Principal Supervisor)

Signature……………....... Date…………………..

Prof. Yarhands Dissou Arthur (Co-Supervisor)

Signature………………….. Date………………….

DEDICATION

This thesis is dedicated to my mother Mrs. Janaba Bakari, who taught me that hard work is the ideal guarantor of true success. It is also dedicated to my father Mr. Joseph Alhassan Batowura, who made me walk the talk of discipline in life. And finally to my lovely daughter Samreen Afeso and her mom Mariama Jimah, who stood for me in all my trying moments.

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ABBREVIATIONS

CAMFED	Campaign for Female Education
CONJUGALES	Conjugate Linear Equations
CRDD	Curriculum Research and Development Division
Df	Degree of freedom
GES	Ghana Education Service
MoE	Ministry of Education
NCTM	The National Council of Teachers of Mathematics
SHS	Senior High School
SPSS	SPSS
Std. Deviation	Standard deviation
t	t-test calculated value
UNGEI	United Nations Girls' Education Initiative
WAEC	West Africa Examination Council
WASSCE	West Africa Senior Secondary Certificate Examination

ABSTRACT

This study sought to find out the viability of the method of conjugate linear equations in solving quadratic equations among senior high school (SHS) one students, by comparing the effectiveness of teaching quadratic equations using the conjugate linear equations method and the traditional factorization method. A quasi-experimental design involving two senior high school form one classes with twenty-eight (n=28) students for the control and twenty-nine (n=29) students for the treatment group and a total of 57 students was conducted. Pre-test, post-test, and retention test items with respective reliability coefficients; 0.80, 0.87 and 0.85 and their test scores were collected and analyzed using independent samples t-tests. At a significant alpha level of 0.05, the conjugate linear equations method showed significant post-test score differences in favour of the conjugales, with gender difference not significantly different. The method of conjugate linear equations also demonstrated higher retention potential. Although no significant time differences emerged between the two groups, the study highlights the conjugate linear equations method's viability and potential benefits. Recommendations include adoption of the method of *conjugales*, training of teachers on the pedagogy of the method of conjugales, further research, long-term assessment, and resource provision.

CHAPTER ONE

INTRODUCTION

1.0 Overview

Chapter one laid the foundation for this study by meticulously exploring the background to establish a comprehensive context. The narrative progressed from an elucidation of the problem statement, wherein the problem at hand as well as the corresponding research gap were explicitly delineated. Subsequently, the chapter unfolded to articulate the purpose of the study, specific objectives, research hypotheses, and key assumptions that underpinned the research. A critical examination of the significance of the study ensued, providing insights into its potential impact and contributions. The chapter further navigated through the delimitations and limitations inherent in the research. Finally, it culminated with a succinct overview of the organizational structure that governed the subsequent sections of the study.

1.1 Background of Study

Mathematics plays an important role in education since its impact on all areas of life cannot be ignored. A disciplined approach towards orderliness in daily life is greatly influenced by this subject's vast applications. In terms of national development, achieving top notch scientific and technological advancements is based on a solid foundation of mathematics (Aikpitanyi, 2017). Moreover, the subject's relevance is not limited to science and technology as it also impacts everyday activities such as markets, transportation or business transactions (Dantzig, 1947). While being of utmost importance to comprehend economics or business in general. History affirms that societies who gave preference to mathematical knowledge made remarkable progress - effectively drawing mathematics into the realm of civilization itself. It's unfortunate then that most students complain about its difficulty. Students often report difficulties with their mathematics instructors who they feel are unable to effectively convey complex concepts. To remedy this situation and improve academic outcomes, educators advocate for the implementation of more efficient teaching techniques in mathematics. One such approach is using the M_R zero-one matrix relation method when solving for the transitive closure of relations instead of relying solely on Warshall's Algorithm. Similar could be said about the *Pascal's triangle* against the *binomial formula*.

Mathematics offers many tools for solving real-world problems - one of which is polynomial equations. Among these formulae sits the general quadratic equation - a widely used model owing its relevance across an array of disciplines - including science, commerce, physics and engineering (Burkhardt, 2006). The peculiar thing about quadratic equations is that their significance transcends centuries. In fact, ancient civilizations like Egyptians or Babylonians applied algebraic principles such as these models on everyday tasks such as finding maximum or minimum values based on profits/losses calculations.

With the high reaching importance of quadratics, the performance of male and female students have also been observed over the years. In studying students' errors in the area of quadratics, Johnson, (2014) found out that female students had more challenges understanding and solving quadratic problems as compared to their male colleagues. Bilson, (2017) came out with similar findings on his study that compared different methods of solving quadratic equations.

All things considered; mathematics stands out as an important field that has far reaching implications on both individual growth and societal development. Its significance lies in providing a solid foundation for scientific and technological knowledge; without which any country's socio economic progress would be stunted. Additionally, mathematical concepts are closely intertwined with several other areas of study such as physical sciences, technology, economics and business Khattab (2018). Despite its importance, teaching mathematical concepts presents challenges. These challenges could be remedied with novel and innovative methods different from how they are traditionally approached by educators. One such approach to finding innovative strategies when dealing with subjects like polynomial equations is the conjugate linear equations method. The rationale for this study is therefore to probe into the viability of the method of conjugate linear equations, the success of which could positively impact students' academic performance as well as foster an appreciation of quadratics and mathematics as a subject.

1.2 Statement of Problem

Performance in quadratics amongst students has become an increasing cause for concern over recent years. According to WASSCE’s Chief Examiner's reports and student complaints alike, there appears to be a widespread failure to grasp interest or understanding around this topic. A continued dearth in offering adequate examination questions regarding quadratics further compounds this scenario - despite being an integral part of applied mathematics (Kurz, 2019).

Research into this area provides key insights into potential reasons behind these less-than-desirable outcomes: namely issues surrounding pedagogy styles; insufficient content knowledge; limitations on available learning timeframes; alongside limiting "entry level" skills afforded by certain individuals (Makonye et al., 2016). For example - extracting required information from word problems or diagram representations pose particular obstacles for many learners who then have trouble applying traditional factorization methodologies towards solving resulting equations.

Additional approaches like general quadratic formulas or method of completing the square are even less feasible, as they are comparatively more complicated and lack the requisite resources in core mathematics syllabi (Saleh, 2017).

Overcoming numerous obstacles has become the driving force behind research into alternative ways of resolving quadratic equations altogether now. One method that surfaced when Gyening J discovered it in 1982 involves applying conjugate linear equations instead. Despite being a more recent development than other methods, it holds potential. Once people understand better how it works with fewer existing cognitive prerequisites, reduced error potential and time activity. Despite these positives there's still research required before we can be certain enough about its reliability and overall success rates.

In spite of prior investigations comparing the efficacy of conjugates versus factorization when it comes to resolving quadratic issues, as studied by Smith (1999), Smith, Johnson and Williams (2018), and Lee and Kim (2020), there’s currently little data regarding their applicability within Savannah Region ─ given its only recently been established as a separate region. As such it is imperative we conduct proper empirical research into these two problem-solving methodologies' practical usefulness within the region. While previous studies focused predominantly on error rates and the nature of mistakes made when solving quadratic problems utilizing descriptive statistics, this study takes a more quantitative approach - to provide a more rigorous and comprehensive analysis of the efficacy of conjugates versus factorization when tackling quadratics. Thus, this study aims to address this empirical gap by providing a quantitative analysis of the effectiveness of these methods in solving quadratic equations in the Savannah Region.

1.3 Purpose of the Study

The purpose of the study was to investigate the relative viability of the method of conjugate linear equations in finding solutions to quadratic equations, compare this method with the factorization method concerning gender ability, retention potential, and time efficacy, and determine whether the introduction of the method of conjugates in teaching would contribute valuable insights into enhancing pedagogical approaches for the instruction of quadratic equations.

1.4 Specific Objectives

The study was guided by the following objectives:

1. To determine whether there is significant difference in performance of students of different groups during the posttest.
2. To determine whether there is statistically significant difference in performance of students of different gender (*this is to establish whether the age old poor performance of females in solving quadratic equations as found out by* Anokye-Poku (2020), *has anything to do with the conventional method).*
3. To determine the difference in retention potential of students for the method of factorization against the method of conjugales.
4. To determine the average difference in time for which students solve questions under quadratic equations using both approaches.

1.5 Hypothesis

H_{01}: There is no significant difference in performance of students of different groups during the posttest.

H_{02}: There is no statistically significant difference in performance of students of different genders.

H_{03}: There is no significant difference in retention potential of students for the method of factorization against the method of conjugales.

H_{04}: There is no average difference in time for which students solve questions under quadratic equations using both approaches.

1.6 Key Assumptions

The students have the same level of Mathematical abilities.

They are also equally motivated in their study of Mathematics.

The School environment will not affect the teaching process.

The students have reasonable knowledge on:

Integers

Factorization of algebraic expressions

Solutions of linear equations

Solutions of simultaneous linear equations

Computation of square roots of numbers

Change of subject

1.7 Significance of the Study

Various researches conducted (Banks, Butler and Wren 1970; Autrey and Austin 1979; Kinney and Purdy, 1957; Baffour-Wuah, 1977) have revealed that the most widely used method of solving quadratic equations (method of factorization) in our secondary schools is bedeviled with a lot of problems. Wayne (2004), after examining methods of factorizing quadratic expressions noted that all is not well with the teaching and learning of the factorization of quadratic polynomials and that there is the urgent need to remedy the situation. This supports the assertion that the method of factorization is posing a challenge to students in solving quadratic equations.

Students report having trouble remembering and understanding factorization techniques. Additionally, the West Africa Examination Council (WAEC) has been troubled by the enduring inaccuracies in the solution of quadratic equations for some time. All of these are cues for math educators to think creatively about how to address the issues that affect teachers, students, and the WAEC. The significance of this study is seen in light of this. If the study's hypotheses are proven correct, then further investigation will be necessary to either confirm or refute the results. Additionally, it will force curriculum designers to reevaluate the grade placement of the SHS core mathematics syllabus's topic "solution of quadratic equations" for its best use in the study of other subjects. Finally, it will demonstrate first-hand whether male and female students adopting this method have the same potential in academic achievement. This might be helpful to organizations like CAMFED, and UNGEI which advocates for female gender education.

1.9 Delimitation

The study covers only the West Gonja Municipality in the Savannah Region with specific interest in the government assisted SHSs. The methods considered were the ***conjugales*** and ***factorization*** because factorization is the method mostly specified by WAEC to solve questions and one of the only two stated in the syllabus. *Graphical method* and *Completing the square* will not be considered in the study. Using the government assisted SHSs is necessitated as they have relatively the same academic facilities and conditions so as to be able generalize the findings to the entire population.

1.10 Limitations of the Study

Research, especially with human subjects, is susceptible to limitations. The use of intact classes does not allow for randomization and the purposive allocation of methods to particular classes affects the research. The various assumptions do not really hold in psychology since individual differences and other limiting factors exist, hence the results may be influenced.

1.11 Organization of the Study

The rest of the research is structured as follows: The review of relevant literature on the theoretical framework, empirical framework, and conceptual frameworks of the study was covered in Chapter two. The third chapter outlines the methodology for the study and focuses on the demographic, sample, and sampling techniques as well as the research design. Instruments, methods of data collection, and statistical techniques for data analysis were also covered in chapter three. The study's results and analysis were given in chapter four. The study's fifth and final chapter provided an overview of the research and came to conclusions based on its major results. It outlined the study's recommendations and proposed areas for more research.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.0 Overview

At various educational levels, quadratic equations are taught since they are a crucial component of mathematics. High school students receive instruction on this topic and acquire a range of problem-solving techniques with variant methods. Nevertheless, it is vital to investigate solution methods that maximize student understanding and memory retention. This literature review investigates different methodologies for solving quadratic equations specifically focusing on traditional factorization versus inventive conjugales approaches along with available research supporting their efficacy in high school contexts. This section considered the theoretical framework, conceptual review, empirical review and conceptual framework of the study.

2.1 Theoretical Framework of the Study

2.1.0 Action Process Object Schema (APOS) Theory

This study followed the paradigm of the **Action-Process-Object-Schema (APOS)** theory. According to the APOS theory, teaching and learning mathematics should focus on assisting students in using their existing mental structures and in creating new, stronger ones for handling increasingly complex mathematical concepts (Arnon 2014). According to the APOS theory proposed by Dubinsky and McDonald in 2001, students approach perceived mathematical problems by creating actions, internalizing processes, acting on object transformations, and creating schemas in order to appraise the circumstances and find solutions.

In the APOS theory, reflective abstraction consists of the mental operations of interiorization, encapsulation, coordination, observation, de-encapsulation, and thematization. This theory is a continuation of Piaget's idea of reflective abstraction adapted to advanced mathematical reasoning (Nisa, 2020).

APOS Theory was developed in an effort to comprehend reflective abstraction, a term that Piaget first used to describe how children develop logical thought and later expanded upon to include more complex mathematical concepts (Dubinsky, 1991).

APOS theory, which stands for Action, Process, Object, and Pattern, is a theoretical framework in mathematics education designed to explain how students learn mathematical concepts. The theory proposes that learners go through four stages in acquiring mathematical knowledge:

The action phase, where learners develop an intuitive understanding of the concept through physical manipulation or experience.

The process phase in which learners begin to recognize patterns and regularities in the actions they perform and develop procedural knowledge.

The object phase, where learners begin to abstract the concept and develop a symbolic representation of it.

The Schema phase, where learners integrate their knowledge of the concept into a more organized and coherent framework.

According to APOS theory, effective teaching should aim to guide learners through these stages and help them make the necessary transitions from one stage to the next.

2.2 Some Mathematical Research Papers that used the APOS theory:

APOS theory has been widely used as a framework for analyzing students' understanding and reasoning in mathematics classrooms. One of the most notable works in this area is the study by Frank (2013), who studied the feasibility of teaching quadratic equations in senior high school one. In his study, he compared different methods of solving quadratic equations using SHS1 students in a government assisted senior high school of mixed genders. He considered the equivalent simultaneous linear equations method, the factorization method, completing the square, direct trial analysis and the d-h theorem. His study adopted the APOS theory where; the action stage refers to the stage where the learner considered the initiative to learn the various methods of solving quadratic equations. The process stage was the stage where the students were practically learning the various methods. The object stage was the stage where students understood the various methods and the schema stage was the stage where students were able to cognitively compare the methods of solving quadratic equations. He found that the APOS theory was practical in establishing learners learning abilities for comparative methods.

A study by Dubinsky, Dautermann, and Zazkis (1994) also examined the development of students' function concepts using APOS theory. They found that students progressed through different phases of the APOS framework (i.e., action, process, object, and schema) as they developed their understanding of functions.

Another study by Radford and Guzmán's (2003) delved into the significance of APOS theory in deciphering students' struggles with numeracy. Their research highlighted four primary APOS frameworks - graphical, numerical, algebraic, and analytical - utilized by students when learning calculus. As per their findings, educators can employ APOS theory to pinpoint and tackle student misconceptions and hurdles in mastering numeracy.

As well as being used as a theoretical basis the APOS theory has been adapted for practical purposes in educational contexts. Lim and Presmegs' (2010) research examined how instructional materials could be designed utilizing this framework to enhance teaching methods for algebraic thinking. Through implementing this approach, they discovered that activities could be created which supported student advancement through the different stages of the APOS framework.

Besides this, investigators have used the APOS theory to examine how individuals comprehend geometric concepts successfully. Mariottis' (2000) research focused on exploring students' development regarding triangle comprehension using this framework for analysis. The outcome demonstrated that pupils went from having an intuitive idea about triangles based on visual cues towards a more standardized form after going through several phases according to this model's protocol.

Overall, these investigations revealed how useful it is when attempting to understand learners' misconceptions or challenges regarding mathematical concepts by providing insights into their development progressions throughout academic pursuits.

2.3 Effective Strategies for Teaching and Learning Mathematics: Insights from Learning Theories and Educational Research

When it comes to mastering Mathematics, a good teacher must be well-versed in both subject matter expertise and effective teaching methods that take into account psychology. In fact, teachers are essential pillars in both imparting knowledge and helping students acquire it. According to

Skemp (1986), successful Math learners draw from a blend of intelligent strategies and habitual techniques. Habitual learners depend heavily on teachers for problem-solving guidance; conversely, for intelligent learners, self-regulated rule-finding supports greater self-assurance when tackling novel problems - an approach recommended at the Senior High School level. Vygotsky's developmental theory provides perspective on how students form concepts over time while Skemp (1976) categorizes instrumental understanding as merely memorizing rules versus relational understanding that emphasizes grasp of underlying rationale. To Hiebert and Carpenter (1992), grasping a mathematical concept entails forming relational connections between it and existing networks within our mental framework. The coherence achieved through these links is greater with stronger or more numerous connections made (p. 67). They also noted that understanding refers to how we represent and structure information in our minds as it pertains to mathematical ideas (1992, p. 67).

As students enhance their level of comprehension over time, they start connecting different bits of information together such as ideas, words graphs etc., creating links between each other and other already developed concepts. The links between these smaller chunks of information hold great significance for forming a vivid conceptualization. Novice learners possess concept images that constitute disorderly arrangements of multiple 'knowledge elements', as theorized by diSessa (1993). These distinct unconnected pieces can clash with each other which is why cues need to be applied by the learners to establish a connection among them within different contexts. The term 'knowledge elements' according to Clark (2006) encompass varied facets including facts, experiences intuitive conceptions along with some mental models concepts at varying levels of expertise.

Constructivist learning theory posits that learners generate fresh insights and knowledge by leveraging their existing beliefs and understanding. In the context of quadratic equations and functions pre-existing experience influences student comprehension significantly (Bransford, 2000). Such prior knowledge can act as either cognitive enablers or obstacles towards problem solving efficacy as well as overall acquisition of new information (Bishop et al., 2013). Brownell cited in Patricia (1935) recommended a teaching approach that focused on cultivating 'meaningful learning' experiences for students grappling with mathematical concepts. For instance, effectively delving into ideas such as arithmetic entails understanding the underlying connection between various processes within this domain. Consequently, when addressing problems entailing quadratic equations across diverse fields of study, students must grapple with grasping these critical concepts needed to successfully tackle these problems (Holt, 1970). Holt further argued that merely teaching formulae or rules doesn't suffice−there’s a need to focus on deeper comprehension of mathematical principles. The ability to utilize gathered knowledge efficiently depends on ones’ understanding of the relevant facts and competencies which must be imparted through education.

2.4 Conventional Methods of Solving Quadratic Equations

Quadratic equations are polynomial equations of the second degree, which falls under the realm of algebra and are a crucial subject matter. To solve these types of equations, there exist various techniques including but not limited to completing the square, quadratic formula, and graphical methods, as mentioned by Hirsch (2010).

2.4.1. Completing the Square

Completing the square is an approach used for finding the roots of quadratic equations that works by manipulating those equations until they fit into "perfect" squares conveniently solvable using basic algebraic techniques.

In any standard quadratic equation represented as $ax^2 + bx + c = 0$ where all three quantities are constants; completing this sequence requires adding or subtracting specific values until you create an ideal quadratic framework that can easily be solved for its roots.

Dividing both parts of your original problem by 'a' is typically one's first step in performing such actions on their algebraic expression before proceeding further towards creating said "perfect" squares formulation.

Next up here comes incorporating our prerequisite constants into our work by adding or subtracting quantities equal to half squared b divided by "a". Finally taking each side's root reveals $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ – one of available quadratic formulas we can use for solving quadratic equations (Dugopolski, 2012).

2.4.2. Quadratic Formula

In particular $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ represents what's named as "The Quadratic Formula." This effective methodology enables us to determine all possible roots of any given quadratic equation, irrespective of the methodology we choose to use. This formula can furthermore be derived using completing the square method in which case its versatility becomes even more apparent. Its ability to handle all kinds of quadratic equations no matter how complex they appear is a testament to its effectiveness and popularity (Larson and Edwards 2013).

2.4.3. Graphical Methods

Solving quadratic equations can be accomplished through different techniques - one being graphical methods whereby an equation is plotted on a graph and its intersection points with the *x* axis are determined by equating y to zero. In reference to Blumans (2011) findings, this approach can provide approximate solutions for challenging quadratics involving complex numbers or irrationals. Alongside other conventional techniques like completing the square and applying the quadratic formula graphical methods remain fundamental operations in solving these types of equations; having knowledge about each approach's pros and cons enables one to choose the most suitable method for a given problem.

2.5 Non-Conventional methods of solving quadratic equations

In mathematics realms where timely and accurate resolution of quadratic equations is crucial to problem-solving scenarios, analysts typically resort to traditional techniques like factoring or tapping into established formulas (the most popular being Quadratic Formula), which have proven their effectiveness over time. However adaptive thinking suggests exploring other options as well which might come in handy when tackling a complex problem containing this element. Some examples of innovative solutions include Equivalent Linear Simultaneous Equations; Direct Trial Analysis; or considering an unorthodox approach in form of d−h Theorem.

2.5.1 Equivalent Linear Simultaneous Equations:

Solving quadratic equation challenges requires not just one way but may take various alternate approaches like what the Equivalent Linear Simultaneous Equations offer. This particular technique transforms individual quadratics into a series of linked or simultaneous linear-equation systems through proper manipulation and coefficient equalization procedures based on corresponding exponents in every variable occurrence in each respective power term set from all expressed polynomial function elements used in formulae and applications to solve math problems. While solving this linear system may need additional algebraic calculations, it helps us obtain a more innovative perspective on quadratic equation solutions that can further our understanding and expand our knowledge (Smith, 1999).

2.5.2 Direct Trial Analysis

Direct Trial Analysis presents an unconventional means towards solving quadratic equations by suggesting direct substitution of possible numerical variables into satisfactory formulas; thereby enabling us to acquire apt solutions through deliberate trial and error techniques implemented via systematic substitution of variable values for x until acceptable outcomes are achieved. The broader aspects surrounding Direct Trial Analysis require one's tolerance towards extensive experimentation which can range from being quite simple to very complex depending on the underlying constraints involved in a given equation. The significance of this approach comes into play when dealing with quadratic equations that display recurring patterns or specific limitations (Brown, 2007).

2.5.3 d−h Theorem:

The concept known as the d−h Theorem emerged from its founders' ingenuity as they endeavored to solve quadratic equations through unconventional means. With these theorems help through a transformation involving two variables - d and h - these complex problems can now be streamlined into simpler versions without losing any vital information required for accurate solutions. Through trial and error with different substitutions of value for d and h the transformed equation can be simplified down to the answer. This innovative approach proves useful in select conditions involving quadratic equations thereby offering an alternative outlook from traditional methods (Jones et al., 2012).

The best mathematicians are those who strive to continuously innovate and expand their knowledge base by exploring new approaches to tackle challenging problems - even when solving familiar formulas like quadratic equations. Instead of relying solely on traditional techniques learned in classrooms or textbooks, try incorporating less-known yet valuable methods such as Equivalent Linear Simultaneous Equations, Direct Trial Analysis, and the d−h Theorem into your repertoire. These unconventional routes may lead to unexpected insights that can transform how you approach future mathematical challenges with more creativity and confidence.

2.6 Historical consideration of the need for comparing methods

Among scholars, a compelling subject has been, comparing how math is taught using various methodologies. It's common knowledge that one problem can often be solved through a multitude of ways depending on who is attempting it. The ancient Chinese text Nine Chapters on the Mathematical Art was explored by Liu Hui in which he highlighted multiple techniques for handling quadratic equations around year 200 CE. Nearly fifteen centuries later (in 16th century) Italian mathematician Scipione del Ferro developed an elegant solution to algebraic cubic equations replacing previous complicated methods, cited in (Stedall, 2004).

In more recent times researchers have taken on the task of determining effective teaching methodologies specifically tailored for mathematics education. The National Council of Teachers of Mathematics (NCTM) is an educational body that urges teachers to incorporate diverse training strategies such as problem based learning and inquiry based learning in order for pupils to acquire a thorough foundation in mathematical concepts. Additionally, other scholastic inquiries involve scrutinizing particular methods like including manipulatives or technology into math lessons which may prove useful indeed. Therefore, it’s understood that the need for comparing methods with respect to teaching mathematical material was deemed relevant even back in history; furthermore, it continues playing an important role currently within mathematics education.

2.7 Feasibility of Quadratics in SHS1 and all the Methods of Solving them.

For Senior High School (SHS) students in Ghana studying mathematics, understanding quadratic equations is fundamental. Learning such content is vital as these mathematical concepts are widely used across different fields such as physics, engineering and economics among others − a fact underlined by both Al-Mamun et al. (2021) and Ayana (2016). To facilitate comprehension of these concepts among students requires the employment of effective teaching methods.

Solving quadratic equations can be done through different techniques such as factoring, completing the square or using the quadratic formula. Another option is utilizing graphical concepts to find solutions. According to research by Kurz and Brousseau (2019), every method has its unique merits and limitations with regards to precision level attained within a certain period. For educators evaluating which tactic is most suitable for SHS students’ needs based on factors such as class size or mathematics abilities within an available timeframe for instruction (Njagi, 2015).

Effective learning outcomes are closely linked with employing appropriate and feasible instructional strategies among SHS students. Teachers play a crucial role in determining which method best facilitates academic progress based on individual learner needs. Henceforth, evaluating diverse practices for teaching quadratic equations amid an SHS curriculum becomes pivotal in determining which technique suits these students best (Njagi, 2015). Therefore, adopting a systematic approach enables educators to identify optimal methods that enhance learning by concentrating on individual capabilities while ensuring efficient course delivery.

A few empirical research works carried out by various researchers have demonstrated that quadratics is feasible with the current SHS curriculum, with varying feasibility, depending on the method.

Bergsten and Engelbrechts (2005) comparative study analyzed grade nine (9) textbooks from Sweden and South Africa to ascertain how the factorization method was depicted. Their findings showed that this methodology was widely used in both countries providing students with a systematic solution to quadratic equations.

The study uncovered that the factorization method was extensively addressed, indicating its practicality for implementation in the SHS curriculum.

In 2004, Louca and Zachariades conducted a study investigating how students tackle quadratic equations with no apparent practical application. They discovered that learners trained on using completing the square exhibited deeper comprehension of these equations than those who were not taught this methodology. Therefore, introducing this technique into the SHS curriculum might be beneficial.

Chicks' investigation (2004) centered on the historical context surrounding quadratic equations and their development over time. An important turning point happened when mathematicians abandoned ad hoc methods in favor of adopting the quadratic formula. As its effectiveness in solving any kind of quadratic equation cannot be denied, incorporating this method into SHS curriculum is highly relevant.

Quadratic equations can also be tackled through graphical methods alongside algebraic ones. In his examination of historical and pedagogical perspectives on diagram usage in quadratic equation problem-solving, Pimm (1987) discovered that these representations provide students with a visual grasp on calculating the roots of quadratic equations and help them conceptualize solutions in a more complete manner.

A wealth of historical research has identified several effective methodologies for solving quadratic equations that could be incorporated into an enriched SHS mathematics education program. The factorization method, completing-the-square technique, methodologies such as calculating roots using Quadratic Formulae or Graphical solutions have all been vigilantly examined across different settings to evaluate their practicality. Incorporating these different techniques into pedagogy will prepare learners with multifaceted tools for understanding and tackling complex mathematical problems on top of traditional classroom instruction.

2.8 Factorization and Difficulty Level of Quadratic Equations

Quadratic equations can also be solved via factorization (Vaiyavutjamai & Clements, 2006). This is often related to initially teaching how to factor quadratic expressions. At the SHS level, factoring quadratic expressions is a frequent subject in mathematics instruction. Garcia and Rodriguez (2011); Liu and Chen (2010); Johnson and Smith (2014). According to research, the two most popular methods for factoring quadratic expressions are to decompose the linear term and make the quadratic unknown's coefficient to become 1 (i.e., $a = 1$ in $ax^2 + bx + c = 0$).

These two methods of factorization rely on multiplication and division abilities, which show pupils' understanding of number theory and mathematics. The first step in resolving a quadratic equation is to factor quadratic expressions. In reality, decomposing the polynomial into a factoring form is what it means to solve the quadratic problem through factorization.

Quadratic polynomial factorization has been described by some authors (Amissah 1991, Amissah 1993, Mitchelmore 1988, Wilson, 2013) as the inverse algebraic expansion of two binomials. The process of multiplying two or more linear terms in x is known as an algebraic expansion (Amissah et al. (1991); Amissah et al. (1993); Mitchelmore and Raynor 1988; Wilson 2013). On this basis, Amissah et al. (1991) pointed out that the algebraic expansion of two binomials such as $5x + 1$ and $x + 2$ is illustrated by $(5x + 1)(x + 2) = 5x^2 + 10x + x + 2 = 5x^2 + 11x + 2$.

The outcome is a quadratic function with roots that can be determined. According to these researchers, factorization is the process of employing the solutions on the right side of the equation to produce the two linear factors on the left. They define factorization of quadratic polynomials as "the process of separating a quadratic trinomial into linear factors" (p. 207; Barnett & Kearns, 1994; Ferris & Busbridge, 1973; Roberts & Stockton, 1957).

According to the link between square factoring and multiplication, factoring the difference of two squares yields two binomials, each of whose first terms is positive and each of whose second terms has a different sign. For instance, according to Robert and Stockton (1957), one must first discover the square root of m^2 before locating n^2 in order to factor $m^2 - n^2$. The results are m and n, respectively. To obtain a factor, combine the two square roots. Similarly, to obtain the second factor, subtract the square roots. The factor for the difference of two squares is therefore $m^2 - n^2 = (m + n)(m - n)$. Consider the fact that, according to Mitchelmore and Raynor (1988), "the identity above allows us to factor any expression that is the difference of two squares" (p. 139). In fact, there will be a gap for students if a math teacher uses such a textbook merely to provide activities for pupils to memorize a rule and assess their comprehension. Students frequently struggle while factoring the difference between two squares as a result of this discrepancy. For instance, Backhouse (1978) noted that "the difficulty is often of the form a^2" in statements like a^2 - 4, 1 - b^2, and $9q^2$ - 16. As a result, SHS students have trouble understanding this form.

Barnett and Kearns (1994) support the method of factoring monomial common factors in incomplete quadratic factorization. For instance, the common factors are discovered first when factoring $3x^2 + 18x$. The unique quadratic polynomial was discovered to factor up to $3x(x + 6)$, and the product of the common factors of 3 and x was determined to be $3x$. The opinions put forth by Barnett and Kearns are shared by a number of authors (Amissah et al., 1991; Butler, Wren & Bank, 1970; Levis, 1961; Mitchelmore & Raynor, 1988).

This procedure is a direct implementation of the distribution legislation, according to Gyenning (1988). The fundamental idea behind "common factor suppression" is presented on page 7. According to Milli (2014), his literature study looked on how students approach quadratic function graphs and what challenges they have when learning about quadratic functions. Solving quadratic equations has been the main focus of research on students' grasp of quadratic functions. Research on how students comprehend quadratic function tables or how quadratic functions operate as shown by graphs and tables appears to be lacking. Emphasis has been made on students' troubles with quadratic functions throughout the literature, some of which may be caused by cognitive obstacles. The topics of squares as functions, creating graphs and equations of quadratic functions, and solving quadratic equations are the focal points of this literature study, which is structured around a subset of the overarching principles and fundamental notions mentioned above.

In Brazil, educators are required to instruct students on how to solve quadratic equations using the three aforementioned symbolic forms. However, since the teachers were already aware of the issues facing their students, they concluded that the general approach to applying the formula would be successful in all circumstances, which is why it is stressed in order to simplify the complexity. Although it was thought that the product of two factors could only be zero if one of them was zero, factorization was not widely advocated because teachers were aware of their students' challenges with algebraic manipulation. Overall, the teachers concentrated on applying a method that they believed would be accurate every time. Similar to before, when solving linear equations, pupils concentrated on the precise processes they utilized rather than overarching ideas. In general, even after these teachers have experienced resolving particular quadratic problems, we will discover that the majority of pupils still believe the formula to be more effective.

Little study, according to Vaiyavutjamai and Clements (2006), has been done on the cognitive difficulties that quadratic equations present for pupils. Written examinations and interviews from students in Thailand and Australia were compared as part of a study on the effect of instruction on students' understanding of quadratic equations. Despite significant disputes, they discovered that the students were still able to arrive at the right answers. In addition, Brown (1995) pointed out that pupils might not meaningfully understand equations containing a "±" sign like $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$,, even if they were able to write the solution.

Using pencil and paper and graphing calculators, Chen (1987) devised a method for teaching quadratic equations. Students were instructed to graph equation-related functions and come up with several solutions. They were capable of doing a variety of discrete tasks, but they lacked the mobility to switch between presentations with ease.

According to Llinares (2015), many high school students find solving quadratic equations to be one of the most difficult intellectual components of the high school curriculum. Many pupils, he noted, struggle to recall crucial multiplication skills, which has a direct impact on their capacity to drill squares. Factoring simple quadratic equations (such as the former. $ax^2+bx+c=0$ a, b, c R and a1) becomes nearly hard since the factoring strategy for solving quadratic equations demands pupils to be able to locate factors fast. In addition, when given quadratic equations in non-standard forms, students find it extremely challenging to factor them. For instance, when the equation is not given in a conventional form, pupils have trouble factoring $x^2+7x+2 = 5x+5$ (Llinares, 2015). Similarly, Johnson (2023) discovered that pupils had difficulty using factorization strategies to solve quadratic equations. They noted that when the leading coefficient or constant squared has more pairs of potential factors, factoring squares might become significantly more challenging for children.

The paradigm provided by Skemp's (1976) definition of instrumental and relational understanding can be utilized to explore the challenges pupils have when factoring quadratic equations. The relational knowledge enables students to quickly apply these rules to various structures, whereas the instrumental understanding of factoring quadratic equations into one unknown necessitates memorization of the rules for the equations presented in each structure (Sfard, 2000). This means that students can apply their knowledge of the rules (and formulae) that worked and the reasons why they worked in one context to another when they have a relational understanding (Skemp, 2002). According to Santos (2016), pupils are able to conceptualize quadratic equations in the same way as they compute them. They could not understand the concepts involved since they tend to concentrate primarily on the symbols needed to carry out operations. According to Vaiyavutjamai and Clements (2006), students' struggles with quadratic equations are caused by a lack of relational and instrumental knowledge of the underlying mathematics. They discovered a number of misconceptions concerning variables that made it challenging to comprehend quadratic equations. For instance, some students believe that the first x stands for one value and the second x stands for a different value in the factored equation $(x - 3)(x - 5) = 0$. Before and after a series of eleven sessions on quadratic equations, the authors examined the work of 231 students at government schools close to Chiang Mai, Thailand, as well as 34 interview transcripts. The interviews revealed that students said things such as "The solutions are 3 and 5 because (3 – 3)(5 – 5) = 0 * 0 which is 0". Incorrect reasoning demonstrated by students can teach us just as much as correct reasoning. For example, in solving for x when given (x – 3)(x – 5), one student arrived at two solutions: both x = 3 and x = 5 must be true simultaneously according to their logic. Unfortunately for them, a function like (x – 3)(x –5) cannot output two values of *y* for any single input value of *x*; in other words, these solutions are mutually exclusive! Research by Özdemir, (2017) who studied over one hundred Turkish high school students written explanations on comparable problems found similar errors indicating poor understanding of factorization techniques.

In a broader sense, quadratics are the first family of functions that students come across that may have one, two, or no real roots, and they may challenge students' comprehensions of how the variable x behaves across the domain. Developing proficiency in quadratics involves gaining insight into how functions produce varying outputs as *x* assumes different values within the set domain. Additionally, this highlights the fact that there can be multiple instances where a function's value equals zero for distinct values of *x*. Such concepts align with understanding functions at large and recognizing that variables depict fluctuating values rather than fixed answers to problems. According to Thompson (2019), students don't comprehend that the solutions to equations with the form $x^2 = \sqrt{a}$ have both positive *a* and negative *a* as solutions. When he asked pupils to solve problems like $x^2 = 100$, he observed that they frequently provided the answer $x = 10$, overlooking the alternative, $x = -10$. In addition to suggesting that students might not completely comprehend the meaning of the plus or minus symbol ($\pm$) in the quadratic formula, Thompson also indicated that students might anticipate equations to have a single solution.

The pedagogy of Mathematics has faced a longstanding challenge in teaching secondary school students how to factorize quadratic trinomials. This problem has been acknowledged globally by academic researchers such as Erisman (1988), Skemp (1966) and Wilson (2013) The issue stems from the fact that mathematical facts are not taught in a cohesive manner, which burdens the student's memory unnecessarily. Gyening (1988) aptly noted this observation.

Upon initial examination, traditional modes employed in teaching Mathematics can come across as only comprising individualized techniques for resolving mundane problems rather than incorporating any fundamental overarching practices meant for broader applications. However, one must appreciate how Mathematics stretches beyond this limited perception by consisting instead of complex systems featuring harmonious structures connecting various aspects comprehensively within each system's domain.

The traditional approach to conveying mathematical facts does not accurately represent them which can impede proper comprehension and utilization of these principles in solving quadratic equations through trinomial factorization. Ferris & Bushbridge (1973) and Mitchelmore & Raynor (1988) share similar opinions on how these methods are taught at the secondary level. Factorizing trinomials becomes more complicated when there are multiple coefficients for x^2 or constant terms with numerous factors making it a tiresome task for students. As Ferris & Busbridge (1973) noted, "factorization is more difficult because of the many ways of pairing numerical factors" (p. 81). These limitations found in learning how to factorize quadratic trinomials highlight existing shortcomings within pedagogical approaches used for this topic.

Wilson (2013) also revealed that many teachers find solving quadratic inequalities quite challenging for average sixth form Math students; around 60% felt this way. The reason behind this difficulty stems from the fact that factorization is deemed the most convenient method - something which many scholars agree with including Davis and Williams (1995). However, it can also be seen as a chore to complete - an opinion shared by Erisman (1986) and Savage (1989). In order to solve quadratic inequalities or equalities its essential for students to apply this method. Moreover, another report from Sfard indicated that roughly 55% of teachers didn't cover or discuss binomial expansion − an essential skill required for factorization − with their students. Consequently, learners were unaware of how multiplying two linear factors relates with factorizing quadratic trinomials based on Robert (1956). As a result, sixth form Mathematics students find themselves struggling. However, it’s worth noting that this report wasn't triangulated and therefore student perspectives weren't taken into account.

For this reason, researchers including Erisman (1986), Gyening (1998), Sawyer (1958), and Steinmetz & Cunningham (1993) have expressed that the approach is lengthy, time-consuming, boring, and a challenging mathematical undertaking for new students.

The procedure of factorizing trinomials has been found to carry some imperfections that have resulted in some secondary school students erring while trying to solve quadratic equations with this method. Robert notes such mistakes in the solution he presents for a particular quadratic equation:

$2a^2 – 10a = –42$

$2a(a – 5) = –42$

Either $2a = – 42$ or $a – 10 = – 42$

Giving $a = –21$ or $a = –32$

Robert & Stockton (1956) reported, “Both results are wrong” (p.254).

An issue plaguing certain students stems from a misunderstanding regarding the principle of divisor of zero - also called the zero principle - and its implications. This concept dictates that when mn equals zero then either *m* or *n* (or both) must be equal to zero as well. Unfortunately, some involved parties have mistakenly associated −42 with being identical to zero instead of accurately recognizing it as an erroneous assumption. Moreover, it should be noted that Lawson’s (1992) research study discovered that science students bore the responsibility for all incorrect responses when trying to solve for roots of quadratic equations. For example, an erroneous answer was reported when attempting to solve $2t^2 – 3t – 5 = 0$:

$2t – 3 = 5/t$

$t(2t – 3) = 5$

$t = 3t -\sqrt{51}$ (after a page of working) (p.863).

Students were either baffled to utilize the method of factorization or the generic quadratic formula to answer the problem, which is why their response was incorrect.

It became evident from their approach that they were unable to apply either method correctly leading them towards an incorrect answer. As Lawson (1992) argue, there exists a significant disparity for many students between what is taught as "Mathematics" versus what is taught as "Science" since language used is different along with various teaching methodologies and solutions leading these learners bewildered or lost completely. To bridge this gap effectively, one needs specialised mathematics instructors instead of having science educators delivering mathematics lectures which creates confusion among learners further highlighted by Johnson's assessment (1988), stating how many students outside mathematics are keen on studying the subject since they require it frequently as a tool yet fail due to weak fundamental knowledge.

In summary findings indicate that there are challenges for students as they delve into the realm of quadratics stemming from the non-one-to-one aspect of quadratic functions. Specifically, issues arise around comprehending how variables operate and incomplete grasp on equations having more than one solution.

2.9 Historical Review of Solutions to Quadratic Equations

Let's examine the development of the quadratic equation in the context of mathematical history as well as our argument. There is no other option, but we must nonetheless make an effort to understand how our perspective differs from that of mathematicians from centuries past. Although it can be challenging to comprehend the challenges of the past due to the way mathematics is taught today, the great brilliant mathematical discoveries of this era frequently appear as singular bursts of brilliant insight. The majority of the time, these discoveries are the result of years of labor by numerous mathematicians, many of whom are novices.

No branch of mathematics has been wholly established by the work of a single person, as Burton (1999) has noted. For instance, it is simple to settle the debate over who discovered infinitesimal calculus first, Newton or Leibniz. Nor because Barrow, Newton's teacher, undoubtedly taught him calculus (Burton, 1999). Naturally, I am not claiming that Barrow discovered calculus; rather, I am merely pointing out that calculus has its roots in a lengthy period of development that began with Greek mathematics. According to Aleksandove, Kolmogorove, and Lavrentiev (1956), the quadratic equation was solved in the early days of civilization. Among the earliest civilizations to study quadratic equations were the Egyptians, Chinese, and Babylonians (Boyer, 1968; Eves, 1964).

Mathematical studies have demonstrated that the exploration of quadratic equations dates back to ancient times whereby Indian mathematicians were some of the earliest scholars to delve into this area around 500BC. Similarly, Greek scholars made significant contributions during Euclids era (Burton, 1999). Surprisingly attempts to develop a more generalized formula for solving quadratic equations relied heavily on geometrical and trigonometrical principles (Smith, 1951). Famous Greek mathematicians such as Pythagoras and Euclid used rigidly geometric approaches in developing solutions for these types of equations while seeking universality (Smith, 1951). In observing discrepancies between the area of squares and their corresponding side lengths known colloquially as square roots Pythagoras noted that these ratios were not always integers. Yet he only entertained rational relationships among these proportions while disregarding others (Smith, 1951).

Instead of performing the operations correctly, this method operates just like the multiplication tables we were taught to remember. So the engineer would head to his workstation and identify the best design if someone with land for a farm desired peace of mind. The engineers lacked the time to calculate all the forms and sides necessary to build a table on their own. Instead, a copy of the primary lookup database was used. Users had little understanding of mathematics, therefore they had no idea if the tools they were using made sense or not (Smith, 1951).

The Egyptian approach appeared to be preferable because it was effective and offered a more comprehensive answer without the need of arrays. The Babylonians then arrived. The fact that Babylonian mathematics employed a number system that is quite similar to the one we use today gave it a significant edge over Egyptian mathematics (Eves, 1964). By 1000 B.C. BC could always double-check the values in their tables because addition and multiplication were considerably easier in this approach (Eves, 1964). They had developed a more comprehensive way for resolving broad area issues termed "Complete Square" around the year 400 BC. (Eves, 1964).

According to Boyer (1968) and Eves (1964), the ancient Babylonians were able to solve issues that were analogous to figuring out an equation's roots. Numerous clay tablets show that the Babylonians were familiar with our method for resolving quadratic problems as early as 2000 B.C. (Burton, 1999). Despite their lack of knowledge of the equations, they followed a process that was comparable to the quadratic formula. However, the concept of the quadratic equation, which is now a crucial tool changing the world, was the result of their work.

In their approach to resolving quadratic problems, they also showed hints of the complete squares method (Boyer, 1968). However, because they typically involved lengths, all of the Babylonian puzzles had solutions that had positive magnitudes. According to studies, the Chinese were working on polynomial equations as early as 100 BC. (Boyer, 1968) AD. While the Chinese, like the Egyptians, lacked a numerical system, the widespread use of the abacus made verifying straightforward mathematical operations surprisingly simple (Boyer, 1968).

According to history, both the Greeks of the Euclid era and the Indians had studied quadratic equations in mathematics as early as 500 BC (Burton, 1999). The search for an all-inclusive formula to solve quadratic equations has been an endeavor since centuries ago. Some of the eminent mathematicians of antiquity like Pythagoras and Euclid attempted to address this issue by employing pure geometric methods. These efforts which date back approximately over two millennia ago bore fruit with the discovery of a general procedure for resolving quadratic equation problems (Smith, 1951). Pythagoras observed that the square root, which represents the relationship between the area of a square and the length of each side, is not always an integer, but he insisted that proportions must be rational (Smith, 1951).

Mathematicians were assigned tasks in the 15th century that required them to determine the roots of polynomials in order to solve them. This caused early mathematicians like Neil Henrik Abel (1802–1829), Cardan A. Magna (1430–1540), and Ludovico Ferrari (1522–1556) to spend restless nights trying to solve such issues (Burton, 1999).

At the start of the 19th century numerous mathematicians became engrossed in tackling algebraic equations (Alesandove et al., 1956). The crux of their dilemma lay in uncovering ways to solve nth degree polynomial equations featuring only one variable. In all probability their quest was propelled by multiple factors - not least amongst which were increasing demands from diverse fields such as mathematics and applied sciences. Polynomials of the form:

$a_n x^n + a_{n-1}x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots\ldots\ldots + a_1x + a_0 x^0 = 0$, where $a_n, a_{n-1}, a_{n-2}, a_1, a_0 \in R$ are the coefficient of $x^n, x^{n-1}, x^{n-2}, \ldots\ldots x, x^0$ and subsequently recognized (Mathew, 1973).

Thomas Harriot proposed writing any polynomial as an equation with a right side of zero in 1621 (Dantzig, 1947). The factor theorem, which asserts that if an is the root of an algebraic equation in x, then x - an is the coefficient of the corresponding polynomial, was developed by Harriot as a result of this clever notion (Dantzig, 1947). In his important works on mathematics, Thomas Harriot (1560-1631) employed the factorization approach to resolve quadratic equations (Sastry, 1988). One of the traditional techniques still taught in schools today for solving quadratic equations is the Harriot method. Descartes is also credited with having written comprehensive instructions on how to use geometric algebra to solve quadratic problems (Sastry, 1988).

A quadratic function in mathematics is a polynomial function with the formula $f(x)=ax^2+bx+c$, where an is a non-zero integer. The Latin word "quadratus" (which meaning square) is where its name originates. In fact, functions show up while calculating the area of squares.

It was also mentioned by Haag and Weisstern (1959) and Budnick (1985) that an equation of degree *n* in *x* is referred to as a polynomial. When the polynomial is devoid of its grouping symbols, its degree is the biggest exponent of *x* that results. Quadratic refers to a quadratic polynomial. An exact definition of a quadratic equation is given by Briton and Bello (1979) on page 303: "a quadratic set whose standard form is $ax^2 + bx + c = 0$, where *a*, *b* and *c* are real numbers and $a \neq 0$." A second order polynomial equation is what the quadratic equation is as a result. The word "solution" is the root term (see Miller 1957, p. 175).

In Egypt's Middle Kingdom (about 2160–1700 BC), the Berlin papyrus has the earliest known example of a quadratic equation being solved.

In Smiths account from 1953 we learn that the Hindus had already discovered a workaround for a $x^2 + bx = c$ which holds striking similarities to today’s method referred commonly as "completing the square." Sadly, though since often times these quadratics can yield two solutions in reality the Hindus were not always able to find both of them. Interestingly the Greeks too used geometric techniques to solve this problem and even Euclid presented three examples involving quadratic equations. Unfortunately, like the Hindus before him he struggled with providing both roots even when they were both positive (Smith 1951 p.134). Fast forward to Viétes’ time it’s clear that he was one of many to abandon geometrical methods and embrace more analytical methods instead for solving quadratics although he had some difficulty grasping the idea of general quadratic equations (Smith, 1953).

Indian mathematicians developed a number of formulas that were equivalent to the quadratic formula in antiquity. Although there are no historical documents that explain how they did it, it is possible that some altar structures built around 500 BC demonstrate answers to the equation (Smith, 1953).

Understanding quadratic equations and their solutions has long been a top priority for mathematicians throughout history. One such individual was Aryahata, a Hindu mathematician whose life spanned roughly between 476 AD and 550 AD.

Aryahata demonstrated an extraordinarily high level of expertise in quadratics. He even developed a rule for the sum of geometric series, which indicates his understanding of quadratic equations with both solutions.

Brahmagupta would explore these principles several centuries later but seemed limited to focusing only on one solution rather than both.

As time progressed other experts were able to build upon Aryhatas' foundational work in this area; Mahàvira had already developed what we would recognize today as the modern rule for calculating positive roots by around AD850. Later experts such as Sridhara would continue to expand upon these breakthroughs.

By around AD1100 or so Persian mathematicians like al Khowarizmi and Omar Khayàm had also made significant progress towards determining positive roots.

In present day schooling systems there has been an increase in utilizing an ancient practice known as the Hindu method that dates back to over a millennium ago in 1025 AD. One of its most prominent techniques is what we commonly refer to now as the quadratic formula - often considered by pupils as the "almighty" formula. Essentially this approach entails solving ax^2 + bx^2 + c = 0 and deriving solutions for x from this equation: $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$. Interestingly enough it was through their innovative technique of completing squares that Hindus first established a unified algebraic solution for quadratic equations (Eves, 1964), recognizing the existence of two formal roots in a quadratic equation having real roots.

$(x_1 \; _{and} \; x_2)$.

Where $x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$, $x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$

The approach of the Arabic mathematician, Mohammed ibn Al-KhowârizmÎ divided them into three fundamental types:

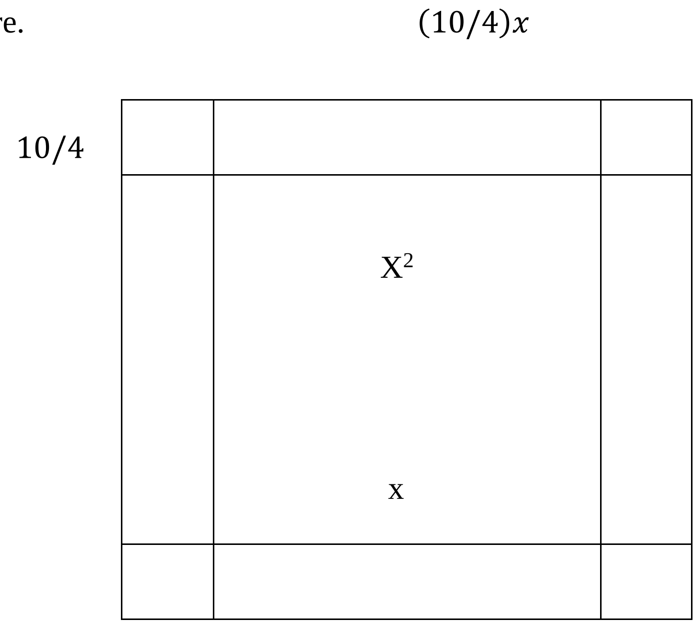
$x^2 + ax = b$

$x^2 + b = ax$

$x^2 = ax + b$

with only positive coefficients allowed (Arabic mathematicians still did not accept negative values by themselves). Every issue was broken down into a few common sorts and solved utilizing certain fundamental principles. To address the challenge of $x^2 + 10x = 39$, the individual utilized two approaches. Firstly, he generated a square with each side measuring x to signify x^2.

He then added 10x to x^2. This is accomplished by divided 10x by 4, each part representing the area $\frac{10x}{4}$ as a rectangle (Burton, 1999), applying these four rectangles to the four edges of the created square.



This resulted in a figure represented by the expression

$x^2 + 10x = x^2 + 4 \left[\frac{10x}{4}\right]$: To make the figure a large square of sides $x + \frac{10}{2}$,

Al-Khowârizmi added four small squares at the corners, each of which has an area equal $\left[\frac{10}{4}\right]^{2.}$

Hence, to complete the square he added 4 $\left[\frac{10}{4}\right]^2 = \left[\frac{10}{2}\right]^2$

which produced ➡ $\left(x + \frac{10}{2}\right)^2 = (x^2 + 10_x) + 4 \left(\frac{10}{4}\right)^2 = 39 + \left(\frac{10}{2}\right)^2 = 39 + 25 = 64.$

The side of the square must be $x\frac{10}{2} = 8$, x = 3.

The method of completing the square is used to solve the general form of this kind of quadratic, $x^2 + px = q$, by adding four squares, each of area $\left(\frac{p}{4}\right)^2$

to the figure representing $x^2 + p$, to get.

$\left(x+\frac{p}{2}\right)^2 = x^2 + px + 4\left(\frac{p}{4}\right)^2 = q + \left(\frac{p}{2}\right)^2$

$\Rightarrow x = \sqrt{\left(\frac{p}{2}\right)^2 + q} - \frac{p}{2}.$

The second method used by Al-Khwarizmi in solving this problem starts with a square with side x and area x^2 and two rectangles, with dimensions x and $\frac{10}{2}$, resulting in the area of the entire figure to be $x^2 + 2\left(\frac{10}{2}\right)x$ To complete the square,he added a smaller square of area $\left(\frac{10}{2}\right)2$. The complete therefore has an area of $\left(x+\frac{10}{2}\right)2$ but $\left(\frac{10}{2}\right)2 = x^2 + \left(\frac{10}{2}\right)x + \left(\frac{10}{2}\right)2 = 64$

The side of the square must be $x + \frac{10}{2} = 8,\ x = 3\left(\frac{p}{2}\right)$, to the figure representing $x^2 + 2\left(\frac{p}{2}\right)2x$ to get

$\left(x^2+\frac{p}{2}\right)2 = x^2 + \left(\frac{p}{2}\right) + 2\left(\frac{p}{2}\right)2 = q + \left(\frac{p}{2}\right)2 \quad x = \sqrt{\left(\frac{p}{2}\right)2 + q} - \frac{p}{2}$

Arabic mathematicians contributed significantly to developing solutions for quadratic equations by bringing unparalleled insights into the field. For instance, Arabs embraced irrational roots for these kinds of equations − something Greek scholars ignored entirely. Furthermore, they acknowledged only positive results when it was clear that there were two roots (Burton, 1999).

They were not cognizant of the reality of the negative solution of the quadratic equation. The Hindus mathematician, Bhaskara was the first to affirm the existence and validity of negative as well as positive roots (Burton, 1999).

Upon scrutinizing the different endeavors, it was evident that the resolution of quadratic equations held a prominent place in Euclid's Elements. The algebraic concepts were invariably expressed through geometric language.

The following methods are for solving quadratic equations and are frequently taught in schools, according to Butler, Wren, and Bank (1970). Which are:

Solution by graphical methods

Solution by inspection (in case of incomplete quadratics)

Solution by the factorization method

Solution by completing the square and

Solution by the quadratic formula

While acknowledging its usefulness in certain cases critics argue that the graphical method cannot be considered a true algebraic approach due to its limitations in providing only approximate solutions. Additionally, this procedure is seen as sluggish and laborious. These concerns were voiced previously by Miller (1957) who pointed out how the graphical method may at times fall short in delivering exact roots for quadratic equations.

2.10 Theoretical bases of the factorization and conjugales methods

Attempts to find a most viable method of solving quadratic equation have proved futile. The search is still on going and the following proposed methods are line up for a test.

2.11 Factorization method

The man who developed this method is known to be Thomas Harriot (Dantzig,1947). It is the most common method used in our school textbooks and prescribed method in the teaching syllabus of the SHS programme. The fundamental idea behind this approach is to transpose all of the equation's terms to one side of the equality sign, giving rise to the equation Q (x) = 0, where Q (x) is a polynomial. The "null factor law" must then be used after factoring the non-zero side of the equality into two factors, according to the principle. According to this rule, if the sum of two numbers (ab = 0) is zero, then either a = 0 or b = 0. implies that either x - m = 0 or x - n = 0 if (x - m) (x - n) = 0. A product cannot be zero unless at least one of its factors is zero, which is the basis for the aforementioned principle.

Hariot's approach to the solution of the quadratic equation is as follows:

Suppose a and β are roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, it follows that the equation whose root are a and β will be $(x - a)(x - \beta) = 0$ or $x^2 - (a_{1+\beta})x + a\beta = 0$……….(1)

Also consider $ax^2 + bx + c = 0$, and divided this equation through by a

i.e. $x^2 + \frac{bx}{a} + \frac{c}{a} = 0$….(2)

Comparing equations (1) and (2), it follows that

$a + \beta = \frac{-b}{a}$……………………………….(3) and

$a\beta = \frac{c}{a}$…………………………………...(4)

But $(a - \beta)^2 = (a + \beta)^2 - 4a\beta = \frac{b^2-4ac}{a^2}$ i.e $(a - \beta)^2 = (a + \beta)^2 = \left(\frac{-b}{a}\right)^2 - 4\left(\frac{c}{a}\right) = \frac{b^2-4ac}{a^2}$

Hence $(a + \beta) = \pm\frac{\sqrt{b^2-4ac}}{a}$ ....(5)

Solving equation (3) and (5) simultaneously, we obtain the solution of the quadratic equation

$ax^2 + bx + c = 0$,

i.e $a = \frac{-b+ \sqrt{b^2-4ac}}{2_a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2_a}$

Hence $3x^2 - 11x + 6 = 0$

$\Longrightarrow 3x^2 - (9 + 2)x + 6 = 0$

$\Longrightarrow (3x^2 - 9x) - (2x + 6) = 0$

$\Longrightarrow 3x(x - 3) - 2(x - 3) = 0$

$\Longrightarrow (x - 3) (3x - 2) = 0$

Applying the null factor law, either

$x - 3 = 0 \Longrightarrow x = 3$

or $3x - 2 - 0 \Longrightarrow x = \frac{2}{3}$

The truth set of equation is $\left\{3, \frac{2}{3}\right\}$.

The equation can also be solved using the following procedure:

$3x^2 - 11x + 6 = 0$

Let the roots of the equation be a and β. Comparing the equation with $ax^2 - bx + c = 0$ implies

that a = 3, b = − 11 and c = 6

$\Longrightarrow a + \beta = \frac{11}{3}$ ……………………………………………………………(1)

and $a\beta = 2$……………………………………………………………..(2)

But $(a - \beta)^2 = \left(\frac{11}{3}\right)^2 - 4(2)$

$= \frac{(-11)^2 - 4\,(3)(6)}{3^2}$

$= \frac{121-72}{9}$

$\Longrightarrow (a - \beta)2 = \frac{49}{9}$

and = $(a - \beta) = \pm\sqrt{\frac{49}{9}} = \pm\frac{7}{3}$ ……………………………………….(3)

Solving (1) and (3)

$2a = \frac{11}{3} \pm \frac{7}{3}$

= 6

$\Longrightarrow a = 3$

Substituting $a = 3$ into (1)

$\Longrightarrow 3 + \beta = \frac{11}{3}$

$\Rightarrow \beta = \frac{11}{3} - 3$

The truth set of the equations is $\left\{3, \frac{2}{3}\right\}$

Barnett, Byleen & Ziegler (1994) observed that the essential principle underlying the technique of factorization is none other than the null factor law. The backbone of this approach hinges on inferences and takes root in the zero property common to complex numbers - a generalization stemming from that found in real numbers. Nevertheless, one must exercise caution as wrongly implementing this critical tenet could lead to misleading outcomes during equation-solving processes.

That is, $x^2 - 5x + 6 = 12$

$(x-2)(x-3) = 12$…………………………………......(1)

$(x-2) = 12$…………………………………………… (2)

or

$(x-3) = 12$…………………………………………… (3)

x = 14 and x = 15.

The equations (2) and (3) are erroneous expressions. Zero is the only number with this property.

Butler et. al., (1970) noted that when a quadratic equation of the form $x^2 + 6x - 40 = 0$ is given in factorized form as (x + 10) (x – 4) = 0, it is always confusing to students. Many of them question why the factors should be reduced to two linear equations. But we notice that they only carry out the operation without necessarily understanding this all important steps. This is in line with Brownell's theory of meaningful learning. This practice is unjustifiable because it lacks generality in terms of real numbers (Butler et al., 1970).

Specialized skills are necessary for students to use the Harroit's method effectively, including algebraic expression factorization - a prerequisite skill highlighted by Kinney and Pudey (1957) and Gyening and Wilmot (1999). Time invested in honing these abilities could be spent on other academic pursuits instead. Adele (1963) further identified three specific areas that frequently lead to confusion when applying the Harroit's method (p. 132). These are:

One side of equation must be $x^2 - 5x = -6$, $x(x - 5) = -6$, does not imply that $x = -6$ or $x - 5 = -6$

The equations $3x^2 = 4x$ and $3x = 4$ are not equivalent. The first has solution set 0, $^4/_3$ while the second has solution set $\left\{\frac{4}{3}\right\}$.

To solve for case two one must divide the equation with respect to x. It should be noted that any resulting value of x = 0 should be excluded from the final solution set. It has been observed that a majority of misinformed students tend to make this particular error frequently.

The equation $px^2 + q = 0$ is a special case and requires the use of the square root sign. That is if $m^2 = n$, then $m = \pm\sqrt{n}$. If only the positive answer is considered, then the answer to the problem will not be complete.

To avoid making mistakes when utilizing the Harriot's method, one must acquire and commit to memory all these skills.

Although the method is popularly used in our schools today, it is claimed to have some limitations. Usiskin (1980) indicated that factorization is not a strong technique for solving quadratic equations of the form $ax^2 + bx + c = 0$. They claim that it takes time to teach. They have suggested the use of the general quadratic formula because it works for all quadratic polynomials and so can be memorized. Cornish-Borden (1999) also shared the same view with Usiskin (1980) when he stated that the method of factorization is easy but a useless method of solving quadratic equations since it cannot be used in solving all quadratic equations (p.152).

From the preceding discussion about factorization not being a strong technique, Smith (1958) contended that the first important treatment of the solution of quadratic equation was by the method of factorization. Similarly, Cundy (1968) stated that the method of factorization is the most convenient to solve quadratic equations. This claim however contravenes the finding of Birken (1986) and Usiskin (1980).

Factorization of the quadratic expression is a specialized skill that takes time to learn. As noted by Crowhurst (1961), the method of factorization involves some trial and error. In support of this, Budnick (1985), and Richardson (1966) all attest to the fact that there are many quadratic equations involving trinomials which cannot be factorized or are factorized only by trial and error.

Hoffman (1976) maintained that factorization requires a very strong skill in multiplication and students who lack this skill are unable to develop strong skills in factorization. Students therefore spend too much time trying to find the appropriate factors needed to tackle other steps in the process of solving the quadratic equation. This is even more tedious and frustrating when the coefficients of x^2, the x terms, and the constant are large numbers or fractional numbers.

Hoffman (1976) recognizes several different methods available for solving quadratic expression such as $3v^2 + 7v + 2 = 0$ among them are the following he discussed:

2.12 The inspection method

$3v^2$ Suggests factors of $3v$ and v.

2 suggests factors 1 and 2, -1 and -2.

Checking of the linear terms (7v) shows that the correct expression $3v^2 + 7v + 2 = 0$ is $(3v + 1)(v + 2) = 0$.

The method of decomposition of the linear terms

Multiply $3v^2$ and 2 to get $6v^2$.

Decompose 7v into the sum of two terms whose product is $6v^2$ this gives 7v = 6v + v.

Factorize $(3v^2 + v) + 2 = 0$ by grouping.

$3v^2 + (6v + v) + 2 = (3v^2 + 6v) + (v + 2) = 0$

$3v(v + 2) + (v + 2) = 0$

$(3v + 1) + (v + 2) = 0$

This method is based on the observation that

$$(ax + b)\ (cx + d)$$

$= acx^2 + adx + bcx + bd$

$= acx^2 + x\ (ad + bc) + bd$

And that $(acx^2)(bd) = abcdx^2 = (adx)(bcx)$

A third method

Multiply 3 by 2 to get 6

Find two factors whose product is 6 and whose sum is 7.

The numbers are 1 and 6.

$$3v^2 + 7v + 2 = \frac{(3v + 6)(3v + 1)}{3} = 0$$

$$= (v + 2)(3v + 1) = 0$$

This method can be justified as follows:

$$3v^2 + 7v + 2 = \frac{1}{3}\ [(3v)^2 + 7\ (3v) + 6]$$

$= \frac{1}{3}\,[(3v)^2 + 7(3v + 1)] = 0$

$= (v + 2)(3v + 1) = 0$ (Hoffman, 1976 p. 54)

It can be challenging for the student to discover the four constants that satisfy these conditions, according to Budnick (1985), who commented on the decomposition method.

Apart from this, the method of factorization will not work with a situation like $x^2 - 5x + 2 = 0$. Hence, this method is handicapped in several ways.

Fremont (1969) observed that all the standard methods including the graphical method are lengthy and tiresome and leave students open to calculation errors (p. 276).

Notwithstanding the criticism levelled against this method, Usiskin (1980) reported that, in most text of Mathematics, factorizations is first applied to help solve quadratic equations. From all indications, the method of factorization is the most popular technique to solve the quadratic equation among the secondary school students. For instance, in a study conducted to compare the methods used in solving the quadratic equation, Cornilious and Gott (1988) reported, 19 used factorization and 3 used the quadratic formula' (p.863). From their report it means that 84.4% used the method of factorization whilst 13.6% used the quadratic formula.

Reeves & Kilmister (1952) says that 'it is impossible to find a quadratic equation in a real life situation that can be solved by factorization' (p.500). So to him, solving equations by factorization is artificial. For example, the equation $x^2 - 6x + 7 = 0$ cannot be solved by factorization method.

Based on these quadratic equations, Gyening (1993) suggested the use of the 'novel method', which he claims can be used to solve quadratic equation, with very little chance of committing errors.

2.13 The Conjugales

In order to achieve the required result using this method, two simple linear equations must also be solved. To make it simple to identify the constants, it is crucial to put the given equation in its standard or canonical form. For instance, the equation $ax^2 + bx + c = 0$ can be restated as follows:

$ax^2 + bx = -c$ (1)

Multiply equation (1) by 4a

$4a^2x^2 + 4abx = -4ac$ (2)

Add b^2 to both sides of the equation (2)

$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$ (3)

The left hand side of equation (3) is now a perfect square

$(2ax + b)^2 = b^2 - 4ac$

$2ax + b = \pm\sqrt{b^2 - 4ac}$

Let d be any number equivalent to $\pm$ (2ax + b) Implying that d = $\sqrt{b^2 - 4ac}$

Hence 2ax + b = -d (4), and

2ax + b = d (5)

The answers to these two straightforward linear equations make up the quadratic equation's solution set.

Illustrative example:

Find the truth set of the equation: $5x^2 + 4x - 1 = 0$

Compare $5x^2 + 4x - 1 = 0$ with a quadratic equation's generic form

$ax^2 + bx + c = 0$, a = 5, b = 4 and c = −1

Substituting these values into d = $\sqrt{b^2 - 4ac}$, d = $\sqrt{4^2 - 4 \times 5 \times (-1)}$, d = 6

The two linear equations required can therefore be formed as,

10x + 4 = -6 (1) and

10x + 4 = 6 (2)

From equations (1) and (2), x = −1 and x = $\frac{1}{5}$ respectively represents the roots of the equation, hence the truth set of the equation is

$\left\{x: x = -1, \frac{1}{5}\right\}$

We notice that the student has recently been reminded of the vital importance in exchanging a complicated problem for an equivalent yet elementary one in order to resolve it more efficiently. As demonstrated through past teachings during their Junior Secondary School days, students have already gained knowledge on solving linear equations; thus, recalling these proficiencies and implementing them should not pose a significant challenge.

2.14 Importance and Applications of Quadratics

Quadratic equations hold immense importance across multiple arenas- be it education, construction or science. Specifically, in science, they serve as a crucial tool for modeling diverse physical phenomena including sound waves, chemical reactions and projectile motion (Söylemez & Koçak, 2018). These models offer researchers a deeper understanding of these events while also facilitating the development of innovative technologies.

When designing physical artifacts with desired shapes and properties- such as those used for satellites or solar ovens- quadratic equations are extremely useful tools. Parabolic reflectors commonly found in these devices rely heavily on such formulas to focus light or radiation onto specific points (Khattab, 2018). By calculating ideal dimensions based on these equations during design phases, accuracy can be maximized. Furthermore, lenses, mirrors, and other optical devices also require similar calculations using quadratic formulas to function optimally (Moussa, 2019).

In the realm of education quadratic equations constitute an essential component of the mathematical syllabus. Their study fosters vital skillsets such as critical thinking prowess effective problem solving techniques and sound logical inference capabilities (Rahman, 2020). As per Rahman’s observation (2020) proficient knowledge in this area imbues students with a deeper understanding of complex mathematical theories while facilitating their judicious application within real world settings.

In conclusion, it is clear that quadratic equations hold great importance across multiple domains including science, physical artifacts, construction, and education. Acquiring proficiency in solving these mathematical functions is pivotal for succeeding in diverse professions and can serve as an asset for individuals looking to advance their academic or professional pursuits.

2.15 Empirical studies on the methods of solving Quadratic Equations

As an essential component of mathematics curricula at all levels, quadratic equations demand thorough understanding for students' progress. Valuable lessons on effective techniques for imparting knowledge on this subject can be obtained by analyzing earlier studies empirically. Over time, several inquiries have investigated diverse pedagogic avenues for quadratic equations education while shaping the conceptual bases driving our project.

Studies are carried out to find a better method free from the numerous limitations of the conventional methods. Essah (1999), Danso- Addo (2000), and Kisi-Twum (2003) used two intact classes each to conduct separate studies on the feasibility of teaching solution of quadratic equations in the first year of the SHS programme. The studies were carried out in different geographical locations. These were Koforidua, Cape Coast and Sekondi. These researchers compared the conventional factorization method with ELSE on the bases of accuracy and retention. They also tried investigating the potential of the method of Equivalent Simultaneous Linear Equations (ELSE) and other methods. In all the cases, achievement tests were the instrument used to collect the data. The data was analyzed using the ANCOVA except the study conducted by Danso-Addo, who used the chi-square, t-test, and Wilcoxon’s test in his analysis. Two first year classes were used by each of the researchers. The elements in these classes were not randomized, so the intact classes were considered.

From the result, there was no significance difference between the pre-test means scores and the post-test means scores in all the three studies conducted by Adams, Eassah, and Kisi-Twum on the factorisation method, indicating that the students were comparable in learning the topic in terms of performance. However, there was a significant difference in the post-test and retention test mean scores between the ELSE and the Harriot's group favouring the ELSE group in all the three cases. This was also the case with the retention test mean scores. Danso-Addo's study however gave a significant difference, from all the three statistical tools, between the frequency distribution of the scores of the sample subjects. They also agreed that the ELSE was a better method of teaching Quadratic Equation than the conventional and most popular Harriot's method. They also noted that students made fewer errors when using the ELSE.

A study by Huang et al. (2016) investigated which approach, between factorization and the quadratic formula, is more effective in terms of helping students understand these mathematical concepts better. Their research indicated that utilizing quadractic equation resulted in greater proficiency among learners than utilizing factoring approach for instruction purposes.

A study was also carried out by Fefoame (1996) to compare the effectiveness of the DTA against the conventional method of factorization in solving quadratic equations. The aim of the study was to find out how the DTA compare with the conventional factorization method of solving Quadratic Equations on the dimensions of accuracy and retention.

This study was carried out using two intact classes selected from the first year classes of Okuapeman Secondary School. The students were from predominantly farming communities and aged between 15 and 18 years. The researcher used an achievement test to collect the needed data for the study. A pretest was first administered to the students followed by the post-test. There was also a fallow period of two weeks of no teaching, after which the retention test was administered.

Finding from the study showed a significance difference in the means post-test scores of the Harriot's method and the DTA, favouring the DTA. It was the same with the retention test mean scores. In his conclusion therefore, he recommended the DTA as a better method for teaching Quadratic Equations than Harriot's method. He observed that DTA was easy to handle by students and gave them a very positive attitude towards the method and topic. This was evident as students were always seen practicing the method in solving Quadratic Equations during their spare time. In his recommendation, he strongly recommended the inclusion of the DTA in the next revision of the current Senior Secondary School Mathematics book as a method for teaching Quadratic equations.

In the same year, Opoku carried out a study parallel to Fefoame's study in Atwimaman secondary school in the Ashanti Region. The findings of Opoku, (1996) were not any different from that of Fofoame. Two Senior Secondary School intact classes were used for the study. An achievement test, comprising a pre-test, post-test and a retention test was the instrument used for the study.

His sample was taken from students whose parent were predominantly farmers in the community with very limited social facilities. Their ages ranged between 15 and 17 years. In an investigation to find out the potentials of the DTA as an alternative method to the Harriot's method, he found out that there was a significant difference in the mean scores at 0.05 significant levels, between the DTA and the Harriot's method on both the post-test and the retention test. He concluded that the DTA was a better method than the retention test. He called on teacher associations to organize workshops to teach its members the DTA.

Bafour-Wuah (1997) also replicated the studies of Fefoame and Opoku. He used two intact classes of seventy-five students. From his findings, he concluded that the DTA was a superior method to the Harriot's method on both measures of accuracy and retention. He therefore recommends that DTA be introduce into the Senior Secondary School Mathematics books. To him this would safe teaching time and provide a breather for the students who have to battle it with the conventional methods to solve Quadratic Equations (Baffour–Wuah, 1997).

Mensah and Abedi (2005) replicated Opoku's (1996) study. They carried out their study in OLA Teacher Training College. Sixty-six female students selected from two intact classes took part in the study. Findings from their study confirmed findings of Opoku (1996). Comparing the performance of the two groups, there was significant difference in the mean scores of both the posttest and the retention test, all favouring the DTA. In their findings, Mensah and Abedi observed that the method created a lot of interest in the teacher trainees. They also recommended the introduction of the method in the teacher training institutions since these teachers would serve as the first point of dissemination of the potential of the DTA.

Similarly, Sari (2018) conducted research exploring whether teaching quadratic equations using either factorization or completing the square methods impacts student comprehension. The data collected revealed that pupils exposed to completing the square had a stronger command of quadratic equations compared to those who received instruction utilizing only factorization method.

In 2013, a research was carried out on the most viable method of teaching quadratic equations. The respective methods considered the d−h theorem and the conventional factorization methods. Emmanuel Dodzi Havi who carried out his study at Akro Secondary Technical School, Eastern

Region in 2013 had his findings in favor of the d−h theorem. A findings using instruments with over 70% reliability coefficient, indicated that students did significantly better in the d−h theorm method compared to the factorization method (Havi, 2013).

A similar study was carried out in the Northern Region by Dramani and Adam, (2017) at the Bagabaga collage of Education. This research considered the Equivalent Simultaneous Linear Equations (ESLE) and the Factorization methods. They came out with the findings that the ESLE method proved more viable in teaching Quadratics as compared to the factorization method.

The empirical data from these studies highlight the importance of identifying effective methods for teaching quadratic equations.

To sum up finding optimal techniques for educating seniors in high school about quadratic formulas is imperative. Based on empirical evidence presented thus far different methodologies produce varying degrees of success while teaching these formulas; hence selection should be based on criteria such as instructional styles or specific course content tailored towards each individuals unique learning style. Results from this literature review informed our research design and contributed significantly towards developing an existing pool of knowledge regarding effective teaching strategies for seniors at the secondary level.

2.16 Conceptual Framework

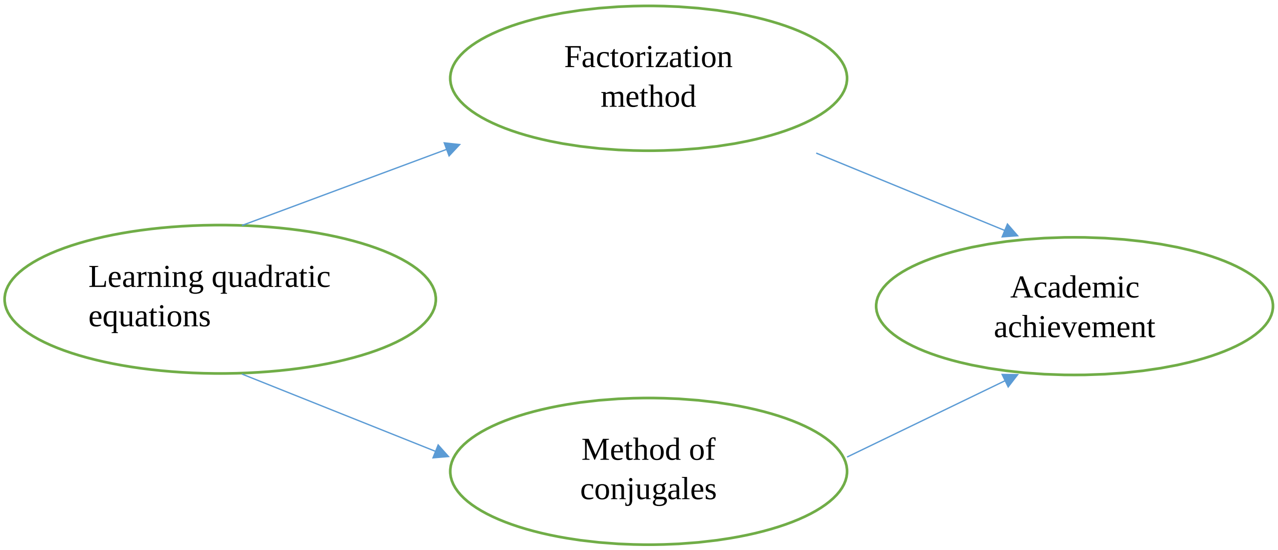


Figure 1: Conceptual Framework

The conceptual framework for this study builds upon the relationship between the independent variables, the teaching methodologies (factorization method and method of conjugales), and their impact on learning quadratic equations. Additionally, it explores the influence of these teaching approaches on students' academic achievement in quadratics, which serves as the dependent variable.

2.16.1. Independent Variables:

a. Factorization Method: The factorization method is a common technique used in solving quadratic equations. It involves breaking down a quadratic equation into its linear factors, allowing students to identify the roots of the equation and understand its behavior. The factorization method emphasizes the concept of factoring quadratic expressions and simplifying their solutions.

b. Method of Conjugales: The method of conjugales is another technique employed in solving quadratic equations. This method aims to transform a quadratic equation into a perfect square trinomial, enabling students to determine the equation's roots. The method of conjugales focuses on utilizing algebraic manipulation to derive quadratic solutions.

2.16.2. Dependent Variable:

Academic Achievement in Quadratics: Academic achievement serves as the dependent variable in this conceptual framework. It refers to students' performance, understanding, and proficiency in solving quadratic equations, their ability to interpret quadratic graphs, and their overall mathematical competence in the subject area.

2.17 Collective summary of theoretical, empirical and conceptual frameworks

The Action-Process-Object-Schema (APOS) theory, is a theoretical framework in mathematics education. The APOS theory emphasizes the importance of helping students utilize their existing mental structures while developing new ones to handle advanced mathematical concepts. The theory consists of four stages: action, process, object, and schema. In the action phase, learners develop an intuitive understanding through physical manipulation. The process phase involves recognizing patterns and developing procedural knowledge. The object phase focuses on abstraction and symbolic representation, while the schema phase involves integrating knowledge into a coherent framework. Effective teaching should guide learners through these stages, facilitating their progression and understanding of mathematical concepts.

Several empirical studies have been conducted to investigate different methods of teaching quadratic equations. Researchers compared the conventional factorization method with alternative approaches such as the method of Equivalent Simultaneous Linear Equations (ELSE) and the quadratic formula. The studies assessed factors like accuracy, retention, and overall performance. The results indicated that while there was no significant difference in performance between the factorization method and ELSE, the ELSE method exhibited better post-test and retention test scores. Another study favored the d─h theorem method over factorization. These studies highlight the importance of selecting appropriate teaching methods tailored to individual learning styles and preferences.

A conceptual framework was developed to examine the relationship between teaching methodologies, specifically the factorization method and the method of conjugales, and their impact on learning quadratic equations. The factorization method involves breaking down quadratic equations into linear factors, while the method of conjugales transforms equations into perfect square trinomials. The framework also explores the influence of these teaching approaches on students' academic achievement in quadratics, measured by their performance, understanding, and proficiency in solving quadratic equations. By investigating these relationships, the conceptual framework contributes to the understanding of effective teaching strategies for quadratic equations.

Combining the theoretical considerations, empirical findings and conceptual framework, it becomes evident that quadratic equation instruction requires careful consideration of teaching methodologies. While the factorization method remains widely used, alternative approaches such as ELSE and the d─h theorem have demonstrated potential for improving students' learning outcomes. These findings emphasize the importance of tailoring teaching methods to suit individual learning styles and preferences. By utilizing effective teaching strategies, educators can enhance students' understanding and academic achievement in quadratics.

In conclusion, this comprehensive overview sheds light on the empirical studies conducted to explore teaching quadratic equations. It highlights the need for effective instructional techniques, as demonstrated by the comparison of various methods. The conceptual framework further contributes to understanding the relationship between teaching methodologies and students' academic achievement in quadratics. By incorporating these insights into mathematics education, educators can foster a deeper understanding of quadratic equations and facilitate improved learning outcomes for students.

2.18 Summary of Review of Related Literature

The study of quadratics is as old as mathematics itself. Because of its importance to humanity, many attempts have been made by mathematicians to develop an effective all-round method to solve problems involving it. The Greeks, the Hindus, the Arabs, the Chinese and the Babylonians in early civilisation strived to solve problems involving quadratic equations with various methods.

The review of Literature has revealed extensive knowledge in the area of trying to find the roots of quadratic polynomials. Early investigations have shown that solving quadratic equations has been a difficult task to many Senior High School students all over the world. Many researchers therefore tried to develop a more efficient method to deal with the situation. The method of factorization is mostly relied on. This method, because of its position among authors, is most widely used as the conventional method in solving examples involving solutions of quadratic equations in the Secondary School Mathematics textbooks (Steinmetz & Cunningham, 1993). However, in recent time, the method of Thomas Harriot (method of factorization) which is very popular with students has been criticised by many meaningful Mathematicians. Many contended that the method places a lot of demands on the student. With this method, the learner is expected to learn some specialised skills to be able to use the method effectively. Factorization must be learnt and the concept of the zero principle well understood before the method can be used effectively. This is only not time wasting but also demands so much from the learner.

The method of completing the square is not only complex, but is also difficult to use by most students. Though the method was used during early civilisation by the Chinese and the Babylonians to solve quadratic equations, present day students find the procedures involved in it very cumbersome and difficult to follow. The quadratic formula, though important that students master it, it is not easy to teach nor understanding it at the SHS level. It is only good for students with good memory and the understanding of its derivation.

Since these methods are plagued with inherent difficulties and thus not easy for students to use, there is the need to search for methods devoid of these limitations. This search has brought to the fore, three methods. These are the Equivalent Simultaneous Linear Equations (ESLE), the Directed Trial Analysis (DTA) and the Conjugate Linear Equations (CONJUGALES). The proponents of these methods claim that they are easy to use, and do not require any specialised pre-requisite skill. They also claim that the methods can be used to solve all types of quadratic equations. The CONJUGALES makes use of skills of solving linear equations, which is already learnt at Junior High School level. Research conducted on most of these methods have proven positive. But in order the make the statistical evidence replete and widely accepted, there’s need to delve a lot more into this area of study.

CHAPTER THREE

METHODOLOGY

3.0 Overview

The methodology employed to accomplish the study's objectives is the underpinnings of this chapter. It addresses the research paradigm, research design, study area, study population, sample size, sampling technique, data collection instrument, and statistical tools for data analysis.

3.1 Research Paradigm

The idiosyncrasies of this research called for the adoption of **positivism** research paradigm. The foundation of an ideal researcher with a positivist paradigm is understanding and articulating belief about the nature of reality, what can be understood about it, and how we go about obtaining this information.

According to Rehman (2016), a research paradigm is a fundamental theoretical framework and belief system that makes assumptions about ontology, epistemology, methodology, and methodologies.

Ontology refers to “the nature of our beliefs about reality” (Richards, 2003). Researchers have perceptions and assumptions about the ideal world, how it works, and what can be learned about it that are primarily implicit.

Epistemology is “the branch of philosophy that studies the nature of knowledge and a process by which knowledge is acquired and validated” Gall & Burge (2003). It is concerned with the nature of forms of knowledge, its acquisition and how it can be communicated to other human beings

(Cohen, 2007). It is the "epistemological question that leaves a researcher to debate the possibility and desirability of objectivity, subjectivity, causality, validity and generalizability" (Patton, 2002).

Positivism as a subfield of philosophy gained popularity around the turn of the 19th century as a result of the works of French philosopher Auguste Comte, (Richards, 2003). According to the positivist philosophy (Job and Schneider, 2014), scientific beliefs are exact theoretical representations of observed reality. Positivism acknowledges that reality exists apart from people and their knowledge. It is unmediated by our senses and subject to unchangeable laws. According to the positivist position, information lacking empirical support is unscientific (Siponen, 2018). As positivists share the same viewpoint as realists regarding intuition against the true nature of reality, their ontological perspective is that of realism (Killam, 2013).

According to positivism paradigm, measurement is crucial in creating a clear understanding of things. Data produced by measurement are organized according to theoretical principles, hypotheses are tested against evidence in order to disprove them, and then those hypotheses are replaced with significantly modified ones (Cupchik, 2001). The positivism paradigm therefore defines the blueprints of the study with experimentation and empirical findings.

Experimentation is central to positivist methodology. In order to explain the causal relationship between phenomena, hypotheses are presented in the form of prepositions or questions. The quantity of empirical data is obtained, examined, and then put into the shape of a theory that describes how the independent variable and dependent variable interact. Deductive methods are used to analyze data; first, a hypothesis is put out, and then, depending on the findings of statistical analysis, it is either confirmed or rejected. The goal is to quantify, regulate, forecast, create laws, and assign causality (Cohen, 2007).

Positivist researchers attempt to control extraneous variables by subjecting two or more groups to the same conditions with the only difference being the independent variable in order to ensure that no other variables produced (influenced) a particular effect. Data in positivist research are often numerical. Positivism-based epistemology is congruent with the use of quantification to represent and evaluate aspects of social reality (Gall, 2003). True experiments, less stringent quasi-experiments, standardized tests, large - or small - scale surveys employing closed-ended questionnaires, and other methods can all be used to gather the quantitative data that positivist researchers need to address research problems and build hypotheses. These techniques produce numerical data that is then analyzed statistically, either in a descriptive or inferential manner. The positivist approach holds that research is of high quality if it possesses internal validity, external validity, dependability, objectivity, and a number of other characteristics that best define the study issue (Guba, 1994). This demonstrates why positivism deserves consideration as a paradigm for the study.

The study is deemed to have internal validity if the researcher can show that the independent variable—and not any other variables—has an impact on the dependent variable. It has internal validity if the conclusions reached are generalizable. It has reliability if different researchers conducted the study in various contexts, locations, and times and came to the same conclusions. Researchers are regarded to be objective if they conduct exceptional research without letting their bias cloud their judgment.

Positivism, suggested as the best paradigm for the study, however has its own shortfalls or shortcomings. It has been labelled with some criticisms of which the post-positivism emerged out of. A predominant phenomenon in all of philosophy.

3.2 Research Approach

The study's chosen research methodology/approach is the quantitative approach. Quantitative research entails the systematic collecting and analysis of numerical data in order to provide answers to research questions, make predictions, or test hypotheses, (Babbie, 2016). It focuses on measuring variables, establishing patterns, and quantifying relationships between variables using statistical analysis techniques (Creswell & Creswell, 2018).

In quantitative research, data is typically collected through structured surveys, experiments, or observations, and is analyzed using statistical methods to generate objective and numerical findings (Creswell & Creswell, 2018). This approach aims to achieve objectivity and generalizability by collecting data from a representative sample and employing rigorous statistical procedures (Babbie, 2016).

The quantitative research approach was therefore adjudged most appropriate for this study because quantitative data was sourced from students using achievement test (pre-test, post-test, retention-test), for analysis that was carried out on experimental and control group in determining the differences in achievement between groups.

3.3 Research Design

It is a researcher's framework for selecting the research techniques and procedures to use in a study. The process for gathering, organizing, and analyzing data is outlined in the research design, which also contains all of the components required to organize the study together (Kombo and Tromp, 2006).

The study is quasi-experimental. It employed a quasi-experimental, pre-test post-test non-equivalent comparison group design. This was because of the nature of the environment in which the study was being carried out. It was not appropriate to disrupt the natural structural setting of the classrooms. Christensen (1980) suggested the adaptation of such methods when it is not possible to change the environment under which a study is carried out**.**

The use of a quasi-experimental design is justified by the idiosyncrasies of the study, which aims to compare two methods of teaching solutions to quadratic equations. Conducting a randomized controlled trial (RCT) with random assignment of students to each teaching method may be challenging due to practical constraints and ethical considerations. Randomly assigning students to different teaching methods could disrupt their learning experience and raise ethical concerns regarding the fairness of instruction (Cook & Campbell, 1979). Therefore, a quasi-experimental design offers a suitable alternative in this context. It allows for a comparison between two groups, one taught using the method of interest (e.g., the method of conjugales) and the other taught using a different method (e.g., the method of factorization). By comparing the performance of these groups on a pre-test and post-test, the study can provide valuable insights into the relative effectiveness of the two teaching methods for solving quadratic equations. The quasi-experimental design allows for a controlled comparison while considering the practical and ethical constraints inherent in the educational setting, making it a justifiable choice for this particular study (Creswell and Creswell, 2018; Shadish, Cook, and Campbell, 2002).

3.4 Population

According to McMillan and Schumacher (2001), a population is a group of factors, including people, things, and events, that meet certain characteristics and to whom researchers want to apply their findings broadly. Osuala (1993) asserts that when the universe to be sampled is not exactly defined, challenging issues occur.

The target population comprised first year students of Damongo SHS and Ndewura Jakpa SHS which is about 1,130 and 300 students respectively, thus a total of 1,430 students.

3.5 Sample and Sampling Technique

A sample is a collection of people, things, or occurrences chosen at random from a larger population. It consists of the components of the population that are really taken into account for research inclusion (Ranjit, 2005).

Sampling technique on the other hand refers to a method of selecting individual members or elements from a population to be used for a study so that statistical inference and estimation of characteristics could be made from the whole population (Creswell, 2015). This study employed two sampling techniques: simple random sampling and purposive sampling.

Simple random sampling is a sampling technique in which each member has equal opportunity of being selected to be part of a study. This method was used to select one school out of the two government based secondary schools in the municipality.

Using elements or cases that provide information pertaining to the study's objectives is possible with the use of the technique of purposeful sampling (Mugenda, 2003). This method was used to select intact classes with one a core-maths based class with the other an elective mathematics class, which meets the needs and requirements of the study.

3.6 Data Collection Procedure

An introductory letter was taken from the department which served as evidence of the researcher's coming from an institution. The letter explained the purpose of the research. Permission was sorted from head of the institution. After purposefully-randomly obtaining the classes to be used for the study, an achievement test was used to determine which class will be the control group with the other as the intervention group. A series of lessons was organized for the students of which a post achievement test was given to each group to check their performance. Finally, a retention test was given to students (after four weeks) to check how well they could remember their individual methods.

During the data collection, students went through series of exam that was invigilated by the researcher in person, with help from two other teachers, one a maths teacher, the other an ICT. This was to ensure maximum diligence in the process. To cater for the third research question, time used by students was taken as and when they claim to have finished or submitting their papers. All those who did not submit their work before time were considered to have used all 60 minutes.

A "blind experiment" was carried out to curb reactivity effect. The term blind experiment means that the subjects will not know whether they belong to the experimental or control group.

3.7 Data Analysis

Collected data was analyzed with the aid of SPSS software (Version 26). To address the first to forth research hypothesis, ***independent sample t-test*** was used. T-test was employed because it's a statistical tool that compares the means of two groups, especially when there's the need to determine whether a process or treatment or intervention actually has an effect on the population of interest (Tim, 2015). This tool assumes independent samples, homogeneity of variances, and a normal distribution. Since the individuals were divided into separate treatment groups, this tool was thought to be useful in this type of study.

3.8 Experimental Treatments

Two groups were taught the factorization and CONJUGALES methods. The method of factorization was used in a Mathematics (Elective) class whiles the CONJUGALES method was treated with a non-Elective Mathematics class, with both classes being general art classes (Geography and Literature). Thus purposively assigned because it is assumed the students in an elective-mathematics class can match the difficulty of the factorization method as a control group. The factorization method was used because it is a main prescribed method in the Core Mathematics syllabus, (CRDD, 2003, 2007, and 2010). WAEC also mostly requests for this specifically to test the attainment of its concept and skills.

3.9 Research Instruments

Research instruments are tools or techniques used to collect data in research. In this study, the research instruments ware test items (achievement tests). These were ***Pre-test*** Achievement test, ***Post-test*** Achievement Test as well as ***Retention-test*** Achievement Test. The pre-test instruments were constructed with fifteen multiple choice questions and five essay questions. The multiple choice was given five options, lettered a – e, this was to rid-off or limit chances of guesswork. The essay questions were altogether inferred from past questions and past literature of similar study.

The lesson notes and test items were developed based on the content of textbooks and past questions. The use of appropriate research instruments ensures the collection of accurate and valid data. (Jones, 2008). See appendix for details of Achievement Tests.

3.10 Validity of the Instrument

The extent to which evidence and theory support the interpretations of test results implied by suggested uses of the test instrument, according to the American Educational Research Association, American Psychological Association, and National Council on Measurement in Education (cited in Zakariah and Cobbinah, 2021), is referred to as validity. In other words, validity describes how well or appropriately one interprets and applies the outcomes of an evaluation.

The term "content validity of items" describes how successfully the subject matter from which inferences are to be formed was sampled by the content of the instruments (test items) and the test-takers' responses. The test items were created using the required Form 1 Mathematics Syllabus and SSSCE/WASSCE previous questions to ensure topic validity. A few seasoned mathematics teachers were given the test items to review. This review was finalized by my research supervisor.

3.11 Reliability

A fundamental component of quality research is ensuring that instruments are reliable - meaning that they yield consistent results when tested under similar conditions over time. A popular approach for measuring reliability is through use of the Cronbach alpha coefficient which evaluates how well an assortment of questions within a survey or scale relate back to one overarching concept or construct. The formula for computing Cronbach alpha calculates the average correlation between all items in an instrument. The resulting alpha value ranges from 0 to 1 representing no internal consistency and perfect internal consistency respectively. Generally, researchers are looking for a Cronbach alpha score of 0.7 or higher (DeVellis, 2017). That said, it is worthy of note that other factors like instrument validity, item difficulty and response bias can also impact reliability.

The reliability of the instruments was tested with the aid of the Cronbach alpha formula, $\propto = \frac{k}{k-1}\{1 - \frac{\sum_{i=1}^{n} p_i(1- p_i)}{\sigma_x^2}\}$. This was done after the research instruments were pilot tested.

For the symbols/variables in the formula, k is the number of items, p_i refers to the item difficulty (the proportion of the test respondents who answered item *i* correctly), and σ_x^2 is the sample variance for the total score.

In the present study, the reliability analysis that was conducted using Cronbach alpha, yielded a value of 0.80, for the pre-test, 0.87 for the post-test and 0.85 for the retention test. This coefficient indicates the internal consistency of the test items, measuring the extent to which they collectively measure the intended construct. According to George and Mallery (2003), Cronbach Alpha values above 0.7 are generally considered acceptable, suggesting that the current test demonstrates an acceptable level of reliability.

3.12 Pilot Study

This pilot study undertook an evaluation of two distinct techniques used in solving quadratic equations at Ndewura Jakpa Senior High School (SHS). These methods included factorization and conjugate linear equations. Through conducting this preliminary investigation, the researcher aimed to not only compare both approaches but also highlight any challenges they may face during data collection when conducting a more extensive study on quadratic equations later down the line. Permission for conducting this research within this educational establishment had already been granted prior by school authorities before proceeding with recruiting thirty (30) SHS1 student participants.

Before the lessons and tests were administered, the instruments used in the study were scrutinized by my supervisor to ensure they were valid and reliable. The test questions were reviewed to ensure they were appropriate for the students' grade/academic level and aligned with the objectives of the study.

3.13 Structural Tools

Structural tools are important in research as they provide a framework for organizing and presenting data. In this study, the structural tools used include conceptual framework, theoretical framework, and research design. The conceptual framework helped define key concepts under investigation while the theoretical frameworks provided logical reasoning for selected study techniques.

The study placed significant focus on different ways of solving quadratic equations at Senior High School level such as conjugates and factorization methods within its conceptual framework. At the same time, the study's theoretical framework offered detailed explanation of these methods, their mathematical formulae and equations, and a comparison of their complexity, accuracy, and efficiency.

A quasi-experimental study was deemed suitable for this scenario involving teaching first-year SHS students from West Gonja Municipality about solving quadratic equations through conjugate or factorization method followed by testing their apprehension levels related to topic comprehension using said techniques. To guarantee valid and reliable results, collecting and analyzing data in a systematic manner is crucial. This is precisely what the use of appropriate structural tools ensured in this project, as noted by research experts Jones (2015) and Smith (2022).

3.14 Ethical Consideration

In conducting the research on the relative viability of the method of conjugate linear equations versus the conventional factorization method for solving quadratic equations, ethical considerations played a pivotal role in ensuring the integrity and fairness of the study. First and foremost, participants were approached with transparency, and permission was obtained before their involvement in the quasi-experimental study. Privacy and confidentiality were diligently maintained throughout the research process, with participant identities anonymized in all documentation and reporting.

Furthermore, adherence to ethical principles extended to the research design and methodology. The choice of a pretest-posttest retention test aimed to measure the effectiveness of the two methods while minimizing potential biases. The application of the Positivism research theory emphasized empirical observation and objective measurement, aligning with ethical standards by fostering a systematic and unbiased approach to data collection and analysis. Overall, the ethical considerations implemented in this research underscore the commitment to upholding the rights and well-being of participants and ensuring the scientific rigor of the study.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

In this chapter, the data was analyzed and the results were discussed. The analysis primarily focused on addressing the research questions and testing the research hypothesis using SPSS version 26, with a significance level of 0.05. The significance level was utilized to determine whether the hypotheses should be accepted or rejected. To test all four null-hypotheses, the t-test statistic (Independent Sample) was employed.

4.1 Demographic Data

Table 1: Demographic data of participants

Control Group			Experimental group		
SN	**Description**	**Number**	**SN**	**Description**	**Number**
1	Male	15 students	1	Male	16 students
2	Female	13 students	2	Female	13 students
3	Age range	14 – 23 years	3	Age range	15 – 21 years
4	Mean age	18 years	4	Mean age	17 years
5	Total	28 students	5	Total	29 students

Source: Field data

Table 1 represents the demographic data of the participants. It entails a total of 57 students with 28 belonging to the control group and 29 for the experimental group

4.2 Analysis and Interpretation of the Data

Table 2 shows the pre-test scores for both the control and experimental groups. This test was carried out prior to the teaching of solutions to quadratic equations with different methods (conjugales against factorization).

Table 2: Independent sample t-test of the pre-test scores

Group	N	Mean	Std. Deviation	t-value	df	p-value
Control	28	22.7241	7.634	-1.850	55	0.072
Experimental	29	19.8710	3.3837			

Source: SPSS Output

The results show that there was no significant difference in the pre-test scores of the Control Group (Mean = 22.724, SD = 7.634) and Experimental Group (Mean = 19.871, SD = 3.383); [t (55) = -1.850, p = 0.072]. An indication that both groups were on the same level prior to the intervention. It is however worthy of note that the experimental group lagged the control group by 2.8531 in pre-test mean score, this indicated that the control group outperformed the experimental group just a little and for that matter, suited for allotting to be the control group.

4.3 Hypothesis One

H_{01} – There is no significant difference in performance of students of different groups during the posttest.

The hypothesis sought to find out if “there is any significant difference in the achievement scores of students taught solution to quadratic equations using the conventional ***factorization method*** and others taught using the ***method of conjugate linear equations***". In testing this hypothesis, an analysis of the post-test mean scores for both control and experimental groups was carried out.

Table 3 shows the post-test scores for both the control and experimental group. This test was carried out after the control and experimental group were taught using their respective methods (factorization and conjugales respectively).

Table 3: Independent sample t-test of the post-test scores

Score	Mean	Std. Deviation	t	Df	Sig (2-tailed)	95% Confidence Interval	
Control	18.714	14.463				Lower	Upper
Experi-mental	39.689	14.072	-5.738	55	0.000	-28.304	-13.646

Source: SPSS Output

From Table 3, the results obtained shows that; Control Group (Mean = 18.714, SD = 14.463) and Experimental Group (Mean = 39.690, SD = 13.075); [t (55) = -5.738, p = 0.000], a p-value of 0.000 was obtained which is less than the alpha-value (0.05). Also, at 95% confidence interval, the lower and upper limits do not include zero (0): [lower (-28.302), upper (-13.647)] and finally, the t-*value* from the statistical table is less than the calculated t-*value* ($T_{calculated} > T_{table}$: 5.738 > 1.671). Based on these, the null hypothesis was rejected in favor of the alternate hypothesis, and it was concluded that there was a significant difference in the scores between students taught solutions to quadratic equations using the conventional factorization method and those taught using the method of conjugate linear equations.

4.4 Hypothesis Two

H_{02} - There is no statistically significant difference in performance of students of different genders.

In testing the above research hypothesis, the post-test achievement scores of male and female students in the experimental group were analyzed and presented in Table 4.

Table 4: Independent sample t-test of the post-test scores of male and female students in the experimental group.

Score	Mean	Std. Deviation	t	Df	Sig (2-tailed)	95% Confidence Interval	
Male	44.000	9.429				Lower	Upper
Female	36.647	14.633	1.644	27	0.112	-1.827	16.532

Source: SPSS Output

From Table 4, the results obtained shows that there was no statistically significant difference in the achievement scores of male and female students taught solutions to quadratic equations using the conjugate linear equation method; Male Students (Mean = 44.000, SD = 9.429) and Female Students (Mean = 36.647, SD = 14.633); [t (27) = 1.644, p = 0.112]. At 95% confidence interval, the lower and upper limits include zero (0): [lower (-1.827), upper (16.532)]. Also, the t-*value* from the statistical table is greater than the calculated t-*value* ($T_{calculated} < T_{table}$**:** 1.644 < 1.703). From these, there was no enough evidence to reject the null hypotheses. It was concluded that there was no significant difference in the scores between male and female students taught solutions to quadratic equations using the method of conjugate linear equations.

4.5 Hypothesis Three

H_{03} - There is no significant difference in retention potential of students for the method of factorization against the method of conjugales.

In testing the above research hypothesis, the retention-test achievement scores of the control and experimental group were analyzed and presented in Table 5.

Table 5: Independent sample t-test of the retention test scores of students in both groups.

Score	Mean	Std. Deviation	t	Df	Sig (2-tailed)	95% Confidence Interval	
Control	16.607	12.89944				Lower	Upper
Experi-mental	35.413	14.72490	-5.134	55	0.000	-26.149	-11.463

Source: SPSS Output

From Table 5, the results obtained shows that; Control Group (Mean = 16.607, SD = 12.900) and Experimental Group (Mean = 35.413, SD = 14.725); [t (55) = -5.134, p = 0.000]. The result indicates that a p-value of 0.000 was obtained which is less than the alpha-value (0.05). Also, at 95% confidence interval, the lower and upper limits do not include zero (0): [lower (-26.150), upper (-11.464)] and finally, the t-*value* from the statistical table is less than the calculated t-*value* ($T_{calculated} > T_{table}$: 5.134 > 1.671). Based on these, the null hypothesis was rejected in favour of the alternate hypothesis, and it was concluded that there was a significant difference in the retention potential of students taught solutions to quadratic equations using the conventional factorization method and those taught using the method of conjugate linear equations.

4.6 Hypothesis Four

H_{04} - There is no average difference in time for which students solve questions under quadratic equations using both approaches.

In testing the above research hypothesis, the time used by students in the post-test achievement test in the control and experimental group were analyzed and presented in Table 6.

Table 6: Independent sample t-test of time used by students in both groups for the post-test.

Time	Mean	Std. Deviation	t	Df	Sig (2-tailed)	95% Confidence Interval	
Control	49.929	11.975				Lower	Upper
Experi-mental	50.828	6.381	-0.352	55	0.727	-6.059	4.261

Source: SPSS Output

From Table 6, the results obtained shows that there was no statistically significant difference in the time both groups used to solve the post-test. Control Group (Mean = 44.929, SD = 11.975) and Experimental Group (Mean = 50.83, SD = 6.381); [t (55) = −0.352, p = 0.727]. At 95% confidence interval, the lower and upper limits include zero (0): [lower (-6.059), upper (4.261)]. From these, there was no evidence to reject the null hypotheses. It was concluded that there was no significant difference in the time used by students to solve post-test questions using either of the methods (factorization conjugate linear equation method).

4.7 Summary of Discussions and Interpretation of Findings

4.7.1 Interpretation of findings

The study aimed to make comparative analysis of the method of conjugales and the conventional factorization method. It sought to compare the post-test performance of students in different groups, difference in performance of different gender, retention potential of the two methods and the difference in mean time spent on each method in solving quadratic equations. The analysis revealed that both methods were applicable to SHS1 students, but the method of conjugales had a better advantage with performance, gender, retention and time. Although it did not show a t-test statistically significant value for all cases, it showed a better mean in all cases and by inferential statistics could be tipped the better method of the two.

These findings go hand in hand with the findings of Essah (1999), Agyapong (2000), Danso-Addo (2000), Kisi-Twum (2003), Bornaa (2007) and Frank (2013), who, among other variables, compared the factorization method against the method of conjugales and in each case found that the conjugales had better post-test scores.

In terms of gender and academic achievement in quadratics, the study found no statistically significant difference in the scores of male and female students of the experimental group. On the other hand, the mean values showed that male students had a slightly higher post-test mean score than their female counterparts. It therefore meant that male students performed equally well as female students after they were taught solutions to quadratic equations using the factorization method and the method of conjugales. This supports Enu's (2013) findings that sex had no effect on students' academic achievement in his study on “achievement test scores of boys and girls taught through a cooperative learning strategy”. Furthermore, the study's findings were coherent with that of Arhin and Offoe (2015), who confirmed that gender has no specific effect on mathematics learning in their study on “gender differences and mathematics achievement of senior high school students”.

For the measure of retention potential, both factorization and conjugales showed a positive correlation in the post-test and retention test scores but that of the conjugales showed a higher correlation [Factorization (0.639), Conjugales (0.861)]. Having confirmed that there is really retention, the researcher then went on to test the main null hypothesis to ascertain whether there is a significant difference in retention between the two treatment groups. The t-test test for significance however showed that the students in the experimental class showed a significant retention potential. The same could not be said for the control group.

In terms of measuring the mean difference in time used by students of both groups during the post-test, the difference in mean was negligibly 0.899, a value that showed that students in the control and experimental groups used about the same time to execute their tasks. Computing the t-test test statistics did not yield any significant difference in the time used by both groups.

The study's findings are in resonance with previous empirical studies that have examined various methodologies for teaching quadratic equations. Comparison of the factorization method and method of conjugales aligns with the prior investigations conducted by Essah (1999), Danso-Addo (2000), Kisi-Twum (2003), and others. This congruence underscores the consistency of identified benefits associated with specific teaching techniques, reinforcing the understanding that effective methods for teaching quadratic equations can indeed be identified and applied.

The results and ensuing discussion also harmonize seamlessly with the **conceptual framework**. By directly comparing the two teaching methodologies – the factorization method and method of conjugales – in the context of quadratic equations, the study reflects the essence of the independent variables articulated in the conceptual framework. Furthermore, the outcomes presented, including the method of conjugales' superiority in terms of post-test performance, gender-neutral effects, retention potential, and time efficiency, align squarely with the conceptual framework's emphasis on how teaching approaches impact students' academic achievement in the domain of quadratics.

The findings lastly align and coalesce harmoniously with the **Action Process Object Schema (APOS) Theory**, which provides a theoretical underpinning for understanding students' mathematical learning progression. The study's relation to the APOS framework is evident throughout the results and discussion:

- The comparison between the factorization method and method of conjugales mirrors the APOS action phase, wherein students actively engage in problem-solving tasks.
- The recognition of patterns, procedural steps, and regularities while working through the methods directly aligns with the APOS process phase, where learners internalize processes and procedural knowledge.
- The effectiveness of both methods aligns with the APOS object phase, where learners abstract and symbolically represent mathematical concepts.
- The superior outcomes observed with the method of conjugales, including better post-test performance and retention potential, harmonize with the APOS schema phase, where learners integrate knowledge into coherent frameworks.

Collectively, the study's alignment with the empirical, conceptual, and theoretical frameworks underscores the robustness of the findings and their broader implications for effective teaching strategies in quadratic equations.

The ***positivist research paradigm***, guiding the study, closely aligns with the summary of the research findings. The empirical investigation into teaching methodologies for quadratic equations resonates with the paradigm's emphasis on empirical evidence, quantitative data analysis, hypothesis testing, and causal relationships. The advantages observed with the method of conjugales, such as improved performance, retention, and time efficiency, reflect the ***positivist*** pursuit of internal validity and objectivity. Furthermore, the consistency of the findings with previous studies highlights the paradigm's concern for generalizability and reliability. This alignment reinforces the robustness of the study's conclusions within the quantitative framework of positivism, although it's noteworthy that the paradigm's limitations as with every other paradigm, are acknowledged as well.

CHAPTER FIVE

SUMMARY CONCLUSION AND RECOMMENDATIONS

5.0 Overview

The research problem and methods are briefly described in this chapter, along with the research findings and their interpretation. Conclusions were reached in light of these observations. The chapter finishes with recommendations for math educators and curriculum designers, as well as suggestions for future research projects.

5.1 Summary

Quadratic equations find broad applications in both mathematics and related disciplines. However, the existing methods for solving these equations have faced numerous challenges and significant criticism. Consequently, teaching and learning this topic can become tedious, particularly when students lack the necessary cognitive experience to rely on. This raises the question of whether there are alternative conventional methods available to address this issue.

In response to this concern, the method of conjugate linear equations, a relatively uncovered method is explored. This study focused on investigating the relative effectiveness of the *conjugales* method, as a new approach, compared to the conventional factorization method in solving quadratic equations.

To evaluate the effectiveness of the novel *conjugales* method and the factorization method for solving quadratic equations, a group of students from Form One classes were selected. The study involved a total of fifty-seven (57) students from two different classes, 28 and 29 respectively.

These classes were both general art classes with one a geography and elective mathematics class and the other a literature with no elective mathematics.

For the assessment, a pre-test was conducted before the main treatment. It consisted of fifteen (15) multiple-choice items and 5 essay-type questions with the instruction of "Answer all Question: Time One Hour". The post-test and retention test were each composed of five (5) essay-type items. The reliability coefficients associated with the pre-test, post-test, and retention test were 0.80, 0.87, and 0.85, respectively. The post-test and retention tests had higher reliability. This could be attributed to being that, the intervention had an impact on the students by the time of the tests.

The achievement-tests were scored using a marking scheme designed for each test. Mean score differences on the post-test and retention test were analyzed using an independent *t*-test. The post-test was administered immediately after the instructional period, while the retention test was given after four weeks.

Considering the study's use of a quantitative research strategy, it's vital to recognize that the results could not be entirely indicative of circumstances in other communities across the nation.

The specific characteristics of the selected school, students, and their individual orientation in the Savannah Region could influence the results. Therefore, to enhance the robustness and generalizability of the findings, it is recommended to replicate the study in various regions of the country, encompassing both urban and rural areas. By including a more diverse range of settings, researchers can make inter-regional comparisons, which would provide a stronger basis for drawing more generalized conclusions about the relative effectiveness of the *conjugales* method and the traditional *factorization* method in solving quadratic equations.

Inherent in any research undertaking, including this study, are certain limitations stemming from the presence of confounding variables, which can introduce complexities and potential biases to the findings. Although rigorous efforts were made to control for these variables, their influence on the outcomes cannot be entirely eliminated. Despite this challenge, it is important to note that the data collected successfully fulfilled the required statistical assumptions, bolstering the reliability and validity of the study's analyses. However, one noteworthy constraint that needs to be acknowledged is the relatively modest sample size. This limited scope in participant numbers could have constrained the statistical power of the analyses, potentially affecting the ability to detect smaller, yet significant, effects. Consequently, while the study's findings offer valuable insights, future research endeavors with larger and more diverse samples could further enhance the robustness and generalizability of the conclusions.

5.2 Major Findings

The following were the major findings:

- There was significant difference in the mean post-test score of students in the control and experimental group.
- There was no significant difference in the test scores of male and female students.
- There was significant difference in retention potential of students in the experimental group.
- There was no difference in time used by students in the post-test for both groups.

5.3 Discussion and Interpretation of Findings

The findings derived from this research, revealed compelling evidence in support of the *conjugales* method's effectiveness. The results demonstrated that the students who underwent instruction using the *conjugales* approach significantly outperformed their counterparts from the factorization group in the post-test evaluation. This notable disparity in performance indicates that the method of conjugate linear equations exhibits a relative superiority and viability in tackling questions and problems related to quadratic equations. The promising outcomes of this study lend credence to the initial postulation made by Gyening (1988), who originally developed and proposed the *conjugales* method, along with other innovative approaches. The positive validation of Gyening's work through this study not only bolsters the credibility of the CONJUGALES method but also highlights its potential as a valuable alternative to conventional techniques in mathematics education. These encouraging results offer a compelling basis for considering the wider integration and adoption of the CONJUGALES method in educational curricula to enhance students' proficiency and understanding in quadratic equation problem-solving.

Another noteworthy finding of the research revealed that there was no significant difference in the test scores between male and female students. Both genders performed comparably well in the post-test, indicating that the CONJUGALES method was equally effective for both male and female students. This gender balance in achievement suggests that the innovative teaching approach can promote inclusivity and equity in mathematics education, fostering equal opportunities for all students to excel in quadratic equation problem-solving.

The study also found a significant difference in the retention potential of students in the experimental group. Students who were taught using the CONJUGALES method displayed higher retention of knowledge and skills concerning quadratic equations. This result implies that the innovative approach not only leads to better immediate performance but also facilitates better long-term retention of the learned concepts, providing a more solid foundation for future mathematical learning.

Interestingly, there was no significant difference in the time taken by students in both groups to complete the post-test. Despite the variations in their test scores, the students from both the experimental and control groups spent a comparable amount of time on the post-test. This finding suggests that the CONJUGALES method's superior performance was not attributed to students rushing through the test, but rather to a more profound understanding and proficiency in applying the concepts taught during the instructional period.

5.4 Conclusion

In conclusion, the research findings provide compelling evidence supporting the superiority and viability of the ***conjugales*** method in comparison to the traditional factorization approach for solving quadratic equations. The significant difference in mean post-test scores between the control and experimental groups highlights the method's ability to enhance students' problem-solving abilities and comprehension in mathematics. Moreover, the study reveals the method's inclusive nature, with no significant gender differences observed in test scores, fostering equal opportunities for male and female students to excel in quadratic equation problem-solving. Furthermore, the ***conjugales*** method exhibits a clear advantage in promoting long-term retention of knowledge, indicating its potential to lay a solid foundation for future mathematical learning. The research also emphasizes that the method's efficacy does not compromise on time efficiency, as students from both groups completed the post-test within similar timeframes. These collective findings underscore the significance of incorporating the ***conjugales*** method into the syllabus, paving the way for improved student outcomes and enriching the teaching and learning of quadratic equations.

5.5 Shortcomings

- Learning Curve: The method of ***conjugales*** might require a steeper learning curve for both students and teachers. Teachers might need more time and training to effectively implement this innovative approach in their classrooms.
- Lack of Educational Resources: Implementing the ***conjugales*** method might require specific educational resources, such as textbooks, instructional materials, or software, which may not be readily available in all educational settings.
- Teacher Preparedness: The successful implementation of the CONJUGALES method relies heavily on teacher preparedness, expertise, and familiarity with the approach. Inadequate training or support could hinder its effectiveness in the classroom.
- Limited Research: Depending on the novelty of the method, there might be limited research on its long-term effects and outcomes. This lack of extensive research may raise uncertainties about its overall effectiveness and sustainability.
- Generalization: While the study indicated positive outcomes in specific contexts, the method's generalizability to different student populations, educational levels, or cultural settings remains uncertain.

In conclusion, while the method of CONJUGALES shows promise in solving quadratic equations, its potential shortcomings should be considered and addressed to ensure its successful integration into mathematics education and to maximize its benefits for students' learning experiences.

Continued research and further exploration of its strengths and limitations will be essential for a comprehensive understanding of its effectiveness.

5.6 Recommendations and Suggestions for further studies

- The post-test mean score of students in the control (factorization) and experimental (conjugales) group favoured the conjugales group. In light of this, the Ghana Education Service, heads of institutions, and various stakeholders should organize workshops and seminars to train teachers on the pedagogy of the ***conjugales.***
- Male and female students had relatively equal scores using the *conjugales* approach in solving quadratic equations. It is therefore recommended that CAMFED, UNGEI as well as all other concerned stakeholders should capitalize on this finding, to advocate to the girl child that, not only in quadratics, but in all other aspects of mathematics, she can perform equally well and probably better than their male counterparts.
- Based on the finding that there was a significant difference in the retention potential of students in the experimental group, it is recommended that educational institutions consider the incorporation of the CONJUGALES method into their mathematics curriculum. This innovative teaching approach has demonstrated its ability to not only improve immediate performance but also enhance the long-term retention of mathematical knowledge. By adopting the CONJUGALES method, schools can better equip their students with the skills and understanding needed for sustained success in quadratic equation problem-solving and other related mathematical concepts.
- Based on the finding that there was no difference in the time used by students in the post-test for both groups, it is recommended that educators and curriculum developers focus on

optimizing the instructional time allocated to quadratic equation problem-solving. Since no significant time advantage was observed between the CONJUGALES and traditional factorization methods, efforts should be directed towards ensuring that students have ample time to engage with and comprehend these mathematical concepts thoroughly. This underscores the importance of efficient time management during instruction to allow for a comprehensive exploration of quadratic equations and related problem-solving techniques.

- Further Research: GES-CRDD, Educational Researchers, tertiary students, mathematics teachers and lecturers alike should conduct further research with a larger and more diverse sample of students from different regions and educational settings. This will provide a more comprehensive understanding of the method's effectiveness across various demographics and contexts.
- Long-Term Assessment: GES-CRDD, heads of institutions and mathematics instructors should work hand in hand to extend the study's assessment to examine the long-term impact of using the ***conjugales*** method on students' mathematical abilities. Follow-up assessments conducted over an extended period will reveal its sustainability and effectiveness beyond the immediate post-test.
- Educational Resources: The Ministry of Education and Ghana Education Service should develop and provide appropriate educational resources, including textbooks, instructional materials, and software, to support teachers and students in their use of the ***conjugales*** method. Accessible resources will aid in the method’s effective implementation.

With these bodies, institutions, and all due stakeholders following these recommendations, educators and researchers can strengthen the implementation and effectiveness of the ***conjugales*** method, ultimately enhancing students' mathematical learning experiences and problem-solving abilities in quadratic equations.

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APPENDICES

APPENDIX A: PRE TEST

Answer all questions in both sections

Multiple Choice

1. Simplify the expression $3x^2 + 4x^2 - 2x - 5$:
a) $7x^2 - 2x - 5$
b) $7x^2 + 2x - 5$
c) $7x^2 - 2x + 5$
d) $7x^2 + 2x + 5$
e) None of the above

2. Factor the expression $x^2 + 5x + 6$:
a) $(x + 2)(x + 3)$
b) $(x - 2)(x - 3)$
c) $(x + 2)(x - 3)$
d) $(x - 2)(x + 3)$
e) None of the above

3. Solve for x: $2x + 5 = 11$
a) $x = 3$
b) $x = 4$
c) $x = 5$
d) $x = 6$
e) None of the above

4. Which of the following is a solution to the equation $x^2 - 9 = 0$?
a) $x = -3$
b) $x = 0$
c) $x = 3$
d) $x = -9$
e) None of the above

5. What is the slope-intercept form of the equation of a line?
a) $y = mx + b$
b) $y = mx - b$
c) $y = bx + m$
d) $y = bx - m$
e) None of the above

6. Simplify the expression $2(3x - 4) - 5(2x + 1)$:
a) $x - 13$
b) $x + 13$
c) $-5x - 13$
d) $5x - 13$
e) None of the above

7. Solve for x: $3x - 2 = 7x + 6$
a) $x = -2$
b) $x = -1$
c) $x = 1$
d) $x = 2$
e) None of the above

8. Which of the following is equivalent to the expression $(x + 3)(x - 4)$?
a) $x^2 - x - 12$
b) $x^2 + x - 12$
c) $x^2 - x + 12$
d) $x^2 + x + 12$
e) None of the above

9. Evaluate the expression $2x^2 - 3xy + y^2$ when $x = 4$ and $y = 2$:
a) 2
b) 8
c) 12
d) 16
e) None of the above

10. Factor the expression $x^2 - 16$:
a) $(x + 4)(x - 4)$
b) $(x + 8)(x - 2)$
c) $(x + 2)(x - 8)$
d) $(x + 16)(x - 1)$
e) None of the above

11. What is the equation of the line that passes through the point (3, 2) and has a slope of 4?

a) $y = 4x - 10$

b) $y = 4x - 5$

c) $y = 4x + 2$

d) $y = 4x + 6$

e) None of the above

12. What is the slope-intercept form of the equation of the line that passes through the point (3, 5) and has a slope of -2?

a) $y = -2x + 11$

b) $y = -2x + 5$

c) $y = -2x - 1$

d) $y = -2x - 7$

e) None of the above

13. Solve for x: $5(x - 3) + 2 = 3x - 4$

a) $x = 1$

b) $x = 2$

c) $x = 4$

d) $x = 5$

e) $x = 6$

14. Simplify the expression: $4x^3 + 6x^2 - 3x^3 + 2x^2$

a) $x^3 + 8x^2$

b) $x^3 + 3x^2$

c) $x^3 - x^2$

d) $x^3 - 3x^2$

e) $x^3 + x^2$

15. Solve for x: $(x + 3)(x - 4) = 0$

a) $x = -3$

b) $x = 4$

c) $x = 3$ or $x = 4$

d) $x = -3$ or $x = 4$

e) $x = -3$ or $x = -4$

Theory

1) Solve the equation: $\frac{(2x + 1)}{(x - 3)} = 5$.

2) Solve the equation: $\sqrt{(2x + 3)} = 4$.

3) Solve the equation: $2x - 5 = 3x + 2$.

4) Simplify the expression:

   $(3x^2 - 2x + 4) - (2x^2 + 5x - 1)$.

5) Solve the inequality: $2x - 5 > 3x + 2$.

APPENDIX B: POST TEST FOR BOTH GROUPS

Instruction: ***To be given by invigilator***

Time: 60 minutes

1. $x^2 + 7x + 10 = 0$
2. $2x^2 + 5x - 3 = 0$
3. $3x^2 - 10x + 7 = 0$
4. $9x^2 - 16 = 0$
5. $x^2 + 2x - 8 = 0$

APPENDIX C: RETENTION TEST FOR BOTH GROUPS

Instruction: ***To be given by invigilator***

Time: 60 minutes

6. $2x^2 + 11x + 5 = 0$
7. $3x^2 - 14x + 8 = 0$
8. $4x^2 - 16x + 15 = 0$
9. $x^2 - 6x + 8 = 0$
10. $2x^2 + 3x - 9 = 0$

APPENDIX D: PRE-TEST MARKING SCHEME

(1). a	(2). a	(3). a	(4). a/c	(5).a
(6). c	(7). a	(8). a	(9). c	(10). a
(11). a	(12). a	(13). 4.5	(14). a	(15). d

1). $\frac{2x+1}{x-3} = 5$

2x+1 = 5x-15

2x = 16

X = 8

2). $\sqrt{(2x+3)} = 4$

2x+3=16

2x=13

X = $\frac{13}{2} = 6\frac{1}{2}$

3). 2x-5 = 3x+2

2x-3x = 2+5

-x = 7

X = -7

4). $(3x^2 - 2x + 4) - (2x^2 + 5x - 1)$

$3x^2 - 2x^2 - 2x - 5x + 4 + 1$

$x^2 - 7x + 5$

5). $2x - 5 > 3x + 2$

$2x - 3x > 5 + 2$

$-x > 7$

$x < -7$

APPENDIX E: POST-TEST MARKING SCHEME _FACTORIZATION

Paper 1

Excell 10

1) $x^2 + 7x + 10 = 0$

$x^2 + 5x + 2x + 10 = 0$ 2

$x(x+5) + 2(x+5) = 0$ 1

$(x+2)(x+5) = 0$ 1

$x + 2 = 0$ 1 or $x + 5 = 0$ 1

$x = -2$ (1) or $x = -5$ (1)

2) $2x^2 + 5x - 3 = 0$

$2x^2 + 6x - x - 3 = 0$ 1

$2x(x+3) - (x+3) = 0$ 1

$(2x-1)(x+3) = 0$ 2

$2x - 1 = 0$ (1) or $x + 3 = 0$ (1)

$\frac{2x}{2} = \frac{1}{2}$ (1) $x = -3$

$x = \frac{1}{2}$ (1) or $x = -3$ (1)

3) $3x^2 - 10x + 7 = 0$

$3x^2 - 3x - 7x + 7 = 0$ 2

$3x(x-1) - 7(x-1) = 0$ 2

$(3x-7)(x-1) = 0$ 2

$3x - 7 = 0$ (1) or $x - 1 = 0$ (1)

$\frac{3x}{3} = \frac{7}{3}$ $x = 1$

$x = \frac{7}{3}$ (1) $x = 1$ (1)

4) $9x^2 - 16 = 0$

$9x^2 + 12x - 12x - 16 = 0$ 2

$3x(3x+4) - 4(3x+4) = 0$ 2

$(3x-4)(3x+4) = 0$ 2

$3x - 4 = 0$, (1) $3x + 4 = 0$ (1)

$3x = 4$ $3x = -4$

$\frac{3x}{3} = \frac{4}{3}$ $\frac{3x}{3} = \frac{-4}{3}$

$x = \frac{4}{3}$ (1) or $x = -\frac{4}{3}$ (1)

5) $x^2 + 2x - 8 = 0$

$x^2 - 2x + 4x - 8 = 0$ 2

$x(x-2) + 4(x-2) = 0$ 2

$(x+4)(x-2) = 0$ 2

$(x-2) = 0$ (1) or $x + 4 = 0$ (1)

$x = 2$ (1) or $x = -4$ (1)

APPENDIX F: POST-TEST MARKING SCHEME _CONJUGALES

Paper 1

1) $x^2 + 7x + 10 = 0$ — V_1

$d = \sqrt{7^2 - 4(1)(10)} = \pm 3$ — F_1

$2ax + b = 2x + 7$ — P_1

P_1

$2x + 7 = 3 \qquad 2x + 7 = -3$ — $P_1 P_1$

$x = -2 \qquad x = -5$ — $A_2 A_2$

2) $2x^2 + 5x - 3 = 0$

$d = \sqrt{5^2 - 4(2)(-3)} = \sqrt{49} = \pm 7$

$2(2)x + 5 = \pm 7$

$x = -3, \quad x = \frac{1}{2}$

3) $3x^2 - 10x + 7 = 0$

$d = \sqrt{(-10)^2 - 4(3)(7)} = \pm 4$

$2(3)x - 10 = \pm 4$

$x = 1 \qquad x = \frac{7}{3} = 2.33$

4) $9x^2 - 16 = 0$

$d = \sqrt{0^2 - 4(9)(-16)} = \pm 24$

$2(9)x = \pm 24$

$x = \frac{4}{3}, \quad x = -\frac{4}{3}$

1.333

5) $x^2 + 2x - 8 = 0$

$d = \sqrt{2^2 - 4(1)(-8)} = \pm 6$

$2x + 2 = \pm 6$

$x = 2 \qquad x = -4$

APPENDIX G: RETENTION-TEST MARKING SCHEME _ FACTORIZATION

Paper II

1) $2x^2+11x+5=0$

$2x^2+10x+x+5=0$ 2

$2x(x+5)+(x+5)=0$ 1

$(2x+1)(x+5)=0$ 1

$2x+1=0$ 1 or $x+5=0$ 1

$\frac{2x}{2}=\frac{-1}{2}$ 1 $\quad x=-5$ 1

$x=-\frac{1}{2}$ 2 $\quad x=-5$ 2

2) $3x^2-14x+8=0$

$3x^2-12x-2x+8=0$ 2

$3x(x-4)-2(x-4)=0$ 2

$(3x-2)(x-4)=0$ 2

$3x-2=0$ 1 or $x-4=0$ 1

$3x=2$ or $x=4$

$\frac{3x}{3}=\frac{2}{3}$

$x=\frac{2}{3}$ 1 or $x=4$ 1

③

3) $4x^2-16x+15=0$

$4x^2-10x-6x+15=0$ 2

$2x(2x-5)-3(2x-5)=0$ 2

$(2x-3)(2x-5)=0$ 2

$2x-3=0$ 1 or $2x-5=0$ 1

$2x=3 \qquad 2x=5$

$\frac{2x}{2}=\frac{3}{2} \qquad \frac{2x}{2}=\frac{5}{2}$

$x=\frac{3}{2}$ 1 or $x=\frac{5}{2}$ 1

4) $x^2-6x+8=0$

$x^2-4x-2x+8=0$ 2

$x(x-4)-2(x-4)=0$ 2

$(x-2)(x-4)=0$ 2

$x-2=0$ 1 or $x-4=0$ 1

$x=2$ 1 or $x=4$ 1

5) $2x^2+3x-9=0$

$2x^2+6x-3x-9=0$ 2

$2x(x+3)-3(x+3)=0$ 2

$(2x-3)(x+3)=0$ 2

$2x-3=0$ 1 or $x+3=0$ 1

$2x=3$ or $x=-3$

$\frac{2x}{2}=\frac{3}{2}$

$x=\frac{3}{2}$ 1 or $x=-3$ 1

APPENDIX H: RETENTION-TEST MARKING SCHEME _ CONJUGALES

Paper II

1) $2x^2 + 11x + 5 = 0$ V1 F1

$d = \sqrt{11^2 - 4(2)(5)} = \pm 9$ P1 P1

$2(2)x + 11 = \pm 9$ P1 P1

$x = -\frac{1}{2}$ $x = -5$ A2 A2

2) $3x^2 - 14x + 8 = 0$

$d = \sqrt{(-14)^2 - 4(3)(8)} = \pm 10$

$2(3)x + (-14) = \pm 10$

$6x - 14 = \pm 10$

$x = 4$ $x = \frac{2}{3} = 0.667$

3) $4x^2 - 16x + 15 = 0$

$d = \sqrt{(-16)^2 - 4(4)(15)} = \pm 4$

$2(4)x - 16 = \pm 4$

$8x - 16 = \pm 4$

$x = \frac{5}{2}$ $x = \frac{3}{2}$

2.5 1.5

4) $x^2 - 6x + 8 = 0$

$d = \sqrt{(-6)^2 - 4(1)(8)} = \pm 2$

$2x - 6 = \pm 2$

$x = 4$ $x = 2$

5) $2x^2 + 3x - 9 = 0$

$d = \sqrt{3^2 - 4(2)(-9)} = \pm 9$

$2(2)x + 3 = \pm 9$

$4x = \pm 9 - 3$

$x = \frac{3}{2}$ $x = -3$

1.5

APPENDIX I: TABLE OF STUDENTS' ACHIEVEMENT SCORES

Achievement test scores					
Control group			Intervention group		
Pre-test	Post-test	Retention-test	Pre-test	Post-test	Retention-test
22	0	0	4	23	15
5	0	0	4	44	33
19	38	35	4	50	50
25	37	40	8	50	46
16	0	0	4	50	40
20	0	0	14	50	50
17	25	29	3	50	50
5	37	30	5	50	40
18	36	19	8	33	17
29	24	10	7	47	50
29	34	30	6	17	16
29	18	19	7	42	39
31	0	0	6	40	30
7	34	22	3	46	42
25	0	2	10	6	9
26	8	2	6	47	40
25	20	12	5	39	43
18	11	23	5	50	50
33	18	20	5	19	23
28	32	41	9	26	22
25	25	22	17	50	50
21	5	15	3	46	27
31	4	5	7	48	44
31	24	16	7	50	48
21	28	33	6	18	5
29	2	9	5	22	10
30	40	20	10	11	12
			10	28	20

APPENDIX J: RELIABILITY TESTS:

Pre-test

Correlation		Control	Intervention
Test	Cronbach Alpha	1	.801**
	Sig. (2-tailed)		.000
	N	20	20
Retest	Cronbach Alpha	.801**	1
	Sig. (2-tailed)	.000	
	N	29	28

**. Correlation is significant at the 0.01 level (2-tailed).

Post-test

Correlations		Control	Intervention
Test	Cronbach Alpha	1	.869**
	Sig. (2-tailed)		.000
	N	20	20
Retest	Cronbach Alpha	.869**	1
	Sig. (2-tailed)	.000	
	N	29	28

**. Correlation is significant at the 0.01 level (2-tailed).

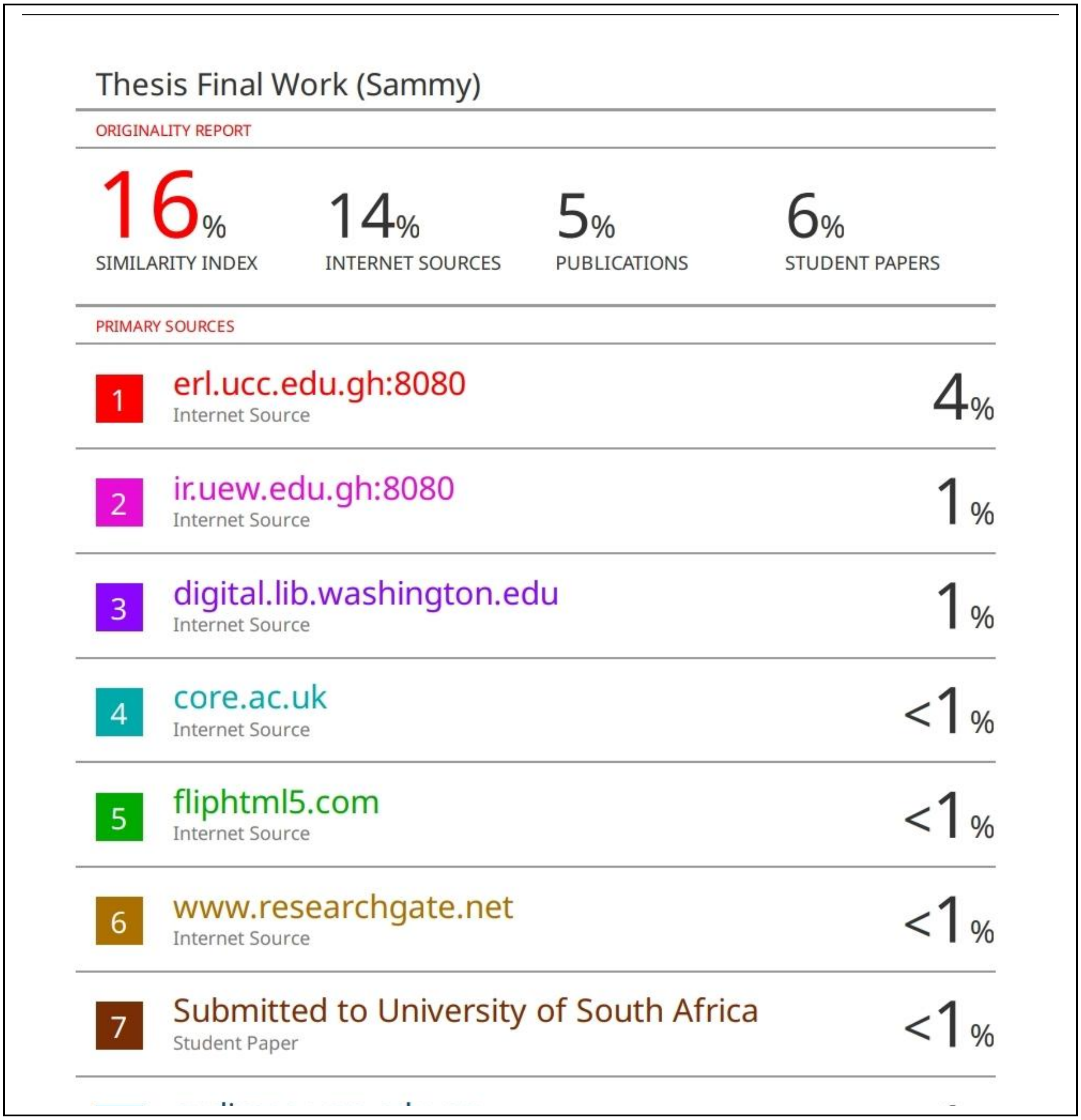
Retention-test

Correlations		Control	Intervention
Test	Cronbach Alpha	1	.852**
	Sig. (2-tailed)		.000
	N	20	20
Retest	Cronbach Alpha	.852**	1
	Sig. (2-tailed)	.000	
	N	29	28

**. Correlation is significant at the 0.01 level (2-tailed).

APPENDIX K

Plagiarism test (turn it in (16%))



Thesis Final Work (Sammy)

ORIGINALITY REPORT

16%	14%	5%	6%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	erl.ucc.edu.gh:8080 Internet Source	4%
2	ir.uew.edu.gh:8080 Internet Source	1%
3	digital.lib.washington.edu Internet Source	1%
4	core.ac.uk Internet Source	<1%
5	fliphtml5.com Internet Source	<1%
6	www.researchgate.net Internet Source	<1%
7	Submitted to University of South Africa Student Paper	<1%