AKENTEN APPIAH-MENKA UNIVERSITY OF SKILLS TRAINING AND

ENTREPRENEURIAL DEVELOPMENT

THE EFFECT OF USING GEOGEBRA ON BEREKUM COLLEGE OF EDUCATION STUDENTS' ACADEMIC ACHIEVEMENT IN CIRCLE THEOREMS

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THE EFFECT OF USING GEOGEBRA ON BEREKUM COLLEGE OF EDUCATION STUDENTS' ACADEMIC ACHIEVEMENT IN CIRCLE THEOREMS

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DECLARATION

STUDENT'S DECLARATION

I, Emmanuel Teku, declare this thesis, with the exception of questions and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature: ………………………………… Date: ………………………

SUPERVISORS' DECLARATION

We hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines or supervision of thesis as laid down by the Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Kumasi.

Prof. Yarhands Dissou Arthur (Principal Supervisor)

Signature: ………………………………….. Date: ……………………….

Prof. Francis Ohene Boateng (Co-Supervisor)

Signature: ………………………………….. Date: ………………………

DEDICATION

To God be the glory, I dedicate this work to my family and friends.

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ABSTRACT

The purpose of the study was to investigate the effect of using GeoGebra software on Berekum College of Education students' academic achievement in circle theorems. The study assessed the usefulness, ease-of-use and challenges of GeoGebra in teaching and learning of circle theorems. The study made use of purposive sampling to select level 100 students as the population and simple random sampling (lottery method) techniques to select participants for the study. The study used mixed method (Embedded) approach and quasi-experimental design. The sample size for the study was 240 students. The two groups, experimental and control were 127 Bachelor of Education (Primary) option and 113 Bachelor of Education (Junior High School) option respectively. The study used pre-test and post-test, questionnaires and interviews to collect data. The pre-test and post-test scores were descriptively (mean and standard deviation) and inferentially (paired sample t-test) analyzed. Data gathered from questionnaires were descriptively (mean and standard deviation) analyzed and the qualitative data gathered through interviews were thematically analyzed. The students who used GeoGebra software in the teaching and learning of circle theorems performed better than their peers who did not use GeoGebra. The study also found that the use of GeoGebra in teaching and learning increased student's interest and motivated them to learn circle theorems. Based on the findings, it was concluded that integrating the use of computers and interactive educational software into the teaching and learning of mathematics will help improve students understanding in mathematics and find the geometry interesting to learn.

CHAPTER ONE

INTRODUCTION

1.0 Overview

The chapter consists of the background to the study, statement of the problem, the study's purpose, objectives of the study, research questions and hypotheses, the significance of the study, its delimitation, limitations, and organization of the entire study.

1.1 Background to the Study

Since the 1980s, several computer applications have been created for use in business, education, society and administration (Fogg, 2002). Computer programs are written to carry out particular functions or to address a set of issues. These software products are intended to facilitate and expedite work. Fogg (2002) said that computing had significantly altered social and cultural dynamics. The employment of computer programs has many uses and advantages. Computers and other technical instruments have been incorporated into every facet of the American educational system (Eady & Lockyer, 2013). In the classroom, new ideas like the inverted classroom and computer-assisted instruction (CAI) are being implemented (Trelease, 2016).

The use of computers in education has sparked a beneficial shift in how teaching and learning are conducted, particularly in mathematics (Gunga & Ricketts, 2008). As a result, developing instructional strategies that would aid pupils in learning mathematics falls under the purview of mathematics teachers. Since the middle of the 2000s, emphasis has been placed on using technology as a teaching tool to improve student achievement (Ross, Morrison, & Lowther, 2010). Thus, programs and teaching methods that significantly affect both instruction and student achievement have been examined by policymakers and educators (Shieh, 2012).

Computer programs are another major influence in the field of mathematics. Algebra, calculus, geometry, trigonometry, statistics, probability, and Microsoft mathematics are just a few of the subjects for which computer programs like MATLAB, Maxima, GraphCalc, Scilab, GeoGebra, etc. have been developed.

The positive impacts of employing interactive software on students' learning have been shown in several studies done in the 1990s and 2000s (Cavanaugh, 2001; Christmann, Badgett & Lucking, 1997; Fried, 2008; Page, 2002). These studies show how technology, such as interactive software, improves student achievement. According to Agyei and Benning (2015), the use of computer (interactive) software in the classroom has helped to advance the technological, pedagogical, and subject-matter expertise of math educators. The essence of using technology to teach and learn any subject is using computers and other technological tools like calculators, projectors, software, and the internet both inside and outside of the classroom context (Chai, Koh & Tsai, 2013).

Hackett and Betz (1989) opined that mathematics self-efficacy (task-specific) was found to be a greater predictor of job choice than examination performance. Additionally, it has been asserted that self-efficacy beliefs and their influence on performance and career choice would be enhanced by perceived parental and instructor support for pursuing mathematics (Hackett, 1995). In a study on mathematics self-efficacy and achievement, Mahmud and Jamil (2019). Mahmud and Jamil, (2019), discovered that great students have a higher level of motivation to solve mathematical problems than mediocre and weak students. Mahmud (2001) finding was confirmed by Faridah (2004) that excellent students have high level of aptitude and skills towards mathematical problem solving.

All levels of education in Ghana place a high value on the study of mathematics. It is a topic that all students in the colleges of education are required to learn. For tertiary education in the nation, a passing grade is needed. For recruitment to state organizations like the police service, military, and immigration services, a passing grade in mathematics is one of the requirements.

In the discipline of mathematics, geometry is a very important component. It is a branch of mathematics that examines connections between and characteristics of solid, spherical, and flat shapes. Understanding the world through geometry is significant (Schopenhauer, 2016). Regarding architecture, machinery, and pretty much everything else that is made with today's technology, it is a crucial component of the modern world of mathematics. Geometry education and learning in Ghana's elementary, secondary and tertiary levels is encouraged and recommended in this regard. According to Noss, Healy, and Hoyles (1997), teaching geometry requires assisting students in developing a clear mental image of objects that they may find challenging to perceive and comprehend. However, it is because of this that geometry is a challenging subject to study.

To aid in the teaching and learning of geometry, a number of resources have been developed (Schopenhauer, 2016). A compass, dividers, protractors, and set squares are a few of the construction equipment items. Additionally, educators employ geometry charts to help students have a better conceptual understanding of the subject (Hooper & Rieber, 1995). The same is true for computer programs, which have developed into useful tools that have facilitated the learning of geometric ideas (Li & Ma, 2010). Many computer programs exist that have improved pupils’ mathematical achievement (Oldknow, Taylor, & Tetlow, 2010). The 2001 invention of Marcus Hohenwarter and Yves Kreis’ GeoGebra software is one example of such a program. Geogebra is intended for use at the pre-tertiary and tertiary levels of mathematics instruction and learning. In other words, the GeoGebra software integrates components from both computer algebra and dynamic geometry systems (Hohenwarter, Kreis, & Lavicza, 2008). It ties mathematics to algebra, geometry, and analytics. A mathematics application called the Computer Algebra System uses mathematical expressions in a manner that is analogous to the way that manual calculations were once performed. It is useful for users who work in a mathematical subject where manipulating mathematical expressions is required. The DGS is a program that may be used to construct and manipulate geometric figures in plane geometry.

For the purpose of making mathematics instruction simple and engaging, GeoGebra was developed for both teachers and students. GeoGebra can be described as a powerful and essential tool for creating links between concepts in geometry and algebra (Pierce & Stacey, 2011). The package is therefore appropriate for kids in elementary school through senior high school and tertiary education. The program can also be used for straightforward structural and functional solutions. Velichova (2011) proposed that students can use GeoGebra to investigate mathematics on their own or with the teacher serving as a guide and further said the program improves pupils’ capacity for visualizing geometrical ideas. The purpose of the current study was to investigate how GeoGebra might be used to teach geometry in colleges of education.

1.2 Statement of the Problem

Mathematicians, scientists, and educational stakeholders have all taken a keen interest in the teaching and learning of geometry (Mesa, Gómez, & Cheah, 2012). Clements (2004) suggested that knowledge and abilities gained from circle theorems assist pupils in developing their capacity for critical thinking and problem-solving.

The computer program GeoGebra has been widely used in mathematics, particularly geometry (Diković, 2009; Hohenwarter & Fuchs, 2004). Velichova (2011) opined that to help children learn geometry better, teachers have adopted GeoGebra in the classroom. By producing two- and three-dimensional geometric visuals, GeoGebra helps students gain comprehension. It has also been demonstrated that using interactive software as a teaching tool in the subject of geometry actually improves students' achievement.

Research indicates that student interest in learning and understanding circle theorems have been unimpressive. Researchers have shown that difficulty in teaching and learning mathematics, circle theorems in particular, have resulted in mass failure in examinations (Adolphus, 2011). The trend of colleges of education students' achievement in circle theorems has been on the decline (Institute of Education - UCC, Chief Examiner's Report for EBS 143: Geometry and Trigonometry, 2018, 2020, 2021) and first-year students of Berekum College of Education was no exception. This poor achievement in circle theorems existed as a result of the traditional approach of teaching and learning which ultimately made students passive learners and deficient in geometrical analysis and reasoning (Mereku, 2010). Mereku, (2010) further added that, method of teaching and learning geometry played more emphasis on how much a student can remember to the detriment of how well the student can reason it also made the teacher dominate the classroom and turns students to mere listeners.

For these reasons students were not encouraged to discuss, interact and explore the content collaboratively, and repeatedly fail to build the exploration and visualization skills. This mode of teaching geometry (circle theorems) led to poor achievement of students as noted by Bittista (2007) and Idris (2006). Again, several reports of the West African Examination Council (WAEC) indicate that students who took WASSCE have been performing poorly in circle theorems questions (WAEC, 2018). In June 2018, the chief examiner for core mathematics stated that most candidates who answered question 8 (a) demonstrated their understanding of geometrical concepts were woefully inadequate. Candidates could not apply the cyclic quadrilateral theory and other geometrical principles to solve the problem. Mostly students avoid circle theorems questions when they have other alternatives. On rare occasions, few students attempted questions on circle theorems and those who attempted most of them demonstrated lack of understanding of the topic (Fletcher & Anderson, 2012). Since students at the colleges of education were the product of these senior high schools, they came with little or no understanding of the circle theorems.

The few studies that have been conducted on the effect of ICT tools (GeoGebra) on the teaching and learning of mathematical concepts in Ghana (Tay & Wonkyi, 2018) focused on the teaching and learning of mathematics at the pre-tertiary education level. This is the phenomenon that is observed from literature on the world stage as well. The characterization of the Ghanaian mathematics classroom is caused by this, since the colleges of education is where teachers for all the basic educational levels are trained.

Owusu, Bonyah and Arthur (2023) conducted research on the effect of GeoGebra on University students' understanding of polar coordinates. They suggested that mathematics teachers at all levels including university lecturers should be encouraged to integrate GeoGebra or similar technological software capable of teaching and learning mathematics concepts.

This demands that the use of ICT tools in learning mathematics be studied at college level to contribute to the advocacy of using ICT tools in teaching mathematical concepts at basic levels. A study at the colleges of education level on the effectiveness of circle theorems however is one of the concepts in mathematics that the effect of GeoGebra has not been felt. It is against this backdrop that this study was designed to look into the effect of using GeoGebra on Berekum College of Education students' achievement in circle theorems.

1.3 Purpose of the Study

The purpose of this study was to investigate the effect of using GeoGebra in teaching circle theorems on the achievement of Berekum College of Education students in the Bono Region of Ghana. The emphasis was to discover whether the method of instruction (using GeoGebra) would motivate students to learn, enhances their problem-solving techniques and ultimately improve their achievement in geometry (circle theorems).

1.4 Objectives of the Study

The following objectives served as the study's guiding principles:

1. to find out the mean difference in achievement between students who were taught circle theorems using GeoGebra and students without GeoGebra.

2. to assess the usefulness of GeoGebra in the teaching and learning of circle theorems.

3. to determine the ease-of-use of GeoGebra in teaching and learning circle theorems.

4. to find out students' perceived challenges of using GeoGebra in learning circle theorems.

5. to explore how mathematics self-efficacy influences students' achievement in circle theorems.

1.5 Hypothesis

In the study, the following null hypothesis was tested:

H_o: There is no statistically significant mean difference between achievement of students taught circle theorems using GeoGebra and those taught without using GeoGebra.

1.6 Research questions

The research questions that underpinned the study were the following:

1. what are the usefulness of GeoGebra in the teaching and learning of circle theorems?

2. what are the ease-of-use of GeoGebra in teaching and learning circle theorems?

3. what are the students' perceived challenges of using GeoGebra in learning circle theorems?

4. how does mathematics self-efficacy influence students' achievement in circle theorems?

1.7 Significance of the Study

The findings of this study will be utilized to design a professional development plan that would educate mathematics tutors in colleges of education on the effect of utilizing GeoGebra to teach circle theorems. This study will therefore serve as a source of reference material to all stakeholders of education including Colleges of Education Mathematics Tutors, Ghana Education Service, various religious education units and other organizations who have interest in mathematics education. The study will also motivate teachers at all levels of education to incorporate ICT into the teaching and learning of mathematics. This will improve the quality of education in Ghanaian schools more especially at the Berekum College of Education where the research will be carried out. The study will also serve as a guide to mathematics educators in finding alternative and /or supplementary ways of teaching circle theorems instead of the usual talk and chalk method of teaching mathematics.

Students will find it easy to solve geometry problems using the software. The community would as well benefit as students will use knowledge gained in solving environmental problems. Lastly, the study will also be used as conceptualization for further research in the area of teaching and learning of circle theorems.

1.8 Delimitation

This study's focus was only on first year students of Berekum College of Education in Bono Region of Ghana. The first-year (level 100) students were used for the study because Geometry was a course of study in first-year and that was where the problem was predominant. Only geometry teaching and learning were the subject of the study. Because of this, the software's application was limited to certain disciplines and academic levels. Although there were numerous computer programs that could be utilized as teaching aids for circle theorems, this study primarily focused on GeoGebra.

1.9 Limitation

The population was all the first-year students of the Berekum College of Education which followed the structured academic curriculum and calendar as prescribed by the affiliate university -University of Cape Coast.

The study was based on a purposeful sample of Berekum College of Education students from Ghana's Bono Region. As a result, there was a restriction because the sampling process was founded on the deliberate selection of a sampling unit. The sampling procedure therefore limits the extent of application of the results.

1.10 Operational Definition of Terms

GeoGebra: GeoGebra is an interactive geometry, algebra, statistics, calculus application and so on, intended for learning and teaching mathematics and science from basic school to tertiary level. GeoGebra helps students to visualize mathematical concepts that can seem abstract at times. It's a tool that allows students to visualize and manipulate the geometry, functions, vectors and so on.

Circle theorems: Circle theorems are properties that show relationships between angles within the geometry of a circle. Students can use these theorems along with prior knowledge of other angle properties to calculate missing angles, without the use of a protractor.

Self-efficacy: Self-efficacy is defined as a person's judgment of his or her own abilities to compose and execute the tasks to achieve optimal performance.

Self-efficacy can be defined as the students' "judgements of their capabilities to organise and execute courses of action required to attain designated types of performances"

(Bandura, 1988, p. 391) and is deemed to have a strong influence on individual choices, efforts, perseverance and emotions related to the tasks. Self-efficacy can be regarded as one part of a comprehensive personal theory about the learner's own learning process, which directs his or her own learning (Bandura 1997).

Mathematics Achievement: Mathematical Achievement is the competency shown by the student in the subject mathematics. Its measure is the score on an achievement test in mathematics

1.11 Organisation of the Study

There were five chapters in the thesis. Chapter One presented the background to the study, problem statement, the purpose of the study, research questions, the significance of the study, delimitation, and limitations of the study, as well as the study's organizational structure. Chapter Two reviews literature on the theoretical framework, empirical review and conceptual framework of the study. Chapter Three discusses the methodology while Chapter Four presents the analyses of the data collected. The last chapter, Chapter Five, which presents the summary, conclusion and recommendations.

1.12 Chapter Summary

This chapter provided a broad background for the study, giving it focus and direction. The chapter began by introducing interactive software as a cutting-edge tool for the teaching and learning of circle theorems. The study's difficulty was then described, underscoring the necessity of doing the study. The study's purpose and its precise objectives were further stated in the chapter. The research hypotheses and questions were then presented. The relevance of the study was then examined. The study's delimitation and limitations were both acknowledged and presented. Before the chapter summary, the final section, which is the study’s organization, was provided.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This would review literature relevant to the study on theoretical framework, empirical review and conceptual framework for study.

2.1 Theoretical framework

According to McCombes (2022), a theoretical framework is a foundational review of existing theories that serves as a roadmap for developing the arguments a researcher uses in a research work.

2.1.1Constructivism philosophical viewpoint

The study was guided by the constructivist philosophical viewpoint, which holds that learners play an important role in learning.

Constructivism is a learning theory that views the learning environment as a “mini-society, a community of which learners engaged in activity, discourse, interpretation, justification, and reflection” (Fosnot, 2013). According to constructivist education theory, knowledge is individually constructed by the student, and learning occurs in a social environment (the classroom), with some experiences that have been carefully constructed by the teacher.

Constructivist teachers encourage appreciation and consideration of the opinions or points of view expressed by others, as well as mutual respect, allowing for the development of creative thinking and independence. Meaning, according to a constructivist, it is the result of individuals (here, teachers) “establishing relationships, reflecting on their actions, and modeling and constructing explanations” (Fosnot, 2013 p. 280).

Contemporary theorists and researchers believe that real learning is about interaction, development, and growth rather than isolated student mastery of concepts (Fosnot, 2013).

Cognitive science holds that pupils learn by gradually structuring and restructuring their knowledge experiences and that profound conceptual learning necessitates structural changes in cognition, without which disorder would follow. In other words, one of the fundamental principles of constructivist philosophy is that knowledge is actively built. This point of view contends that a teacher's communication with and listening to their pupils helps them develop into readers and writers who think deeply and critically about what they read and write (Fosnot, 2013).

Collaboration between teachers and students helps them better understand each student’s learning style and, as a result, respond to each student’s needs. In most schools, Gablinske (2014) said that constructivist theorists’ focus on subject matter material has led to “a situation in which affective or relationship development is adversely influenced.” He contended that the creation of a beneficial network of interpersonal connections, which will rule the student’s academic experience, should be the main objective of constructivist-based education. “Interpersonal relations are the context for the child’s construction of the self, of others, and of subject-matter knowledge,” they claimed.

In agreement with the constructivist position, Bruner (1977) asserted that in order for schools to effectively serve as educational institutions, they must also actively support students' emotional and social growth. One of Bruner's four key learning themes is "stimulating the desire to learn, creating interest in the subject being taught, and what he termed intellectual excitement" (p. 11). Bruner (1977) also identified four other key learning themes. He stresses the significance of investigating the strategies employed by "successful" teachers in order to identify useful procedures.

Constructivist scholars are interested in the interactions between people in a particular situation, hence constructivism offers a natural framework and a central perspective for this study.

2.1.2 Representation Theory by Bruner

Bruner's Theory of Representation asserts that learning is reinforced when it is designed to follow steps to facilitate understanding. Manipulative materials are frequently used in elementary school but are rarely used in tertiary mathematics classes, although they have benefits.

Bruner's Theory of Representation has been used in the classroom to teach pre-algebra and algebra (Gningue, Menil, & Fuchs, 2014). According to this theory, a child experiences three stages of representation when exposed to new information: an active, an iconic, and a symbolic representation. During the active stage, the child needs to interact with materials to understand a concept. At the iconic level, a child visualizes the objects without actually manipulating them.

Last but not least, the child strictly manipulates symbols at the symbolic level and does not necessarily move the objects (Gningue et al., 2014). This theory can be used to explain how children develop and learn, as well as how students learn new material. When applying mathematical ideas that support Bruner's theory, three stages are described in the article, Research on the Benefits of Manipulative Materials. Then there is the concrete stage, where students are exposed to a manipulative and use it to explore a concept.

The mathematical concept is then represented using pictures to stand in for the manipulative in the representational stage, and students should show that they can both visualize and communicate the concept at a pictorial level. Symbols (such as numbers, operation signs, etc.) are then used to express the concept at the abstract level (‘Research on the Benefits of Manipulative Materials,’ 2017). With the help of manipulative materials, students can have a concrete experience and grow in their ability to reason abstractly. Students can improve their mathematical abilities and make connections between mathematical ideals using manipulative materials. Constructivist theory serves as the foundation for Bruner's theory. Students actively construct new ideas by drawing on their prior knowledge during the process of learning (Noreen, & Rana, (2019).

Schemas are cognitive structures that aid in organizing the world, and constructivist theory has something to do with them. People construct their schemas of a subject as they learn it and create new connections between what they have learned. In mathematics, students must develop ‘well-connected schemas’ and sense-making skills. This is done by not jumping straight to a formula but through a process where one can develop ideas from a concrete example to an abstract and symbolic idea.

In a study by Palatnik and Koichu (2017), students looked at a certain algebraic thinking problem about cutting up a pizza. They were to find the largest number of pieces that could be obtained by n straight cuts. They were first given the specific value of n. They went through this example, where they drew pictures and tables. They went through steps that allowed them to find patterns, generalize, and come up with a formula for this problem. When they had a discussion later about their findings, they were happy and excited to explain why it works. One student explained that if you only have a formula and you do not understand it or know its meaning, it is not interesting (Palatnik & Koichu, 2017).

In another study on a schematic-theoretic view of problem-solving in math, Steele and Johanning (2004) looked at students with what they called ‘well-connected schemas’ and ‘partially formed schemas.’ When going through a series of problems that on the surface do not seem to have a lot to do with each other, students with well-connected schemas were able to apply what they did in the last problem to the next to help them. When given a few concrete examples on the first problem that wanted to know how many squares there were in a border, they were eventually able to make connections with what they were doing and generalize it into a formula. In the next problem, students were able to take the formula generated in the last problem and apply it to the next, and so on for the next problems.

Without their knowledge of how they got the formula or why it worked, they would not have been able to connect it to the next problem. Those with partially formed schemas did not make those connections in the beginning, so even if they had a formula, it was hard for them to connect to the next problem what they just did (Steele & Johanning, 2004). This study hinged on Brunner’s theory of representation because the theory provided the needed context for students’ learning using the manipulative material.

2.1.3 Theory of Cognitive Development by Piaget

Piaget and Bruner, developmental theorists, provided the theoretical underpinnings for the use of manipulative material in the classroom. They contend that the capacity for abstract thought is not innate in children. Instead, they develop abstract ideas through interaction with the objects in their environment (Piaget & Bruner, 1966)

Children should therefore engage in physical, hands-on experiences with manipulative material to ingrain new ideas into their cognitive structure (Fennema, 1973). The idea of manipulative material is rooted in the constructivist theory of Jean Piaget (Ojose, 2008). The use of concrete manipulative materials can promote content mastery by aiding the growth of abstract reasoning (Carbonneau, Lezzoum, Voix & Gagnon 2013; Piaget, 1962). When students are allowed to use manipulative material, they are giving meaning to something that they may have only seen as abstract. Their senses are stimulated, as they are able to learn kinesthetically and make tactile connections (Castro, (2017).). Piaget's theory speaks to the idea that children should learn through their senses and, that it is necessary for them to experience and manipulate the ideas that they are learning. Increased student engagement in the learning process can lead to higher achievement (Ross & Willson, 2012). The continued drill-and-practice method of teaching children, especially with higher-level mathematics, is no longer regarded as best practice (Heddens 1986).

Students can learn mathematics by doing it themselves, but they cannot learn it by listening to a teacher lecture about it. Using manipulative material to involve students in their learning allows teachers to turn them from passive bystanders into active participants.

Through their active participation and the use of manipulative material, students can learn without teacher guidance and instead through discovery (Carbonneau et al., 2013). Additionally, using manipulative material gives students the chance to relate the math they are learning to their own meaning and experiences.

Additionally, it might assist in laying the groundwork required for students to access more abstract lessons (Ojose, 2008). Teachers have discovered that even the simplest manipulative material can help students learn more about beginning algebra (Allsop & Calabrese, 1999).

According to learning theories created by Dienes, Piaget, Skemp, and Brownell, if children's mathematical learning is based on experiences with manipulative materials, they may be able to bridge the gap between the real world and abstract mathematics (Kennedy, 1986). That is, when the children use manipulative materials, they can understand mathematical ideas meaningfully and transfer these ideas to real life situations easily (Yıldız & Tuncay, 2012).

Piaget (1973), in his Theory of Cognitive Development, described four stages of children's cognitive development: the sensorimotor stage (birth to age 2), the preoperational stage (ages 2 to 7), the concrete operational stage (ages 7 to 11), and the formal operational stage (age 11 onwards). While going through these stages, children first use physical actions and then use symbols to create schemas. In the concrete operational stage, children can organize data only if concrete objects are presented. Piaget (1952) stated that children cannot understand abstract mathematics through explanations and lectures and should have experiences with models and materials.

In order for children to understand mathematical concepts, Sowell (1989) contends that they must first engage in concrete, concrete-abstract, and pictorial-abstract learning experiences before engaging in strictly abstract ones. As a result, learning opportunities should be planned in accordance with the stages of cognitive development. Concrete and symbolic models must be used in the learning environment in accordance with cognitive development theory so that students with varying levels of development can benefit (Fennema, 1973).

2.1.4 Dewey’s Perspective on Constructivism

Dewey (1938) claimed that progressive education, which employs adult standards, curriculum, and teaching strategies, is “a consequence of disillusionment with traditional education.”

In his opinion, young students could not gain from traditional education in the manner I have just explained. According to Dewey’s concept of progressive education, young children should participate in socially engaging learning activities (Dewey, 1938).

Dewey thought that the key to good teaching was social contact and that schools should be seen as social institutions (Flinders & Thornton, 2021). According to him, education should be seen as a “process of living rather than a preparation for future life” (Flinders & Thornton, 2021, p. 355; Gutek, 2014). Dewey was distinguished from philosophers who favored the traditional classroom setting by this set of views. Dewey believed that schools and classrooms should be more like real-life situations than conventional classrooms, allowing kids to engage in learning activities interchangeably and adaptably in a range of social settings (Dewey, 1938; Gutek, 2014). In his view, it was unethical for teachers to abruptly introduce too much academic material out of context with children's social life (Flinders & Thornton, 2021). This idea would offer a source of disagreement in the current educational system because it differs noticeably from what is occurring in classrooms with the great emphasis on implementing the Common Core requirements.

John Dewey's theories are less frequently found in schools due to the emphasis on raising academic achievement through the adoption of Common Core standards in today's classrooms (Theobald et al, 2009). Many of the educational theories held by learner-centered teachers are supported by Dewey's work, according to them (Schiro, 2012).

John Dewey's social learning theory and educational philosophies are prevalent in learner-centered classrooms. He saw the classroom as a place where kids could interact socially while learning and solving problems as a group. In these classrooms, students construct their own knowledge through personal meaning rather than through teacher-imposed knowledge and teacher-directed activities because they are recognized as unique individuals (Schiro, 2012). In these classrooms, kids will be seen learning by doing and applying practical problem-solving techniques. When teachers plan lessons, they will take student interests into account and integrate the curriculum with a focus on project-based learning. The educational process includes not only academic growth but also the whole child's intellectual, social, emotional, physical, and spiritual development (Schiro, 2012).

2.1.5 Bandura's view on social cognitive theory

Social cognitive theory was based on the work of Bandura (1971), who first proposed a theory in which behavior is not only learned through a person's own experiences but can also be learned vicariously through witnessing the experiences of others. Later, Bandura (1977) expanded this theory to emphasize the importance of self-efficacy on behavioral change. Bandura (1977) explained that self-efficacy is a person's beliefs in his or her ability to perform a task or to learn a topic. Researchers have supported Bandura's contention that self-efficacy and performance modify each other to help individuals build an appraisal of their competence toward a mathematical task (Williams & Williams, 2010). Students with higher levels of self-efficacy have higher levels of general achievement in mathematics, more easily overcome negative outcomes, display more positive attitudes towards mathematics, and possess a more comprehensive understanding of mathematics (Phan, 2012a; Tariq & Durrani, 2012) and are more likely to enter and eventually graduate from college (Larson, Stephen, Bonitz & Wu, 2014; Parker, Marsh, Ciarrochi, Marshall & Abduljabbar, 2014).

The internal principle of social cognitive learning theory explains that people are only partial products of their environments (Bandura, 1997).

2.1.6 Rogers' Diffusion Innovation Model

Diffusion, according to Rogers (2010), is the "process by which an innovation is transmitted over time among the participants in a social system through particular channels" (p. 37). According to Rogers, public perception of a new technology's debut has a big impact on how quickly it spreads. GeoGebra is a novel technology (innovation) that was used in this study, and its effects on students' learning are pertinent. Since Rogers conflated innovation with technology, the diffusion of the innovation paradigm seems especially well suited for the study of ICT diffusion.

In order to facilitate the diffusion of innovations, Rogers proposed characteristics of innovations. Relative advantage, compatibility, intricacy, trialability, and observability are among these traits. In order to determine the effects of teaching the circle theorems using GeoGebra software, this study considered these aspects to be important.

2.2 Empirical Review

Empirical review analyses previous empirical studies in order to provide an answer to a specific research topic. The goal is to provide data that can be quantified using established scientific methods as reviewers thoroughly examine all findings of other authors before drawing any conclusions in a study.

2.2.1 Activity- Based Teaching and Learning

Activity-based Teaching and Learning: Learning by doing is very important in successful knowledge because it is proved that more the senses are inspired, more a person learns and longer he/she retains. Activities bring activeness and smartness among the learners. Because we know that education means all round improvement of the child, therefore we have to organize numerous activities to build up the learner's personalities in several ways. Activity-based instruction technique acts as a dynamic problem solver for the learners. It improves innovative part of experience and gives reality for learning. It gives various experiences to the learners to encourage the acquisition of information, experience, abilities and qualities. It builds the students self-confidence and creates understanding through works. It creates cheerful relationship and enthusiasm for them. If child is given the chance to investigate by his own and gave an ideal learning environment, then the learning gets to be cheerful and durable. It inspires the learners to apply their innovative ideas, information and minds in solving problems. Under Activity-based learning instruction key focus is on child or we can state that it is one of child focused approach. It creates self-learning ability among the students and allows a student to learn according to his or her ability. As noted in Johnson, Johnson and Smith (1998) (as cited in Ahlfeldt, Mehta, & Sellnow, 2005, p.52), "It is the old pattern to give all the resources to the inactive learner by the teacher. The innovative pattern is to dynamically connect learners with the resources and each other. According to Hussain, Anwar, & Majoka (2011) activity-based learning integrated with peer instruction creates an ideal situation for teaching science and mathematics subjects and specially mathematics. In an activity-based learning class, students are actively involved in hands-on experiences and get chance to relate abstract ideas and theories with concrete observations. This helps them to make deep understanding of scientific concepts. Çelik (2018) describes, it was seen that activity-based learning activities improve students' academic achievements and attitudes towards activities. According to Shah and Rahat (2014), activity-based learning teaching method generates an ideal situation for mathematics teaching especially at Elementary level. In activity-based teaching methods, learners are involved actively in hands-on minds on experiences and acquire an opportunity to relate intangible concepts and theories with actual observations. Activity-based teaching method helps learners to understand the scientific concepts. Students actively involved in teaching learning process and activities help them in application of scientific knowledge in various real-life situations. Activity-based mathematics instruction is based on activity by involving learners in reading, discussion, practical activities, engagement in solving problems, analysis, synthesis and evaluation (Festus, 2013). Innovative teaching methods that provide positive mathematical learning experiences could help to enhance students' achievement in mathematics (Riley et al., 2017). If the learner is provided with the opportunity to explore their environment and provided an optimum learning environment then the learning becomes joyful and long lasting. This learning strategy means reversing the traditional teacher-centered understanding of the learning process and putting students at the center of the learning process (Golji & Dangpe, 2016). As per Fallows and Ahmet (1999), education is best when learners' association, contribution and collaboration are maximized.

Noreen and Rana (2019) stated that activity-based learning technique is diverse from conventional technique of instructing. Learners take active part in it. Activity-based learning is such education in which learner is dynamically involved in doing or in considering something prepared. As Churchill (2003) said, such learning helps learners to make intellectual models that take into consideration higher-order presentation, for example, applied critical thinking and exchange of data a skill. Learners' inspiration by interfacing with learners in instinctive activities is a feasible and useful technique for instructing difficult ideas. They described the significance of various activities correlated to the thoughts being displayed. Learners' inspiration is high if these activities are face-to-face to the learners (Hug, Krajcik & Marx 2005). In lab strategy learning by doing may be possible as in activity-based teaching/learning.

2.2.2 Technology in Education

The use of technology in education began with the first Computer Assisted Instruction (CAI) developed by researchers at the IBM in the 1950s (Reiser, 2001). In 1959, Donald Bitier of the University of Illinois employed the first large scale use of computers in education (Choi, Dailey-Hebert, & Estes, 2016). During the 1980s, educators and researchers' interest in the use of technology in education initiated the use of certain applications in some subject areas (Reiser, 2001). One technological tool that that was of particular interest was the microcomputer (that is, personal computer) since it was inexpensive and could perform several functions that other large computers performed. By the 1990s, technological advancements started to have a significant impact on the nature and content of instructional practices (Spector & You-qun, 2016). Technological tools helped instructors to direct learning outcomes to include the development of problem-solving skills and creating the "independent learner". Juniu (2011) observed that the introduction of word processors, databases, spreadsheets and other tools helped both teachers and learners to adopt easier way of teaching and learning without relying on the use mental capacity on trivial tasks. Technological tools are now integral parts of modern educational systems. Krueger and Kumar (2004) have observed that in developed countries new technologies have integrated into the structure, concepts and methods of how teaching and learning are done.

Pilli and Aksu (2013) identified two broad categories of computers application in education. According to the authors, a computer can be an object of instructional lesson or process, or can be used as an instructional device. In the first case, computer literacy courses or programmes are essentially concerned with how the computer or other related technologies become objective of instructional lesson. In the second case, the computer as an instructional device is concerned with how the computer or other related technologies can be used as aid or tool to help achieve instructional goals. The idea of using a computer as an instructional device is termed as Computer Assisted Instruction (CAI) or blended instruction (Tayebinik & Puteh, 2013).

Computer-Assisted Instruction or blended instruction is essentially the use of computers or other related technologies to express any subject matter. Kaur (2013) defined blended instruction as the approach to teaching and learning that mix the conventional method of teaching with the use of technological tools. A blended instruction or CAI is used as a medium to deliver teaching and learning content and activities in a classroom setting. With new advancements in technology, the structure and nature of CAI have become a less rigid system of classroom teaching. However, certain core components are necessary for implementation. Cotton (2008) suggested that three components of a CAI or blended instruction should include a competent teacher or instructor, the learners or students and the computer interactive environment. Cotton (2008) asserted that the components of the blended instruction or the CAI is a union between the elements of the conventional method of instruction and the computer interactive environment.

The use of technology in education has had numerous benefits in teaching and learning. One main benefit of using technology in teaching and learning is the speedy processing of information. The capacity of a computer used in learning makes it easier for the learner to receive instructional information in either textual, diagrammatic or animated form (Berney & Bétrancourt, 2016). Roehl, Reddy and Shannon (2013) asserted that technology in a learning environment allows for interaction between learners and teachers, connectivity of ideas and concepts during the instructional process; it also enables the teacher to effectively control the learning environment. Another advantage of technology in education is that it improves students' interest in learning. Several studies have shown that technology used in teaching and learning increases learners' enthusiasm which results in improved student achievement (Martin & Ertzberger, 2013; Cifuentes, Maxwell, & Bulu, 2011). The use of technology in education ensures that student achievement can be measured in a number of ways and provide instant feedback (Pellegrino & Quellmalz, 2010).

Although the use of technology in blended instructions has been very beneficial to the teaching and learning process of a number of subject areas, there have been some limitations that have hindered the effective implementation some of these technologies. Arkorful and Abaidoo (2015) for example, have pointed out certain causes as the limitations to the use of technology in education. They include the lack of funding for acquiring technological tools, teacher or instructor incompetency in the use of the technology, and the absence of a curriculum with technology integrated into its instructional objectives and processes. Arkorful and Abaidoo (2015) have also pointed out that one of the major hindrances to implementing technology in education, especially in developing countries, is how to fund it. Most poor schools cannot afford to buy technological tools because they are expensive; meanwhile facilities such as expensive building blocks for well-endowed schools are put up. Arkorful and Abaidoo (2015) have therefore opined that with the provision of computers, many learners could have more interactive periods of learning. Another major concern is teacher incapability or incompetence in the usage of technology in classrooms. Even with the presence of the computers and other technological tools in the educational settings, the absence of skilled teachers remains a significant restrictive factor in using technology in teaching and learning (De Grove, Bourgonjon & Van Looy, 2012). This implies that the teacher factor remains a very important aspect of a blended instruction or CAI.

2.2.3 The use of Technology in Education

The first Computer Assisted Instruction (CAI), created by researchers at IBM in the 1950s, marked the beginning of technology's application in education (Reiser, 2001). Donald Bitier of the University of Illinois implemented the first significant use of computers in teaching in 1959. (Choi, Dailey-Hebert, & Estes, 2016). The usage of specific applications in a few topic areas began in the 1980s as a result of educators' and researchers' interest in the use of technology in education (Reiser, 2001). The microcomputer (also known as a personal computer), which was affordable and capable of carrying out a number of tasks that other large computers could, was one technical advancement that attracted significant attention. The nature and substance of instructional practices began to be significantly impacted by technological breakthroughs by the 1990s (Spector & You-qun, 2016). With the aid of technological tools, educators were able to focus learning outcomes on problem-solving abilities and the creation of "independent learners".

Juniu (2011) observed that, the advent of word processors, databases, spreadsheets, and other tools allowed teachers and students to adopt simpler methods of teaching and learning without having to squander their brainpower on pointless errands.

Modern educational systems increasingly include technological resources on a regular basis. In industrialized nations, new technologies have been incorporated into the framework, ideas, and procedures of how teaching and learning are carried out, according to Krueger and Kumar (2004). Pilli and Aksu (2013) divided computer applications in education into two major areas. They asserted that a computer can be utilized as an instructional tool or as an object of a lesson or instructional process. In the first scenario, computer literacy classes or programs primarily focus on how computers or other related technology become the lesson's primary learning purpose. The second situation is using a computer as an instructional tool, which focuses on how a computer or other related technologies might be utilized as an aid or tool to assist in achieving educational objectives. Computer-Assisted Instruction (CAI) also known as blended instruction, is the concept of using a computer as a learning tool (Tayebinik & Puteh, 2013).

2.2.4 The Use of Technology in Mathematics Education

New approaches to the teaching and learning process have been made possible by technological developments. These developments necessitated adjustments in society and individual capacity. According to Butcher (2015), the purpose of new technologies in education, is to change how we practice and learn mathematics. In addition to providing learners with possibilities for proper development, the use of ICTs in the learning environment aids teachers in meeting the objectives of the instructional time.

In a meta-analysis on the impact of CAI on college student performance, Timmerman and Kruepke (2006) conducted the research. They discovered that pupils who received computer-assisted training generally performed better. According to the data, students' performance was significantly more affected by CAI packages designed specifically for one course than by CAI packages in general. Additionally, they found that results were better when the technology was used consistently over the life of the project rather than just as a single instance. Not only the two above but also, the authors contrasted CAI software programs that could offer students feedback with those that could not. According to the findings, there was no proof in favour of the claim that CAIs that gave students feedback affected their performance more noticeably. The effects of technology use in the classroom have been the subject of numerous studies. Badu-Domfeh (2020) investigated how the interactive tool “Critical Learning Instructional Paths Support” (CLIPS) was used in a few Grades 11 and 12 classes to improve knowledge of fractions in one of these experiments. With the help of videos and assigned readings, this program was designed to give pupils individual lessons. Following the reading were quizzes and other exercises. The outcomes demonstrated that when a learning tool was introduced in class, pupils benefited the most. It is implied that integrating technology into a lesson plan helps pupils learn by giving them the support they need. For this reason, colleges of education in developed nations use ICT tools including computers, web-based applications, graphic calculators, and dynamic mathematics/geometry software. In order to assess the usefulness of technology in mathematics education, numerous studies have been conducted in those nations (Skryabin, Zhang, Liu & Zhang 2015). Amarin and Ghishan (2013) contributed on the effect of educational technology on learner interactions, that students’ interest in and enthusiasm for learning are boosted when educational technology is introduced into traditional teaching practices. Technology-enhanced lesson presentations and make lessons more constructivist. The way that educational material is provided to pupils typically determines the requirement to increase students’ mathematical achievement (Pierce & Stacey, 2011). Chang and Lee (2010) noted that when instructional courses are well organized, learners are able to have a broad comprehension of the concepts and theories that are applicable to mathematical solutions. According to some academics, utilizing technology tools to teach mathematics in a classroom is a successful way to improve students’ performance in the subject (Cheung & Slavin, 2013; Eyyam & Yaratan, 2014). These researchers have supported the integration of electronic resources into traditional classroom settings by educational institutions and mathematics teachers.

From the explanation above, it is clear that technological innovation is crucial to mathematics instruction. It also motivates mathematics teachers to better engage students' attention so they can learn mathematical topics (Khouyibaba, 2010).

2.2.5 Teaching and Learning of Geometry

Different activities can be used to teach geometry. The lesson's goals should be taken into consideration while selecting an activity. These exercises typically have an effect on how pupils learn geometry. Diagrams are frequently used to teach geometry. Diagrams serve to clarify ideas and convey a general relationship. According to Schwartz and Heiser (2006), using diagrams can help students show their spatial reasoning. In this sense, the use of diagrams in teaching and learning geometry requires the teacher to be able to graphically depict geometric drawings and assist students in recognizing graphical or geometric relationships from the diagrams. The effects of geometric teaching and learning activities on students' learning have been the subject of several research. Yerushalmy, Chazan, and Gordon (1990) conducted research on the impact of geometry diagram use among high school pupils. From 1984 to 1988, high school geometry classes were taught using an inquiry technique and the "Geometric Supposers". The authors found three barriers that prevent students from properly analyzing and comprehending graphics by using "Geometric Supposers". The three elements were specialized geometric diagrams; employing diagrams frequently leads to misunderstanding with traditional diagrams, and lastly, one diagram can be visualized in various ways. Information gathered on students from various colleges of education who used "The Supposer" to learn and pupils who did not. The findings showed that students made use of the diagrams in their work after the introduction of "The Supposer". The researchers concluded that students that used "The

Supposer" had a better understanding of geometric diagrams than the students that did not use it.

Another study was conducted by Noss, Healy, and Hoyles (1997) to show the value of spatial thinking in geometry learning. The purpose of the study was to examine the connection between learners' behaviors, visualizations, and the methods used to explain them. The researchers used Microworld Mathsticks, a program that links students' actions to the symbolic representations they create to help them establish mathematical meanings. They applied a case study involving two pupils. The findings showed that visual representations assist pupils in drawing connections between spatial and real-world situations. The study also showed that students can create algebraic relationships from diagrammatic representations with the use of visualization.

The improvement of pupils' deductive reasoning skills is a key objective of geometry instruction. In this regard, Jones, Fujita, and Ding (2006) performed research on the methods of geometry teaching and learning. They looked at methods of instruction used in China and Japan. They recommended that students should learn geometry at the University of Cape Coast, utilize problem-solving techniques, deductive reasoning, and the use of modeling to apply geometric principles in a range of applications. They came to the conclusion that creating effective pedagogical models with the use of thoughtfully crafted learning challenges and resources is an important strategy to improve geometry instruction.

Another method for developing geometric activities that support teaching and learning in geometry is by using games. Herbst, Gonzalez, and Macke's (2005) investigation into how a teacher might lay the foundation for pupils to accurately characterize a figure served to illustrate this point. 53 students from two geometry classrooms in high school were employed by the authors. “Guess My Quadrilateral” was a game that the pupils played. The aim was to find out the students’ prior knowledge of quadrilaterals. Before the lesson, students were given a survey to complete. Over the course of three weeks, the researchers prepared the training and put it into practice. The setting of the game was a neighborhood of unique quadrilaterals. Students were required to focus on each quadrilateral and determine how it differed from its neighbors during the game. The results of the students’ responses showed that when they discussed the characteristics of figures, pupils were able to draw such figures rather than just describing them. The authors came to the conclusion that students were able to evaluate the characteristics of the quadrilaterals they examined using the data from the game.

Foster and Shah (2015) conducted a second study to see how games might support learning in a classroom environment. The teaching method employed by the researchers was the Play, Curricular Activity, Reflection, and Discussion (PCaRD) model. An experimental group and a control group were used in this mixed-methods study, which was carried out at a senior high school. For a period of one year, three games were implemented using the PCaRd model. In order to calculate the accomplishment gains, Foster and Shah (2015) employed pre- and post-tests. They discovered that the PCaRD paradigm helped students learn geometry. According to Foster and Shah (2015), the PCaRD paradigm helped teachers include games in their lesson plans. The way that pupils display understanding when learning geometry depends a lot on their cognitive processes. An investigation of the methods students employs to comprehend geometrical ideas was carried out by Moscucci, Piccione, Rinaldi, Simoni, and Marchini in 2005. Investigating students’ perceptions of isosceles triangles was the aim of the study. The study included 105 participants from six third-grade classrooms in Italy. The researchers looked at how students' perceptions of isosceles triangles varied depending on the drawing's "orientation". Researchers noticed that different naive techniques of measurement in geometry were displayed in students' approaches to solving problems. Researchers found that brief activities helped children learn more effectively. The thought processes of pupils in connection to the area and perimeter of geometric forms were also examined by Marchett, Medici, Vighi, and Zaccomer in 2005. 130 Italian primary school pupils in their fourth and fifth years participated in the study. Conflicting concepts of area and perimeter were examined in the study. To examine students' reasoning abilities, the researchers employed two worksheets. They discovered that students were better at comparing areas than perimeters. In this regard, the pupils were able to attempt problems involving areas with more appropriate strategies. It was also found that students were tricked by how they visually perceived the geometric figures. The researchers also realised that students like working on single geometric shapes rather comparing them.

Every educational level offers geometry instruction. For instance, at the tertiary level, non-Euclidean geometry can be taught via geometric visualizations and exercises. In one study, Kaisari and Patronis (2010) looked at how college students create elliptic geometry models. Finding out how context and usage may shape geometrical interpretations was the study's main goal. The scientists believed that reformulating Euclid's axioms and creating models for elliptic geometry would show pertinent connections between elementary and higher geometry. Students were required to work on one of these for the duration of the semester and present their findings in groups. The pupils had the chance to connect with one another and share their own opinions about various assignments. According to the study, students can engage with one another and affect their peers' comprehension of geometrical concepts regardless of how they use them.

The study of circles is one area of geometry that has a significant impact on how students understand and use geometrical ideas. The literature has extremely few works on circular geometry. The building of geometrical objects and shapes as well as the usage of geometrical puzzles to foster learners' reasoning are some of the topics covered in these circle geometry studies. Canada and Blair's (2006) investigation of the intersections of a circle and square is an example of such a study. How many points of intersection there are between a circle and a square was the subject of Canada and Blair's (2006) research. Students and pre-service teachers both took part in the study. The study's authors devised exercises aimed at assisting students in formulating mathematical arguments. The researchers came to the conclusion that when diagrams were drawn accurately, students were able to make precise discoveries and conclusions about geometric ideas. In a study by Neel-Romine, Paul, and Shafer (2012), participants were asked to write and evaluate definitions of circles. Students from one school in the sixth grade participated in the study. Students were given a task to complete in order to define circles, according to the researchers. The assignment to define a circle was given to groups of students. Counter examples were used to check students' understanding of terminology and to direct them as they investigated how to form a circle. Neel-Romine, Paul, and Shafer (2012) observed that students were able to write down definitions such as "round shape" and "looks like an orange or a coin". In addition, the authors noted that students could accurately define a circle in their writing by using phrases like "all diameters are the same length" and "a circle contains a diameter and a radius". The study also showed that students found it difficult to define the radius as equidistant, even with the use of pencils and paper clips. The authors came to the conclusion that, after a series of exercises, pupils could define circles correctly and list their attributes in writing.

González and DeJarnette (2013) carried out another study on circular geometry with the aim of using problem-based learning to improve reasoning abilities. 22 students were involved in the study, and they were required to take part in a variety of tasks. In a session that was problem-based, the students were given a circular problem to solve. The lesson's goal was to get the kids to see that the issue called for more than just a mathematical solution. Students were given the problem after they had studied all the necessary foundational mathematical ideas. Students were not restricted to using only certain theorems and notions to solve the problem because the problem allowed them to construct their own strategies. According to the study, practically all students applied several different solutions to the problem in order to solve its major components, including its diagram. The Pythagoras theorem was applied, helping lines (dotted lines) were used, and right angles were marked out using these techniques. The task of solving the problem, according to González and DeJarnette (2013), allowed students to analyze the issue, come up with a plan of action and put it into practice, connect various mathematical ideas and theorems, and lastly reflect on the resolution.

In conclusion, improving knowledge of geometry requires an emphasis on teaching students to visualize geometric concepts and analyze the connections between them. With the aid of interactive software, this study employs visualizations and diagrams to enhance colleges of education students' mathematical performance.

2.2.6 Dynamics Geometry Software (DGS) in Mathematics Education

Computer tools called Dynamic Geometry Software (DGS) offer a hands-on interface where users may measure and sketch geometrical shapes. Additionally, it enables users to measure and compute geometric figure variables and create relationships between geometric figures (Hollebrands, 2007). While maintaining the mathematical relationships on the image, DGS allows users to adjust the orientation of geometric figures and their look (Hollebrands, 2007). Students can better learn mathematical ideas and create problem-solving strategies by using these computer programs to add dynamic motions to the visual representations. Geometric reasoning often involves visualization (Presmeg, 2006). In order to see changes in the relationships between the geometric shapes, students can drag and shift the points of a figure using DGS. GeoGebra, Cabri 3D, and Geometer's Sketchpad are some examples of DGS environments.

In order to explore three-dimensional geometrical figures, Cabri 3D, a dynamic geometry program, is employed. A French business named Cabrilog created it in 2004. Cabri Geometry is a commercial product used for geometry and trigonometry. Compared to geometric figures created on a piece of paper, the software enables geometric figure animations, which is a considerable advantage. A geometric figure's points can be related to one another using the program. Good graphing and display features are also offered by the software. User links between geometry and algebra are made possible by these functions. On either Windows or Mac OS, the software can be used.

The Geometer's Sketchpad is another piece of geometry education software. It is a piece of interactive commercial software that allows users to explore many planes of geometry and other mathematical topics. The conventional Euclidean tools for geometric structures are included in Geometer's Sketchpad. Figures like the pentagon or decagon can be created using the software just like they can be with a compass and ruler. The user can make modifications that might be challenging to build using a compass and ruler without the aid of the software. Additionally, it enables object animation. The software enables the user to develop a variety of tools that can be used to tackle exceedingly challenging mathematical issues. Midpoints and mid segments of objects can be calculated using the application. The fact that the software must be purchased is one of its key drawbacks. As a result, both teachers and students cannot simply access it. GeoGebra is an open-source mathematics learning software. GeoGebra that consolidates elements of different dynamic representations and provides a rich variety of computational tools that can be used for modelling and simulations. GeoGebra has a user-friendly interface and has a web platform that makes it convenient for users across the world to access a number of mathematical resources.

In the mathematics classroom, GeoGebra can be used to teach a geometric concept. The process of using GeoGebra in an interactive geometric classroom starts by introducing the concept to be taught. The teacher or instructor then used the tools available in the GeoGebra software to draw or animate the concept for clear visualization.

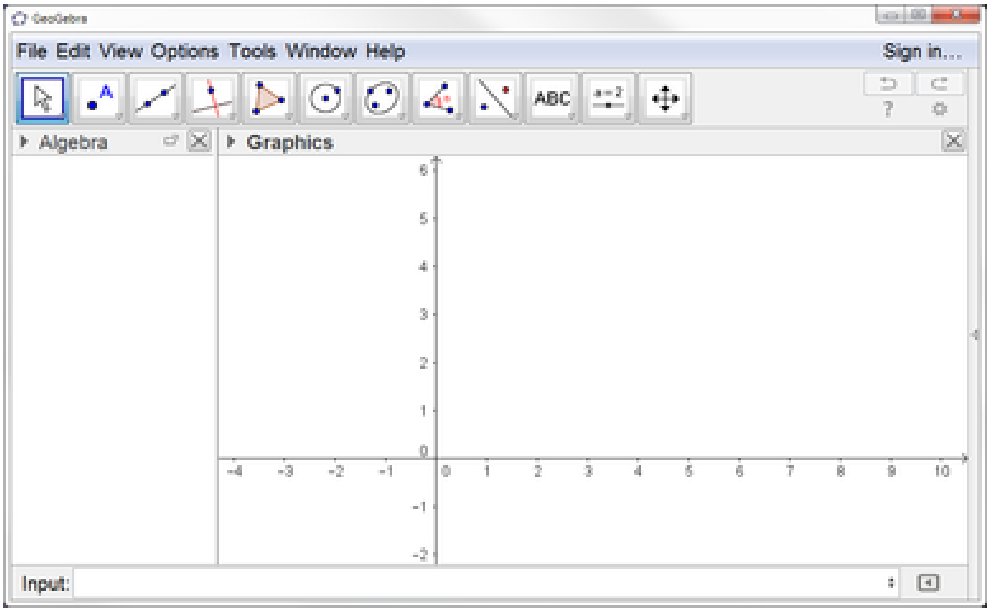

Figure 1: Interface of the GeoGebra software.

The GeoGebra program shows a figure that clearly illustrates (various orientations) the mathematical topic the teacher wants to convey. Among the tools included in the software are the point with circle tool, the ruler, the arc, the angle, the angle with given size tool, the line, the line segment, the line segment with given size tool, the line vector, the bisector, the perpendicular bisector, and so on.

Other tools are available in GeoGebra to enable manipulation of the drawn concept into different examples. Students should then practice using these examples by saving them as GeoGebra files. When students are able to depict different examples of the same idea, it shows that they have understood it. To provide accurate software presentations, the teacher's proficiency with GeoGebra is essential. Due to the GeoGebra Wiki online and local and international professional conferences, GeoGebra has grown internationally. With the use of a variety of representational and modeling tools, users of GeoGebra can create mathematical relationships for real-world situations and create their own diagrams.

This enables students to create highly abstract mathematical concepts. These benefits led to the integration of GeoGebra software in geometry classroom instruction in the current study.

Numerous studies have concentrated on incorporating DGS in mathematics teaching and learning. In a study by Strausova and Hasek (2013) investigated visual proofs using DGS. The researchers hypothesized that diagrams are crucial in assisting students in conceptualizing different mathematical features. They also stated that a diagram may be used to demonstrate a geometrical property or theorem. Strausova and Hasek (2013) proposed that pupils prefer geometric proofs to those that rely solely on words. Geometry courses optimization was also examined by Karaibryamov, Tsareva, and Zlanatov (2012) using the DGS. The researchers developed a fresh approach to teaching geometry at the tertiary level with the help of DGS and identified the most efficient methods for instructing geometry courses. Karaibryamov, et al (2012) asserted that the use of DGS helped optimize the teaching process by ensuring that there was adequate time for drawing, generalizing a large number of issues, and assisting in the formation of creative ways of reasoning. The researchers discovered that students who were taught using the Cabri 3D were more proficient in recognizing special planes and the related normal vectors. Additionally, creating comparable schematics for the special planes was more successful for the students who were taught using Cabri 3D.

Donevska-Todorova (2015) carried out a second study that concentrated on how well students understood the scalar product. The goal of the study was to help students understand the dot product by utilizing DGS in a dynamic geometry environment. Students in the 12th grade who were studying geometry in a dynamic environment made up the study's sample. The study's conclusions demonstrated that DGS aided students in developing a deeper comprehension of dot products. According to Donevska-Todorova (2015), using a variety of diagrammatic representations allowed pupils to better understand a certain mathematical idea.

From the previous description of the DGS and the material that is currently accessible, it is clear that using the DGS properly has a big advantage in mathematics instruction. The section that follows provide an overview of the empirical evidence surrounding GeoGebra's usage in mathematics instruction.

2.2.7 Teaching and learning mathematics using GeoGebra

Numerous studies have shown that GeoGebra can be utilized to raise student achievement. These studies included one by Saha, Ayub and Tarmizi (2010). The goal of the study was to determine how utilizing GeoGebra to teach coordinate geometry affected students' performance. 53 secondary school students were divided into the experimental group and the control group for the study. While the control group did not utilize the software, the experimental group received instruction using GeoGebra. The outcome was that the experimental group outperformed the control group by a large margin. The conclusion was that employing GeoGebra in a mixed learning environment was a powerful addition to traditional teaching techniques.

Selçik and Bilgici (2011) investigated the efficacy of GeoGebra in the teaching and learning of polygons with 32 seventh-grade students from two different schools. An achievement test was used as the research tool in the study. In an elementary school, the sample was divided into two groups: the experimental group (17 kids), who received their instruction using the program, and the control group (15 students), who received their instruction without using it. The study's findings demonstrated a considerable distinction between the experimental group and the control group. This result shows that utilizing GeoGebra improved students' achievement scores more than using traditional education. It was additionally discovered that the experimental group learnt more effectively using GeoGebra and retained what they learnt more than the students that learnt in a computer-free environment.

The findings of Selçik and Bilgici (2011) were comparable to those of Zengin, Furkan, and Kutlucaa (2012) study. The goal of Zengin, Furkan, and Kutlucaa's (2012) study was to ascertain how GeoGebra affected students' trigonometry performance. In the study, 51 students were divided into two groups: a control group and an experimental group. According to Zengin, et al (2012) findings, GeoGebra was able to raise students' trigonometry proficiency levels more effectively than the conventional teaching and learning approach. Zengin, Furkanb, and Kutlucaa (2011) pointed out GeoGebra use in teaching and learning was cited as a successful constructivist instructional technique.

Zakaria (2012) conducted a second study to examine the effect of GeoGebra use on kids' arithmetic performance. 284 students in their sophomore and junior years made up the study's participants. The students were divided into two groups: one received instruction using GeoGebra, while the other received regular classroom instruction. According to the study's findings, students in the experimental group outperformed their counterparts in the control group on the post-exam. The results also showed that there was no discernible difference in the performance of the boys and girls. According to Zakaria (2012), incorporating GeoGebra into instruction and learning supports students' conceptual and procedural learning.

A study by Shadaan and Leong (2013) examined how GeoGebra affected students' comprehension of the teaching and learning of circles. 53 Year 9 students were divided into the treatment group and the control group for the study. To ascertain the impact of GeoGebra on the students' performance in learning circles, the study used a pre-test and post-test. The study's results revealed that pupils who used GeoGebra to learn about circles outperformed their colleagues who did not utilize the program. Shadaan and Leong (2013) also revealed student perceptions of the use of GeoGebra were generally good.

Mukiri (2016) looked into how GeoGebra may be used to teach and study mathematics in Kenyan secondary schools. Mukiri (2016) also looked into how teaching with GeoGebra affected both boys and girls. The study used a mixed-methods strategy that combined qualitative and quantitative techniques. 270 pupils and 33 teachers participated in the study. The study's findings showed that math teachers adopted technology slowly. Additionally, the study discovered that students who used GeoGebra performed higher than those who did not. After using GeoGebra in the classroom, Mukiri (2016) found that pupils' performance was unaffected by gender differences.

Chimuka (2017) also carried out research on how incorporating GeoGebra affected kids' mathematics skills. Students in the senior high school in the study learned about circles using GeoGebra. The study employed a quasi-experimental non-equivalent control group design to compare the academic progress of pupils using GeoGebra software with students receiving instruction using the traditional style of instruction. 47 participants were divided into control and experimental groups for the study. The researcher discovered that on average, pupils who used the GeoGebra software performed better than those who did not. Moreover, Van Hiele's idea of geometric thinking was the main topic of Chimuka's (2017) research. The results also indicated students taught with the GeoGebra perform significantly than their peers that did not use the software at Van Hiele's Levels 1 and 2. However, there was no significant difference in achievement of the two groups at Van Heile's Levels 3, 4 and 5.

Arbain and Shukor (2015) asserted that learning and teaching of Mathematics should not be focused on purely theoretical, but also a variety of learning approaches that involve the use of teaching aids proven to help stimulate students' interest in Mathematics. The mathematics software available in the market or even online has facilitated the task of the teacher to impart knowledge beneficial to the students. However, it depends on the teacher to utilize existing materials without the need to allocate extra time to developed other teaching aids. Conclusively, this study has shown that GeoGebra software has a positive impact on students' achievement in the topic Statistics. The students also have positive perceptions on GeoGebra software in terms of enthusiasm, confidence, and motivation. This software should be introduced to mathematics educators so that students can explore the world of Mathematics in a wider and make the students able to think critically and creatively.

Martinez (2017) also carried out a study on the impact of the iPad software GeoGebra on students' academic performance. A treatment group and a control group were used in the study's experimental, non-equivalent pre- and post-test designs. There were 56 students in the study. According to Martinez (2017), using GeoGebra in the classroom increased kids' test scores.

Tay and Wonkyi (2018) investigated the impact of GeoGebra on senior high school students' GeoGebra performance with an emphasis on circles. Additionally, 49 pupils from a quasi-experimental non-equivalent control group were used in the study. The children were divided into two groups from two schools in the study using purposive sampling. One group received traditional instruction while the other received GeoGebra instruction. Tay and Wonkyi discovered that pupils who used the GeoGebra software to learn geometry showed improvement. GeoGebra-based instruction, according to Tay and Wonkyi (2018), made lessons in the classroom more engaging and useful for pupils.

A study was carried by Aizikovitsh-Udi and Radakovic (2012) showed how GeoGebra helped pupils better comprehend probability and the Bayes Theorem. The goal of the study was to ascertain how GeoGebra affected students' understanding of abstract mathematical topics. An accomplishment test served as the study's primary research tool. According to the study's conclusions, using GeoGebra in the classroom helped students become more adept at using their critical thinking skills to answer probability-related issues. Researchers came to the conclusion that students' grasp of mathematical ideas was improved by the visual representations created by using GeoGebra. Aizikovitsh-Udi and Radakovic (2012) utilized the GeoGebra program to aid in the development of learners' critical thinking abilities, in contrast to all the previously discussed studies, which looked at the impact of incorporating GeoGebra into classroom teaching and learning on students' performance.

Taking into account all of the previous research on the use of GeoGebra, its use has a significant benefit in mathematics education. Therefore, it can be stated that the usage of DGS in a learning environment can be a practical technical instrument that improves the effectiveness and interest of teaching and learning.

Agyei and Benning (2015) researched on the topic: Pre-service teachers' use and perceptions of GeoGebra software as an instructional tool in teaching mathematics. The study made use of 85 final year pre-service mathematics teachers (74 males and 11 females) in Bachelor of Education (Mathematics) programme at the University of Cape Coast, Cape Coast, Ghana. Questionnaire, interviews and lesson artefacts developed by the teachers were the main source of data for the study. All 85 participants responded and completed a questionnaire survey which was administered to them before and after the instructional technology course. Four teams of preservice teachers whose lesson artefacts were sampled and analysed were also interviewed after the course. Descriptive, t-test and effect size statistics were used to analyse the quantitative data whereas the interview data and lesson artefacts were analysed qualitatively. Statistical analysis confirmed that the use of the GeoGebra helped pre-service teachers expand their own understanding of mathematical concepts as well as their knowledge of instructional strategies. Asare (2019) examined the impact of using GeoGebra software in teaching and learning rigid motion on senior high school students in Ghana. The objective of this study was to assess the applicability of GeoGebra in teaching and learning of mathematics in senior high schools in Ghana. The study population was students in the New Juaben Senior High School in the Eastern Region of Ghana. The design adopted for the study was the mixed methods design of qualitative and quantitative approaches. Here two groups of students were taught, one with the use of GeoGebra and the other with the traditional approach. The data collected from the study were analysed with the help of descriptive and inferential statistics. The result of the study indicated that GeoGebra helped improve the students' understanding of the concept 'rigid motion'. The study concluded that GeoGebra is useful in improving performance of secondary school students. Prodromou, Lavicza, & Koren, (2015) reported the various opportunities that teaching mathematics with technology (GeoGebra) offer. In their study, GeoGebra was integrated into the teaching and learning of introductory statistics and the results indicated that college students exposed to this software were able to perform key statistical investigative tasks, such as (1) managing data (2) understanding specific statistical concepts (3) performing data analysis and inferences. As a result of the study, the national and professional standards for teachers of Australia (AITSL, 2014) admonished teacher education graduates to demonstrate technological, pedagogical and content knowledge (TPACK) in mathematics teaching and learning. Reis and Ozdemir (2010) used GeoGebra as a technological tool for instructional method for parabolas. The outcome from the data analysed shown that students can learn meaningfully with GeoGebra. Bhagat and Chang (2015) also examined the impact of using GeoGebra on 9th grade students' achievement in learning geometry. The study utilized a quasi-experimental research design. A total of 50 middle school students were selected from a government school in the eastern part of India as the sample for the study. This sample was subdivided into control and experimental group. Each group consisted of 25 students. Before instruction students in both groups were assessed to find out their knowledge about circle theorems. The actual instruction underwent three phases. In the first phase, students in the experimental group were trained on how to use the GeoGebra software. The second phase, the experimental group was taught circle theorems using GeoGebra while the control group was taught using the traditional teaching method. In the final phase, both groups were assessed to find out whether or not the GeoGebra has had any impact on students' mathematical achievement. Data collected from both the pre-test and the post-test were analysed using the one-way analysis of co-variance (ANCOVA). All analysis was done using the Statistical Package for the Social Sciences (SPSS). At the end of the treatment, students' mathematics achievements were measured using a post-test. The results indicated that the experimental group performed much better than the control group. It was also clear from the study of Bhagat and Chang (2015) that teaching and learning Geometry with GeoGebra, helped students to improve their reasoning, visualization skills and representation of mathematical concepts in diverse ways. This notion was also supported by Dikovic' (2009) that by using GeoGebra, true exploration and visualization are possible, leading to an understandable mathematics solution to both the instructor and the learner (Fahlberg-Stojanovska & Stojanovski, 2009). Zengin et al. (2012) conducted experimental research in order to identify the effect of using the dynamic software GeoGebra towards achievement in the subtopic of trigonometry function and graphs of trigonometry functions. The findings showed that learning using GeoGebra gives meaningful impact for students who learned in the experimental group. Overall, the research concluded that students who learned trigonometry with GeoGebra had better achievement than those who learned with a constructivist approach. Pavethira and Leong (2017) also did a study on students' performance in geometrical reflection using GeoGebra. The research utilized an experimental research method. A total of 24 year one students were randomly selected from an international school as the sample for the study. This sample consisted of three groups namely Extension, Core and Support. The sample selected were taught and learnt by using GeoGebra software after a pre-test. Then a post- test was given. The results indicated a significant difference between pre-test and post-test results. Similarly, the results also found a statistically significant difference in scores among the three student ability groups. In conclusion, the study implies using GeoGebra enhances students' performance in geometrical studies. Dogan and Icel, (2011) also conducted a quasi-experimental research study on the topic “The role of dynamic software in the process of learning: GeoGebra example about triangles”. The aim of the study was to observe the possible effects of computer-based learning environment (GeoGebra software) on students’ achievement. Two eighth grade classes from primary school were selected as the experimental and control group. The experimental group was made up of 9 females and 11 males while the control group consisted of 7 females and 13 males. Before the classroom activities, a pre-test was applied to both groups to determine the students’ attainment levels. The control group were then taught triangles with the conventional teaching method whereas the experimental group were instructed with the GeoGebra instructional teaching approach. A post-test was then applied simultaneously on both groups to find out the effect of the instructional approaches on students’ achievement. Possible comparisons between the tests and the groups were performed. The results showed that dynamic software (GeoGebra) had positive effects on students’ learning and achievements. It was again observed that the GeoGebra instructional approach improved students’ motivation with positive impact. Khor and Md-Ali (2017) examined the effect of using GeoGebra dynamic geometry software on students’ ability to confront geometry problem solving. A total of 102 Form Two students participated in the study. The research participants were divided into three groups, namely Experimental Group 1 (n = 33), Experimental Group 2 (n = 35) and Control Group (n = 34). A guideline book on the usage of GeoGebra dynamic geometry software in learning shapes and space, developed by the researchers and validated by a panel of experts, was used by the teachers and students in the experimental groups while the control group learn geometry with the traditional approach. The research instruments used in the study were the Topical Test (TT), Spatial Visualization Ability Test (SVAT), and the

Teaching and Learning Observation Checklist. The quantitative data obtained via Topical Test (TT) and Spatial Visualization Ability Test (SVAT) were analysed using MANOVA while the qualitative data collected via interviews, teaching observations, video recordings and students' work were analysed thematically. The research findings indicated that the experimental groups' TT and SVAT post-test mean scores for both experimental groups were significantly higher than the control group's TT and SVAT post-test mean scores. The learning of shapes and space using GeoGebra dynamic geometry software enabled students to produce works with evidence of critical, creative and innovative elements in their solution. Students in the experimental groups agreed that the use of the GeoGebra dynamic software was something new to them and was indeed an attractive way to learn mathematics because they had had the opportunity to experience hands-on learning in mathematics using ICT. Shadaan and Leong (2013), investigated the effectiveness of using GeoGebra on students' understanding in learning circles by using quasi-experimental design. Fifty-three form three students from two intact classes selected from a population of 133 students participated in the study. Out of this number, 28 were assigned into the experimental group and 25 into the control group. The instruments for the study were the teacher made achievement test and questionnaire. Before instruction pre–test was administered to both the control as well as the experimental group. The reason for administering the pre-test was to determine the baseline knowledge or preparedness for learning the topic of circle geometry and to compare differences between experimental and control group before treatment. Afterwards the experimental group underwent instruction where they were taught circle geometry using the GeoGebra software while the control group underwent instruction without the use of the software. An independent t-test was used to determine whether there was a significant difference between the pre-test and post-test mean scores of both the control and the experimental group. The result indicated that students in the experimental group not only outperformed those in the control group in academic achievement but also in levels of learning of transformation geometry. This study shows that when technology is effectively integrated into the teaching and learning of mathematics, students turn to think critically and therefore perform better. Numrich (2010) quoted in the National forum for teacher education journal emphasize that only those who can "think" through content truly learn it". What this means is that for the learner to truly learn a concept of mathematics, he/she should be able to think critically. Another study done in Malaysia to evaluate the impact of GeoGebra in learning transformations by Bakar, Ayub, Luan and Tarzimi (2010) revealed that secondary school students achieved better using the software. Adelodun and Akanmu (2016) also did a study titled "GeoGebra; The third millennium device for mathematics instruction in Nigeria. The study adopted the nonequivalent pre-test post-test control group design. The study population comprised secondary school mathematics students in Ogbomoso North L.G.A. of Oyo State, Nigeria. SS2 mathematics students from two intact classes from each of the two purposively selected schools in the area constituted the sample. The classes were assigned into experimental and control group using simple random sampling technique. The experimental group was taught using GeoGebra, while the control group was taught using the conventional method. In the experimental group, the students interacted with different kinds of GeoGebra tools to solve problems in geometry, algebra, introductory calculus, among others. The control group was exposed to the conventional method and taught the same topics. The two main instruments used for data collection were Students Achievement

Test in Mathematics (SATM) and Mathematics Attitudinal Scale (MAS). The two groups were pre- and post-tested using SATM, after which MAS was also administered to them. Data collected were analysed using mean and t-test statistics. The study concluded that the incorporation of GeoGebra and other ICT packages improved the students' learning outcomes in mathematics, while their attitude towards mathematics was also positively enhanced. A study entitled "GeoGebra assist discovery learning model for problem solving ability and attitude toward mathematics" was a study conducted by Murni, Sariyasa and Ardana (2017). The targeted population in this study was all the 6 classes totaling 181 students of class VIII junior high school in Indonesia. The researcher used cluster random sampling to select 4 classes as a research sample with 2 classes as the experimental class and the other 2 classes as the control class. The research design for the study was a quasi-experimental and post-test only control group design. Problem solving ability data was collected through a description test and that of the attitude was collected through questionnaire. The questionnaire data was measured using the Likert scale. Data collected from the study were analysed using the one-way MANOVA. The results of the data analysis showed that the utilization of GeoGebra in discovery learning led to solving problem and attitudes towards mathematics better. Onaifoh and Ekwueme (2017) also conducted a study entitled: Innovative strategies on teaching plane geometry using GeoGebra software in secondary schools in Delta State. The design for the study was quasi-experimental with non-equivalent pre-test and post-test control group design. The population of the study consisted of all private senior secondary form two (SS2) students in the 15 private schools in Oshimili-South Local Government Area of Delta State. Two private secondary schools were selected for the study by purposive sampling technique.

The total number of students who participated in the study was 59. The instrument for the study was a Performance Mathematics Ability Test (PMAT), which included pre-performance test and post-performance test. A pilot study was carried out in two secondary schools in Oshimili-South Local Government Area of Delta State which were not part of the sample. The two intact classes (sample) were then put into the control and the experimental group. Both groups were given pre-performance test after which the control groups were taught plane geometry using the conventional method while the experimental group was taught using the GeoGebra approach. The two groups were again assessed to find out the effect of the instructional approach on their performances. Data from both performance tests were then collected and analysed by SPSS Version 21. The research questions were also answered using mean and standard deviation. The results showed that GeoGebra application was more effective in improving students' understanding in mathematical plane geometry than the problem- based learning approach; however, there was no significant difference in male and female performance. Hutkemri and Sharifah (2016) also investigated the effectiveness of the GeoGebra software: The intermediary role of procedural knowledge on students' conceptual knowledge and their achievement in mathematics. The study employed a quasi-experimental approach on 345 Form Two secondary school students in Riau, Indonesia. These 345 students were randomly selected from two secondary schools in Riau, Indonesia. These students were further grouped into two, that is control group (n = 176) and the experimental group (n= 169). Students in both groups were given a pre-test. The students in the treatment group were then taught functions using GeoGebra while the control groups were taught functions with the conventional method. Students were then given a post-test to find the effect of the instructional approaches on their achievement. The data collected through conceptual and procedural tests, and from students' achievement on the topic: functions were then analysed using SPSS 22, AMOS 18.0 and ANATES v4 software. Findings of the study showed that students who used GeoGebra to learn Mathematics had higher mathematical conceptual and procedural knowledge compared to those who learnt mathematics through the conventional methods. Both experimental and control groups showed that procedural knowledge was a significant mediator between conceptual knowledge and students' achievement in mathematics. The study concluded that GeoGebra software was capable of enhancing students' conceptual and procedural knowledge and at the same time significantly improved students' mathematics achievement. In Malaysia, Noorbaizura and Leong (2013) studied the effect of using GeoGebra to teach students' learning of fractions. The purpose of the study was to investigate the effect of students' achievement in fraction. A quasi-experimental research design was used. A pre-test was administered to each group after which the experimental group underwent an intervention where they learnt fraction using GeoGebra while the control group on the other hand learnt fraction by the traditional approach. The study showed that the use of GeoGebra to teach fractions is very effective. This was shown through the improved scores of the students in the experimental group. The findings highlighted that students in the experimental group performed better than those in the control group that were taught fraction using the traditional learning method. The software also enhanced visualization and understanding of the fractions concept of both the teacher and students. Diaz-Nunja, Rordriguez-Sosa and Lingan (2018) examined teaching of geometry with GeoGebra. The purpose of the study was to evaluate the effects of the use of GeoGebra software in the teaching of geometry with high school students in the development of their capacities for reasoning and demonstration, mathematical communication and problem solving. There were 48 students who participated in the study. Of the 48 students, 40% were men and 60% were women. These 48 students were put into two groups, that is 24 each in the control and experimental group. Both groups were evaluated with test that was applied before and after treatment. The experimental group were taught geometry with the use of the GeoGebra software while the control group were instructed with the normal classroom instruction. Due to the sample size and sampling characteristics, the Wilcoxon T-test was used for the intragroup analysis, while the Mann Whitney U-test was used for the intergroup analysis. The results suggested that the use of the GeoGebra software had effects in the strengthening of the three capacities, with improvements that were significant at high levels in the experimental group as compared to those taught geometry without the GeoGebra software. A study entitled: The effects of using GeoGebra teaching strategy in Malaysian secondary school: A case study from Sibu, Sarawak was conducted by Rohaidah, Ting, Nor'ain and Zamzamin (2016). The study examined the effects of using GeoGebra teaching strategy in learning circle 111 topics on Malaysian secondary Form Four students' performance and attitudes towards this teaching strategy. The targeted population of this study was Form Four students in national secondary school in Sibu, Sarawak. The sample selected for the study were Form Four students from a randomly selected school. In all a total of 46 students, made up of 17 students in the control group and 29 students in the experimental group. A quasi-experimental non-equivalent pre-test post-test control group design was employed in the study. The experimental group underwent learning using GeoGebra teaching strategy whereas the control group underwent learning using conventional teaching strategy. The circle 111 achievement test and the attitude questionnaire were used as instruments in the study. Data collected from the study were analysed using one-way ANCOVA and one sample t-test. The analysis showed that there were no significant differences between mean performance scores of students in the experimental and control groups. However, the experimental students showed positive attitudes towards using GeoGebra software while learning circle 111 topics. This means that not only could the GeoGebra strategy be utilized in learning mathematics but also in enhancing students' performance in learning mathematics in the long run. Syamsiah and Jasni (2018) also examined the effectiveness of using GeoGebra software in teaching angles in Balik Paulau Polytechnic in Malaysia. The research was a quasi-experimental involving 53 randomly selected students from 107 first semester students who underwent diploma in digital technology program. Before the study both the control and the experimental group underwent pre-test after which the control group was taught angles using the conventional teaching methods while the experimental group was taught the same angles but with the use of GeoGebra software. Data collected from the study were analysed using Statistical Package for Social Sciences (SPSS). The result of the study shown that there was no statistically significant difference between students taught angles using the conventional approach and those that were taught angles using the GeoGebra software. However, it was found that the experimental group shown better learning gains than the control group since the experimental group's average score was higher than control group. The study concluded that using GeoGebra in teaching can help students learning about angles in circle Nazihatulhasanah and Nurbiha (2014) also investigated the effects of GeoGebra on students' mathematics achievement in Malaysia. The sample involved 62 students selected from Form 4 students at a secondary school in

Malaysia with 32 students in the control group and 30 students in the experimental group. The sample was selected through the purposive sampling technique. The two research instruments utilized in the study were the performance test and questionnaire. Before instruction both groups were given a pre-test to find their knowledge level before instruction. During instruction, the experimental group were taught mathematics using the GeoGebra software whiles the control group were taught without the use of GeoGebra. Again, students' perception about the use of the GeoGebra software was determined through questionnaire. Data collected from the study were analysed using Mann-Whitney Utest and simple percentages. The results of the Mann-Whitney U-test showed that the experimental group performed better than the control group. The results again indicated that students had positive perception towards learning and had better learning achievement using GeoGebra. This means that GeoGebra can benefit students' mathematics learning and diversify learning in mathematics classrooms. Effect of GeoGebra on senior secondary school students' interest and achievement in statistics was the subject of investigation by Emaikwu, IJi and Abari (2015). A sample of two hundred and forty-two (242) participated in the study. This sample was selected from a population of 2,412 senior secondary one (SS1) students in 18 government co-educational secondary schools in Makurdi Local Government Area of Benue State by multistage sampling. Two research instruments were developed for the collection of data namely; Statistics Achievement Test (SAT) and Statistics Interest Inventory (SII). The test instruments were validated by 5 experts. The researchers and the research assistants administered the pre-SAT and pre-SII to both groups. That is the GeoGebra group and the non-GeoGebra group. The post-SAT and the post-SII were also administered to both groups after treatment. Data collected from both pre-test and post-test were analysed using descriptive statistics of mean and standard deviation to answer the research questions asked while the hypotheses were tested at 5% significance level using the analysis of covariance (ANCOVA). Results from the study revealed that students taught statistics using the GeoGebra teaching method achieved higher and also showed greater interest in learning statistics than those taught using the conventional teaching approach. The study also revealed that both male and female students in the GeoGebra group achieved the same and also showed similar interest in statistics. The study however, recommended among others that teacher education institutions should be encouraged to include GeoGebra as a method in teaching secondary school statistics because of its numerous advantages. Triwahyuningtyas, Rahayu and Agustin (2019) also did a study entitled “the impact of GeoGebra Classic application on learning geometry”. A total of 50 fifth graders selected purposively from elementary school in Malang, Indonesia served as the sample for the study. These samples were further divided into two groups namely, experimental and control group. The main research instrument employed in this study was the teacher-made achievement test. Before treatment each group were given pen and paper test on geometry to find out their entry knowledge and to compare the groups. The experimental group then received treatment which was the use of GeoGebra Classic applications during learning sessions while the control group used Microsoft Power Point. The study used quasi-experimental non-equivalent control group as the design for the study. The groups were again tested to find out the impact of the instructional approaches on students’ geometrical learning. The data collected were then analyzed using SPSS Version 21. The results of the analysis showed that there were significant differences in performance between students taught with the

GeoGebra Classic application and those taught using the Microsoft Power Point in favour of the GeoGebra Classic application group. This indicated that there was a positive impact of the use of GeoGebra Classic application on geometry learning outcome of the fifth graders. Therefore, it can be concluded that GeoGebra Classic application was able to help students understand geometry-related learning materials and its use must be sustained. A study conducted by Mwingirwa and Miheso-Connor (2016) on “status of teachers’ technology uptake uses of GeoGebra in teaching secondary school Mathematics in Kenya” through training thirty-three Mathematics tutors on GeoGebra use. They also tried to implement what teacher had learnt from the training. The outcomes from Mwingirwa and Miheso-O'Connor's (2016) work uncovered that the prepared educators appeared to be excited about utilizing GeoGebra in their classes. This was because GeoGebra instruction in their classes enabled students to grasp difficult and unique concepts in Geometry and also saved teachers time whereby they were able to cover the syllabus more effectively. The outcome of Mwingirwa and Miheso O'Connor‘s study also pointed that GeoGebra was the most appropriate software or teaching learning resource for teaching Geometry due to its abstract nature. Again, teachers’ responses indicated that GeoGebra was perceived as useful for teaching and learning Mathematics. They also found out that teachers usually had trouble teaching geometry when they were solicited to demonstrate areas from mathematics, they discovered hard to teach. The instructors accentuated these challenges they experienced because of absence of assets for teaching, the unique idea of Geometry and students’ failure to envision geometrical objects. Effect of GeoGebra-Aided REACT strategy on understanding of geometry concepts was a research conducted by Jelatu, Sariyasa and Ardana (2018). The aim of the study was to examine the effect of GeoGebra-aided REACT strategy on students understanding of geometry concepts and to investigate the interaction between learning strategy and spatial ability on the understanding of geometry concepts. The study involved 60 grade 8 students of a private Junior High School in Borong, Indonesia. The research design the study utilized was the quasi-experimental design. The instrument for the study was an achievement test. The experimental group was taught using GeoGebra while the control group was taught without GeoGebra. Data from the study was analysed using two-way ANOVA. The result of the study was that:

(1) the GeoGebra-aided REACT strategy led to higher achievement of the students in the understanding of geometry concepts if compared to students in the conventional group (expository), and

(2) there was no interaction effect between the learning strategy and spatial ability on students' understanding of geometry concepts. The research recommended that GeoGebra-aided REACT strategy can be used in mathematics teaching in Junior High School to improve students' conceptual understanding of geometry concepts.

A research study on the subject of "Integration of GeoGebra in the Teaching and Learning of Geometric Transformation" was also carried out by Dahal, Shresthaand and Pant (2019). The study used a quasi-experiment as its research design. All secondary schools in Nepal's Karmandu Valley made up the study's sample. 16 kids were purposefully chosen for the study's sample from a secondary school in Nepal's Karhmanda Valley. These samples were split into control and experimental groups. GeoGebra was used to teach geometric transformation to the experimental groups, but it wasn't used to teach it to the control groups.

Both before and after treatment, students in each group were evaluated. Using SPSS, descriptive analysis of the study's data was performed. The findings of this study demonstrated the value of GeoGebra in the teaching and learning of transformational abstractions. According to study findings, pupils could actively develop their own knowledge if GeoGebra is used in math classes. The researchers suggested that GeoGebra be employed as a crucial instructional tool to augment the conventional lecture method of teaching mathematics.

Sohelia and Kumalludeen (2018) also did a quantitative study on the intention of using GeoGebra in the teaching of mathematics among Malaysian teachers. All mathematics teachers who have attended a workshop organised by a government agency like the Malaysian Ministry of Education or a non-governmental group like the GeoGebra Institute of Malaysia were included in the study's population. A total of 132 teachers were chosen as the study's sample. The 132 instructors in Malaysia who have already taken part in GeoGebra sessions were given an online survey. A cross-sectional survey methodology was used in the study's design. 132 teachers out of this total—76 female teachers and 56 male teachers - participated in the study. 83 GeoGebra users and 49 non-users made up the sample of respondents.

An independent t-test was used to analyze the data gathered from the online survey. Through the Cronbach alpha coefficient, the perceived reliability of current competencies was evaluated. A favorable link between instructors' assessed abilities and their intention to use GeoGebra in mathematics instruction was found by independent t-test and correlation analysis. The findings also showed that there was a considerable difference in the intention to employ GeoGebra in classrooms between GeoGebra users and non-users. These findings demonstrate that teachers and students both benefit from the use of GeoGebra in mathematics teaching and learning.

Dogän and Icel (2010) conducted an experimental design study to evaluate the success of students' learning using GeoGebra. The result show that the GeoGebra software encouraged higher order thinking among students. The software was also observed as having a positive impact in motivating students to learn and retaining their knowledge for a long time (Dogan, 2010, Erhan, Andreasen & Janet, 2013). Hutkemri and Zakaria (2012) also did a study on the effect of using GeoGebra on conceptual and procedural knowledge of high school mathematics students. The purpose of the study was to determine the effect of GeoGebra on conceptual and procedural knowledge of functions. The sample for the study was 124 high school students selected from Ujung Batu Rokan Hulu, Riau, Indonesia. The design for the study was a quasi-experimental made up of 60 students in the experimental group and 64 students in the control group. Data collected through the conceptual and procedural knowledge test of function were analysed using SPSS and the results indicated that students taught with the GeoGebra software performed better than students taught without the software. This study therefore show that GeoGebra is an effective tool for teaching and learning of mathematics and its usage should therefore be encouraged in mathematics classrooms. Sudihartinih and Purniati (2019) also conducted a study on using GeoGebra to develop students' understanding on circle concept. The study utilized the quasi-experimental design. The sample for the study was selected by purposive sampling method, made up of two classes. Class A (experimental class) and class B (control class). The experimental classes were taught circle using the GeoGebra approach whiles the control class were taught circle using manipulatives. Students were assessed before and after instruction. Before instruction, all students were given written test on circle using paper and pencil. After instruction students were again tested using paper and pencil, in addition, attitude test was given. The data collected were descriptively analysed. Again, the Man Whitney U-test was conducted to test the null hypothesis that the distribution of control and experimental class was the same across categories. The analyzed data showed that students' understanding of concept of circle in the experimental group (use GeoGebra) was higher than the control group (using manipulative). Students' attitude towards learning using GeoGebra was positive. Vasque (2015) also did a study entitled: Enhancing students' achievement using GeoGebra in a technology rich environment. The study involved 112 high school students. Out of this number, 53 were males and 59 were females. The control and the treatment groups were randomly selected from four periods of geometry classes. Periods two and five were randomly selected to be the treatment group, whereas periods three and six were selected to be the control group. The study employed the mixed method design. The control group were taught geometry using the normal classroom instructions while the treatment group were instructed using GeoGebra. Students' performance before and after instruction were noted. Data collected from the study were analysed descriptively and inferentially using SPSS. The findings and analyses from the qualitative and quantitative data indicated that the use of the GeoGebra software improved students' level of understanding of abstract concepts, increased students' comprehension and retention of geometric transformations, and had a positive effect on students' attitudes towards mathematics, thus enhancing their learning and achievement. The effects of an inductive reasoning learning strategy assisted by the GeoGebra software on students' motivation for functional graph 11 topic, was the study by Abdullah, et al. (2020). The research design was quasi-experimental which involved 94 Form 4 students from a secondary school in Johor. The research sample was further divided into three groups: (1) study group 1 (an inductive reasoning strategy assisted by GeoGebra); (2) study group 2 (an inductive reasoning strategy without GeoGebra); (3) control group (a conventional strategy). The research instruments consisted of a motivational questionnaire set and an inductive reasoning strategy assisted by GeoGebra and without GeoGebra. The MANOVA test results shown that the overall motivation level for study group 1 was higher in terms of attention and relevance. With regard to confidence, the results indicated that control group and study group 1 shown the same motivation level. The study concluded that learning through an inductive reasoning strategy assisted by GeoGebra can increase the students' motivation in mathematics specifically for Functional Group 11 topics. Azizul and Din (2016) also conducted a quantitative study on the use of GeoGebra in teaching and learning geometry via Massive Open Online Course (MOOC). The purpose of the study was to develop mathematics teaching and learning materials for the topic geometry based on GeoGebra for Form Four students of Sekolah Menengah Kebangsan Bandan. This study was a developmental one that used quantitative method for data collection. In all a total of 23 students were randomly selected to participate in the study. The selected students were then instructed using the learning module prepared by the researchers. The research instrument used in the study was the questionnaire. The questionnaire was in three parts. Part 1 was for demographic data, part 2 for usability of the GeoGebra module, while part 3 for elements of MOOC. The questionnaire used a five Likert scale for all the items. Data collected from the questionnaire was descriptively analysed using the SPSS. The results shown that the software was not only easy to use but also helped students understand the basic concept of geometry. Again, the students agreed that GeoGebra was an effective tool for teaching geometry related topics such as straight lines, circles and trigonometry. Kutluca (2013) also studied the effect of Geometry instruction using GeoGebra on Van Hiele Geometry Understanding Levels of students. The quasi-experimental design was employed. A total of 42 students were chosen for the study, made up of both the control and the experimental group. Kutluca (2013) found out from his study that GeoGebra instruction employed on the experimental group was better on increasing Van Hiele geometry thinking levels of students than the traditional approach of teaching geometry on the control group. He indicated that the GeoGebra helped students in creating their own geometric shapes, trying different things on the shapes, testing and constructing their own knowledge. In addition, students in the experimental group also had the opportunity to participate actively to the instructional process, share ideas comfortably, and discuss results obtained with friends and to construct their own knowledge. It is obvious from Kutluca's (2013) study that when GeoGebra is fully utilized in classroom, it will enhance better teaching and learning, especially in Mathematics. Also, according to Ugorji and Chimuka (2017) it is virtually impossible to have passive students when computer technology, such as GeoGebra, is used in the teaching and learning process. According to the same researcher, GeoGebra changes passive students to independent explorers and the role of the teacher in the teaching and learning process is to direct and monitor students' work. Despite the benefits, this software (GeoGebra) is not widely used among mathematics teachers in Ghana (Tay and Mensah-Wonkyi, 2018) and more effects should be taken to encourage the use of GeoGebra for personalized learning.

2.2.8 Difficulties in learning circle geometry (circle theorems)

Idris (2006), studied the causes of students' difficulties in learning geometry and came out with the following as the main causes; geometry language, visualization abilities and ineffective instruction by teachers. She also highlighted that spatial visualization has been linked with geometric achievement because geometry is visual in nature. Geometry is the study of shapes and space; it requires visualizing abilities but many students cannot visualize three–dimensional objects in a two–dimensional perspective (Guven & Kosa, 2008). Other factors include: non–availability and obsolescence of instructional materials, gender differences, poor reasoning skills, inadequate skill, inadequate time, inadequate school curriculum and lack of proof by students (Mason, 2002; Uduosoro, 2011). All these contribute to student's difficulties in learning circle geometry.

Johnson–Wilder and Mason (2005) have also blamed students' lack of interest and understanding of geometry on teachers' poor teaching skills and lack of resources for presenting geometrical shapes to students. According to them, the ordinary primary tutor has an anxiety of the very word 'geometry' and therefore do not handle the concept well. Again, textbooks for basic schools devote little attention on geometry. Students, who enter Colleges of Education, therefore have very weak foundations in geometry in general and circle theorems in particular. Adolphus (2011) also studied the problem of teaching and learning of geometry in secondary schools in Rivers State, Nigeria. The sample for the study was 300 students and 30 teachers drawn from a population of 10 secondary schools in River State. The research instrument for the study was questionnaire. The research also adopted the descriptive survey method as the design for the study. The data collected by the means of the questionnaires were analysed using simple means. The following were identified as some of the causes of students' difficulties in learning geometry; poor foundations of most mathematics teachers in geometry, poor foundations of students in mathematics in general, poor teaching and learning environment of most schools, poor attitude of students towards learning of geometry, lack of commitment on the part of some teachers due to lack of motivation. Based on the findings of the study, the study recommended that:

(a) The State government should as a matter of urgency send mathematics teachers for training and seminars for effective teaching and learning.

(b) The government should endeavor to provide the necessary infrastructures and facilities that will motivate teaching and learning of mathematics.

Adegun and Adegun (2013) also carried out a study on students' and teachers' view of difficult areas in mathematics syllabus: basic requirement for science and engineering education. The population of the study consisted of all mathematics teachers and all the senior secondary III students in all the 18 secondary schools in the Local Government Area. A sample of 15 mathematics teachers and 180 senior secondary school III students were selected through simple random sampling techniques. Survey questionnaires were designed and administered by the researcher to elicit information from both students and teachers on difficult areas in mathematics. The analysed data indicated that geometry was one of the topics students perceived as difficult and the reasons assigned were: poor knowledge of the subject matter by teachers, low level of commitment by teachers and poor attitude towards and teaching and learning of geometry by students. Suredra (2016) also conducted qualitative research on the topic: "Problems of teaching and learning mathematics in geometry at the grade IX". The purpose of this study was to identify the problems faced by teachers and students in teaching and learning geometry. The sample for the study was made up of two mathematics teachers, five students, a head-teacher and five parents. The sample was purposively selected from Shree Rajaji, Tulilal, Jonchhe & Janta higher secondary school Siswa-Belhi, Saptari District. The main research tools used for data collection were observation, recorded history and interview. The collected data were descriptively analysed using SPSS. From the study, the researcher identified that the teaching-learning environment of school and home, pre-knowledge of students, learning activities which seems to be examinations oriented rather than practical oriented, poor evaluation techniques, students' weak pre-knowledge about geometry, lack of appropriate teaching methods and materials, complex and voluminous syllabus in secondary level mathematics curriculum, and no-effective management related problems faced by teachers and students in teaching and learning geometry. Egwu, Asuque and Ofori (2018) investigated the topic: Geometry viewed as a difficult mathematics. The study focused on 450 SS 2 students, made up of 230 females and 220 males from 30 senior secondary schools in Cross River State which were randomly selected within the three senatorial District. The perceived difficult mathematics concepts were study through the research instrument of 20-item questionnaire. Data collected from the study were analysed using frequency counts and percentages. From the analysis, eight out of 20 concepts were perceived difficult by the students, these include coordinate geometry, circle theorems, construction etc. and the reasons given for viewing geometry concept as difficult is as a result of irregular class practices, unavailability of instructional materials, teachers' method of teaching, bad and inadequate timing etc. However, student gender had a great influence on the learning concepts on geometry at 0.05 level of significant in favour of female students. Based on the study, the researchers recommended appropriate teaching methods and effective instructional material if teachers want students to derive better understanding on the identified difficult geometry concepts. Fabiyi (2017), studied geometry concepts in mathematics perceived difficult to learn by senior secondary school students in Ekiti State, Nigeria. 500 senior secondary school two (SS2) students made up of 228 males and 272 females from thirty (30) co– educational schools in Ekiti State State, Nigeria constituted the sample for the study. The proportionate and random sampling methods were used to select the sample for the study. A 23–item questionnaire on Goemetry Concept in Mathematics Perceived Difficult was used as the instrument for the study. The research questions were analysed by using frequency counts and percentages while the only hypothesis was tested using chi-square statistics The findings revealed that, out of 23 concepts, eight concepts were perceived difficult to learn by students which included: Construction, Coordinate geometry, circle theorems and so on and reasons given for perceiving geometry concepts difficult includes: unavailability of instructional materials, teachers' method of instruction and students attitudes towards the teaching and learning of geometry. It was also revealed from the analysis that students' gender had great influences on the learning of concepts in geometry at 0.05 level of significance in favour of female students. Identification and remediation of students' learning difficulties in geometry in River State was the subject of investigation by Ejiofor-Chima and Accra (2019). The population of the study consisted of 7,719 SS2 students in Port Harcourt Local Government Area, Rivers State. A sample size of 314 was drawn from four schools out of sixteen public schools in PHALGA using purposive sampling technique. The study employed a quasi-experimental design. The instruments used for the study were Learning Difficulties

Identification Test on Geometry (LDITOG) and Remediation Test on Geometry (REMTOG). The reliability coefficient of the instruments was determined using test re-test. Before treatment, the two groups were given pre-test on the identification of presence and types of learning difficulties experienced. A post-test was given after treatment to determine the effect of remediation. Data collected for the study were analysed using percentages, frequency counts, mean and standard deviations to answer the research questions while the hypotheses were tested at 0.05 significant level using Chi-Square and Analysis of Covariance (ANCOVA). The findings of the study revealed the presence of adaptive reasoning, procedural formulation, strategic competence and conceptual understanding learning difficulties among students. Again, the study revealed that students' performance and mathematics ability levels improved after remediation. Sulistiowati, Herman and Jupri (2019) conducted a research study entitled: Student difficulties in solving geometry problem based on Van Hiele thinking level. The aim of the study was to analyse students' difficulties in solving geometry problems based on Van Hiele thinking levels.

Descriptive qualitative research was employed in this study. The main research instruments used in the study were the Van Hiele geometry test and problem-solving test, followed by interviews. The subjects of the study were 38 students grade VIII in one of the secondary schools in Bandung and 6 of them were interviewed afterwards. The results of the study showed that the main difficulty of students at level 1 (visualization) is interpreting problems into a mathematical model. While the main difficulty of students at level 2 (analysis) and level 3 (deduction informal) is in the solution processes. The study concluded that problem-solving ability on geometry is important to be taught to all students even though they are at different Van Hiele levels. Ntshengedzeni (2015) did a study on the topic “Enhancement of learners’ performance in geometry at secondary schools in the Vhembe District of Limpopo Province. The purpose of the study was to enhance learners’ performance in Euclidian Geometry as a branch of mathematics in the Further Education and Training (FET) band of secondary schools. Two sampling techniques were used to determine the sample. Purposive sampling were used to select the participants for the study while cluster and simple random sampling were used to classify schools into three categories; low, average and high performing. Each cluster had three schools. In all nine schools were selected for the study, made up of 405 students, 6 school principals, 6 heads of departments and 6 teachers. The study adopted the mixed method design. The main instruments used in the data collection were the questionnaire and the interview. Both teachers and students were asked to response to the causes of difficulties in teaching and learning geometry as well as how these causes can be overcome. The data collected from the instruments were analysed using SPSS Version 22. The data analyzed revealed the following as the causes of students’ difficulties in geometry: poor learning environment and large class size, lack of resources for teaching geometry, poor attitude towards the teaching and learning of geometry. On the other hand, the following were identified as strategies to enhance teaching and learning of geometry: provision of adequate instructional materials to teach geometry, applications of geometry in real life situations, frequent monitoring by supervisors, frequent helpful feedback to students, spending more time on the task will make the learner master geometric skills, making learners more involve in practical work than theoretical work, parents buying the necessary reading materials for students and learners’ willingness to learn on their own. Shankar (2016), carried-out a research titled “Problems faced by secondary level mathematics teachers and students in geometry. The participants of the study consisted of ten secondary level mathematics teachers, ten guardians and two hundred secondary level students. In all 220 candidates were taken as the sample for the study. These sample was selected by simple random sampling method from a population of all secondary level schools, mathematics teachers and students of grade 10 (also guardians) of Tokha. The main data collection instruments for the study were observation, interview and achievement test. The data collected from the study were analysed by survey design. Responses obtained from class observation and achievement test were analysed by three Likert scale with the help of mean weightage, content of text books. The study identified school administration, parents, teachers, publications, students and academic policy of the nation as responsible for students’ difficulties in learning geometry. Mifetu, Kpotosu, Bessah and Amegbor (2019) researched on geometry topics in mathematics perceived difficult to study by senior high school students in the Cape Coast Metropolis. Using the descriptive survey design, the researchers collected data from 300 senior high school Form two students comprising 200 males and 100 females using simple random sampling technique. The two main research instruments employed in this study were questionnaire and teacher-made achievement test. Participants were asked some of the geometry topics they perceive as difficult to learn. In addition, 30-items multiple choice achievement test on geometry were designed for the study. Data collected from both the survey questions and the teacher made achievement test revealed that, four of the geometry concepts perceived difficult to learn by students are: circle theorems, perpendicularity of tangent and radius of a circle, angle between tangent and chord and tangent from an external point. The main reasons given for perceiving geometry concepts difficult include: unavailability of instructional materials to make the teaching of geometry real and lack of understanding of geometrical concepts by teachers. However, students' gender had no influence on the learning of geometrical concepts. Bosson-Amedenu (2017) also examined remedial students' perception of difficult concepts in senior high school core mathematics curriculum in Ghana. The study employed the survey design. A sample of 112 remedial students was obtained by convenience sampling for the study. This number consisted of 62 females and 50 males' remedial students graduating from various secondary schools across Ghana who had been unsuccessful (obtained grades in the range D7-F9) in the WASSCE core mathematics. The instrument used for the data collection was a 38-item questionnaire tagged Difficult Concept Identification Questionnaire in Mathematics (DCIQM). The data obtained from the study were analysed using mean with the criterion mean set at 3.05 for identifying difficult topics and 3.6 for identifying possible causes of the perceived difficulty. The findings of the study revealed that students identified some mathematics topics such as circle theorems, ratio and proportion, plane geometry, trigonometry and bearing, business mathematics and coordinate geometry as difficult. The most difficult topic identified by these students was circle theorems followed by plane geometry and the reasons assigned for these difficulties were;

(1) some mathematics teachers had difficulty with some topics in mathematics themselves,

(2) poor mathematics foundation by students,

(3) lack of resources for teaching mathematics by instructors

(4) Large class size in most senior high schools and

(5) lack of motivation in learning mathematics because of its abstractness. The study recommended amongst others that workshops should be organized to train mathematics teachers on the effective and efficient strategies that should be adopted for the teaching of the identified difficult mathematics concepts.

Tatlah, Amin and Anwar (2017) also did a study on students' learning difficulties in mathematics at the secondary school level. A sample of 60 mathematics teachers and 300 students were selected through simple random sampling techniques to participate in the study. The findings of the study revealed that both teachers and students were of the view that students find it difficult in learning geometry concepts.

2.3 Conceptual Framework

A conceptual framework is a representation of the relationship you expect to see between the variables, or the characteristics or properties that you want to study. Conceptual frameworks can be written or visual and are generally developed based on a literature review of existing studies about the study and below is conceptual framework for the study.

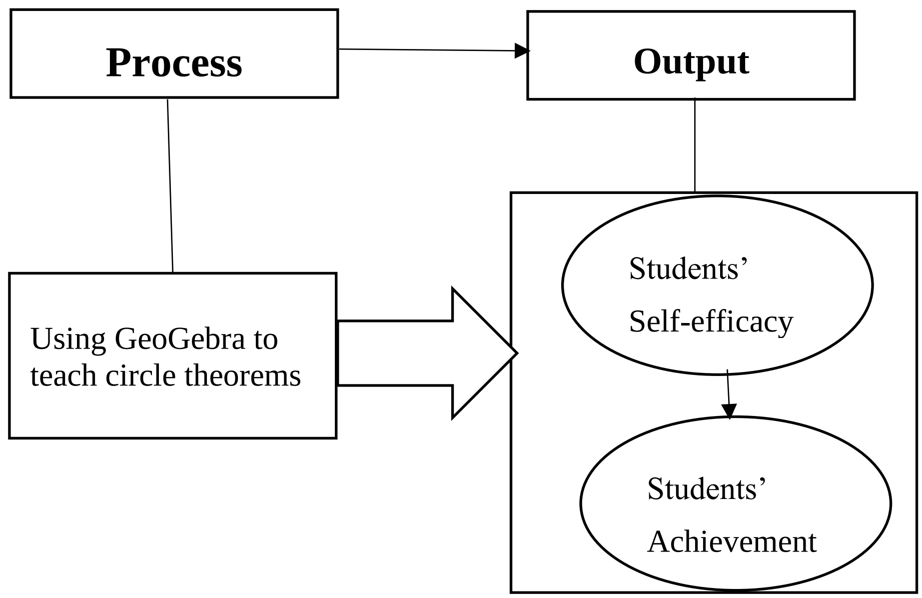

Figure 2: A process-output model showing the connections in using GeoGebra to teach circle theorems, students' self-efficacy and students' achievement.

The model shows the use of GeoGebra in teaching circle theorems approach as the process variable and self-efficacy and achievement as the output variables. The model explained a process (using GeoGebra to teach circle theorems) initiated to result in an output students' self-efficacy and achievement. In other words, there was a teaching method manipulated to cause-and-effect on students' achievement. The model also showed the relationships between the input and output variables.

2.3.1 Mathematics self-efficacy and achievement

According to Bandura (1997), the belief or perception that one is capable of planning and carrying out the actions required to succeed at a specific activity is known as self-efficacy. People might have a variety of self-efficacy ideas about themselves at any given time since self-efficacy can be either general or task-specific. How someone feels, thinks, and motivates themselves may be influenced by their ideas about their own levels of self-efficacy. Individuals with varying levels of self-efficacy may exhibit noticeable behavioral differences as a result of this. Students who have a strong or high sense of self-efficacy have a strong feeling of self-belief and view obstacles as tasks to be conquered rather than dangers to be avoided. Students with high self-efficacy put considerable effort and commitment into their work. They are able to quickly rebound from setbacks and use them as lessons. According to Stevens, Olivárez and Hamman (2006), general mental ability was a weaker predictor of mathematical accomplishment than self-efficacy and the sources of self-efficacy.

Self-efficacy beliefs are significant outside of the current learning setting. The choice of a student's educational path and job may be influenced by how confident they are in their ability to solve mathematical problems (Hackett, 1995). Self-concept is a notion associated with self-efficacy and is defined as "beliefs, hypotheses, and assumptions that the individual holds about himself" by Coopersmith and Feldman (1974).

Mathematical achievement is the level of proficiency a student exhibits in the discipline of mathematics. It is determined by the results of a mathematics achievement test.

The results of a study by Tooke and Lindstrom (1998) revealed that exceptional accomplishment depends on students having confidence in their ability to master mathematics and solve mathematical problems. It follows that confidence must play a significant part in achieving success in mathematics.

In addition, Andrew, Salamonson, and Holcomb (2009, p. 218) cited Bandura's (1977) assertion that a person's self-efficacy expectation of their individual ability to successfully complete a given task is a reliable predictor of whether or not they will attempt the task, the amount of effort they will put forth, and their level of perseverance in the face of unforeseen difficulties. Previous research demonstrates that self-efficacy has been used to assess performance in a range of academic discipline, but a main focus has been on mathematical abilities (Kranzler & Pajares, 1997).

2.4 Chapter Summary

In the current study, a number of studies that have provided insight into the use of technology, particularly interactive software, in the teaching and learning of geometry were analyzed. Although there has been a lot of interest in the teaching and learning of geometry in general, there has not been much focus on using interactive software to study the circle theorems specifically.

When using computer programs to solve geometry problems, studies have focused on improving students' spatial and logical thinking skills as well as their ability to identify geometric objects. The use of interactive software to improve the teaching and learning of circle theorems principles as well as students' grasp of these concepts was not clearly emphasized. This made the current study unique because it concentrated on Ghanaian college students studying education and examined how well they performed in circle theorems.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter outlines the methods utilized to produce and analyze the data for the study. The description consists of the study area, theoretical stand, research philosophy or paradigm, research approach, research design, study population, sample and sampling technique, as well as the data collection instrument(s), data collection procedure, data, data analysis, and ethical consideration.

3.1 Study Area

The study was conducted in Berekum College of Education located in the Berekum East districts in the Bono Region of Ghana. The Bono Region is located in the Mid-West part of Ghana, bordered to the north by the Bono East Region and to the south by Ahafo and Western Region and to the West, Cote D'Ivoire.

The people of Berekum are predominately farmers and there are seven Senior High Schools and a Nursing and Midwifery Training College owned by Catholic Church.

3.2 Research Philosophy

Creswell & Creswell (2017) defined paradigm as the fundamental set of beliefs that directs a researcher's methodology. The philosophical underpinning of pragmatism allows and guides mixed methods researchers to use a variety of approaches to answer research questions that cannot be addressed using a singular method.

The paradigm used in this study is pragmatism because pragmatism research philosophy can integrate the use of multiple research methods such as qualitative, quantitative and action research methods in order to arrive at accurate answers to the research questions in the study.

3.3 Research Approach

The study used a mixed-methods approach in order to collect data and test the hypothesis and also answer research questions accurately.

Creswell (2013) asserted that mixed-methods approach enables the researcher to gather both qualitative and quantitative data on the phenomenon under study to allow for proper understanding of the study's problem. Four categories of mixed-methods approach as described by Creswell, Klassen, Plano-Clark, & Smith, (2011) are convergent parallel approach, exploratory approach, embedded approach, and explanatory approach.

The researcher used embedded approach in this study. In an embedded design, researcher collects and analyzes both types of data at the same time, but within a larger quantitative or qualitative design. The embedded approach is appropriate when one type of data clearly plays a secondary role and would not be meaningful if not embedded within the primary data set. This approach is used because there were different questions which required collection of both quantitative and qualitative at the same time.

3.4 Research Design

A quasi-experimental design was used in the study. A quasi-experiment is an empirical investigation that examines the cause-and-effect relationship of an intervention (treatment) on a target population without using random selection to divide participants into groups (Creswell, 2014). The participants were non-randomly divided into the treatment (experimental) and control groups according to the research design. This approach gave the researcher the opportunity to investigate the causes-and-effects of using the GeoGebra software into instruction and learning to raise students' performance (Creswell, 2014). The purpose of the study was to determine the effects of teaching circle theorems to students in colleges of education using the GeoGebra software. Lessons were prepared and given to the students in that regard. GeoGebra-taught students (experimental group) performance was compared to that of students (control group) who were not utilizing the GeoGebra software but traditional or convention method (marker-board method).

3.5 Population

The population for the study comprised of all 600 students in first year of Berekum College of Education in the Bono Region of Ghana. Students were put into two groups upon admission into the college, students were either admitted to offer Bachelor of Education (Primary School) or Bachelor of Education (Junior High School).

3.6 Sample and Sampling Technique

Taherdoost (2016) defined sampling as taking a subset from chosen sampling frame or the entire population.

Two hundred and forty (240) level 100 students from two intact classes were selected from a population of 600 students who participated in the study based on Yamane's sample size formula (See Appendix B) as cited in Adam (2021).

Psathas (1973) contributed to qualitative research and said that qualitative research in education can continually be found asking questions of the people they are learning from to discover “what they are experiencing, how they interpret their experiences, and how they structure the social world in which they live”. Qualitative research set up strategies and procedures to enable them to consider experiences from the informants’ perspectives. For some, the process of doing qualitative research can be characterized as a dialogue or interplay between researchers and their subjects.

Additionally, Merriam (2015) suggested that for qualitative analysis, purposive sampling is most fitting. The researcher further explained that this method of sampling is based on the premise that the researcher needs to understand and gain knowledge, so a sample from which the most can be learned must be chosen. Based on this assertion from Merriam (2015), four (4) students two from each the experimental group and control group were purposively selected for an interview session.

3.7 Data Collection Instruments

The researcher sort approval from the college’s administration before conducting the research.

The quantitative data were collected using pre- and post-tests as well as questionnaires. Additionally, interviews were held to collect information for the qualitative phase.

3.7.1 Pre-test and post-test

Pre-test consisted of five (5) questions of which students in both experimental and control groups were supposed to answer. The geometric concepts and ideas covered in the colleges of education’s first-year curriculum were the basis for the pre-test questions. There were

10 total points available for the pre-test, with 2 points allocated for each question. The pre-test was used in the study to determine whether the background knowledge and comprehension levels of the students in the two groups were comparable enough to understand the idea of the circle theorems. The researcher administered ten (10) questions on circle theorems as a post-test to both the experimental group and control group after teaching the experimental group using GeoGebra and the control group with the traditional or conventional method (marker-board method). Total marks allocated for post-test was also 10 marks. The main reason for conducting the post-test was to determine the treatment impacts and effects on the academic achievement of first-year students of Berekum College of Education. This test was conducted at the end of the treatment. The test was conducted under the supervision of the researcher and some colleague's mathematics tutors in the Mathematics and ICT Department. The questions were structured sequentially from the very easy to the most difficult.

3.7.2 Questionnaire

A questionnaire, according to Roopa and Rani (2012), is a list of inquiries sent to people in order to gather statistically valuable data about a particular subject or occurrence. They contended that every questionnaire should have a clear objective connected to the study's goals. In this study, students' opinions about the usefulness of GeoGebra in learning circle theorems, the ease-of-use of GeoGebra in circle theorems, and students' perceived challenges in using GeoGebra in the teaching and learning of circle theorems at Berekum College of Education were gathered through a questionnaire.

The purpose of the research questionnaire was to gather data on students' attitudes and motivations toward circle theorems as well as their attitudes toward using the software.

Students in the experimental group were the ones who received the questionnaire. Five-point Likert scale was utilized in the questionnaire. Given the five-point Likert scale, the average score is 3 ([1+2+3+4+5] ÷ 5 = 3). In this study, the determination of respondents' level of agreement with items in the questionnaires was done using mean and standard deviation such that mean < 3 indicated respondents' disagreement and mean > 3 also indicated respondents' agreement. Students' conceptual comprehension and capacity for problem-solving of the circle theorems using GeoGebra was the subject of the questions which sought information on their understanding. Students' perceived challenges with the GeoGebra-based classroom was also captured in the questionnaire.

The questionnaire was used because it enabled the researcher to generate data specific to his research questions and also for easy comparisons. The questionnaire was again employed because it protected the privacy of the respondents as participants responded honestly to the questionnaire because their identity was hidden and their confidentiality was also maintained

3.7.3 Interview

Interviews were intended to find out the experiences, understandings, opinions, or motivations of participants. Interview was conducted for the qualitative phase. Interviewing particularly is an effective technique for collecting data about the lived experience of participants in a phenomenological study (Van den Berg et al 2005). The researcher used semi-structured interviews to collect qualitative data for the study. Semi-structured interviews were defined by Merriam (2009) as interviews in which "the questions are more flexibly worded and consist of a mix of more or less structured questions", allowing the researcher to respond to and be shaped by the situation as it unfolds, and fully explore the emerging worldview of the respondent.

Seidman (2006) proposed that exploring personal perspectives requires the use of interviews, both in focus groups and individually, as it provides the research with the anecdotal and contextual evidence that is vital to an understanding of these interactions. Creswell (2012) had it that one-on-one interviews were primarily used with participants who are "not hesitate to speak, who are articulate, and who can share ideas comfortably". Four (4) students two from each of the experimental group and control group were purposively selected for interview to gather qualitative data for the study. The researcher constructed the semi-structured interview guide under the guidance of research question four. The researcher made it abundantly clear to the study's participants the form it would take and that they could opt-out at any time. Before each interview began, participants were told that their participation was entirely voluntary, that they had the option to end the interview at any moment, and that they had the opportunity to ask questions. Students were interrogated and audio taped while following the interview protocol. The interview methodology included questions that were created from a variety of linked research literature to support the study.

3.8 Validity of Research Instruments

The test items were pre-tested and post-tested on twenty (20) first-year students from St. Ambrose College of Education, the closest sister college in Dormaa Akwamu in the Bono Region, who were not participating in the study. This was done to make sure the test items were valid and relevant to the goal of the investigation. The current study guaranteed the validity of the pre-test and post-test content tests. Polit and Beck (2006) defined content validity as a method for assessing or determining an expert consensus regarding the worth of a certain test item. They pointed out that using expert judgment is the main technique for determining content validity. Following Polit and Beck (2006), three experienced mathematics tutors from Mathematics and ICT Department in Berekum College of Education in the Bono Region assessed the pre-test and the post-test in this regard and made improvements to the instruments.

3.9 Reliability of Research Instruments

To ensure reliability, three experienced mathematics tutors were tasked to score the students on the pre-test and post-test. The three tutors' scores did not differ in any way. The Cronbach's Alpha test was used to assess the reliability of each of the three constructs. In general, it is thought that scores with high alpha coefficients (above 0.7) indicate acceptable internal consistency (Bryman & Cramer, 2005). The components in the three constructs of Cronbach's Alpha are displayed in Table 1.

Table 1: Reliability of the questionnaire items leading to their construct

Construct	No. of Items	Cronbach's Alpha (α)
Respondents' opinion on the usefulness of GeoGebra in teaching and learning circle theorems.	10	.81
Respondents' views on ease-of-use of GeoGebra in learning circle theorems.	9	.77
Respondents' perceived challenges of using GeoGebra in learning circle theorems.	6	.72

Source: Field Work, 2022

Table 1 revealed that the three constructs were judged to be reliable and that the constructs could be used for the study.

3.10 Trustworthiness of Qualitative Data

The researcher again ensured trustworthiness on the part of qualitative date by considering transferability, confirmability, credibility and dependability of the study's findings.

3.10.1 Transferability

This pertains to the extent to which the results can be applied to different contexts or utilized in a broader sense. It indicates the extent to which the findings of one study can be generalized to other studies with comparable characteristics (De Ceunynck et al., 2013). The purpose of qualitative research is not to generalize its findings, but it is thought that other researchers will be able to gain a lot from the results if they conduct similar research with similar questions. De Ceunynck et al. (2013) added that, transferability as opposed to generalization, which states that the results of a particular study can be applied to all contexts relevant to the context under study, transferability focuses on researchers making connections between the results in contexts and circumstances that are related but outside the purview of the initial study.

3.10.2 Confirmability

The unique viewpoint of the researcher may bias or distort the data gathering or analysis. If the study's conclusions required to be confirmed by other researchers, interpretive validity (Elo et al., 2014) helped to produce objective and consistent results. The level of interpretative validity of a study is the extent to which the researcher correctly analyzes and presents the opinions, intentions, and experiences of the participants. The researcher actively participated in critical self-reflection and used the reflexivity technique to prevent biases or predispositions. The researcher was able to control and keep an eye on biases by keeping track of personal assumptions through memo writing. The researcher kept a "paper trail" of the procedures used to confirm and revise the data by documenting the methods on an ongoing basis.

Furthermore, the researcher actively searched for and recorded any negative incidents that contradicted previous interviews. Similarity between the student's interview responses was examined. To put it another way, the researcher looked at how well each student's self-report from the interview matched the nominators' descriptions of them. The most importance was given to the consistency of the students' self-reports regarding the differences in circle theorems between students who utilize and do not utilize Geogebra.

3.10.3 Credibility

Credibility is proving that the study's conclusions are believable from the participants' point of view. Because their participation was the only means to ascertain the effect of employing GeoGebra on students' achievement in circle theorems, the study's participants were the only ones equipped to evaluate the validity of the findings. Member checks, which entail showing research materials to research subjects, are the most crucial strategy for establishing trustworthiness (Daniel, 2019). The opportunity to express agreement or displeasure with the researcher's portrayal of them is provided for participants. The students were provided with the transcripts of their interviews by the researcher so they could do accuracy checks and make any necessary modifications or amendments to accurately represent their viewpoints.

3.10.4 Dependability

This talks about how data remains stable over time. The idea of dependability is how research findings will be reproducible or replicable over time (Coryn, 2007). The researcher ensured dependability by making sure researcher adhered closely to the guidelines when performing the research. Every respondent received assurances regarding their anonymity. Once more, the investigator gathered sufficient evidence from related research to support the findings and conclusions of this investigation. The reference section has correctly cited all of the authors that were cited.

3.11 Data Analysis Procedure

Statistical Package for the Social Sciences (SPSS) version 25 was used to analyze the pre- and post-test scores using descriptive and inferential statistics. The mean and standard deviation were used in the data's descriptive analysis. The paired sample t-test was additionally employed for inferential data analysis to draw conclusions from the data. Additionally, the effect that the treatment had on the experimental group was measured using the effect size statistic (Eta squared).

3.12 Ethical Consideration

Participants of the research work were ethically taken care of by ensuring trustworthiness in their response in the questionnaire. Permission was taken from students who were interviewed. Also, prior notice was given concerning the date and time of the interview. To keep responses of the interviewees private, anonymity and confidentiality were assured and that interviewees were coded.

3.13 Chapter Summary

At the end of this study, the researcher was expected to find out effects of using GeoGebra in teaching circle theorems on Berekum College of Education students' mathematics self-efficacy and achievement. Teachers in this municipality realized the positive impact of the use of activity-based teaching approach precisely GeoGebra software in the teaching and learning of mathematics and that the conventional method of teaching mathematics would be minimized drastically if not totally eradicated.

On the part of the students, using GeoGebra software during mathematics lessons helped improve students’ mathematics self-efficacy and achievement and that student structured mathematical information by themselves on mathematics concepts. These helped students to develop love for mathematics and pursue it in higher educational ladder and ultimately solve their daily life problems using mathematics.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

This chapter presents the results and discussion of the findings obtained from the analysis. The response rate was discussed at the beginning of the chapter, and then the demographic details of the respondents were discussed. The research questions were then analysed, and the discussion of the findings was then presented.

4.1 Response Rate

A total of one hundred and twenty-seven (127) questionnaires were distributed to the respondents. All the questionnaires were retrieved from the respondents, representing a response rate of 100%.

4.2 Demographic of the Respondents

The demographic characteristics of the respondents were examined under the following: gender, age, and programme offered and Table 2 shows the Demographic Characteristics of the Respondents.

Table 2: Demographic Characteristics of the Respondents

Variable	Category	Frequency	Percentages
Gender	Male	138	57.5
	Female	102	42.5
	Total	**240**	**100.0**
Age	15 – 18	7	2.9
	19 – 22	168	70.0
	23 – 26	56	23.3
	27 and above	9	3.8
	Total	**240**	**100.0**
Programme Offered	B.ED (Primary School)	127	52.9
	B.ED (JHS)	113	47.1
	Total	**240**	**100.0**

Source: Field Work, 2022

As indicated in Table 2, there were more male respondents than female respondents in the study which showed a clear study of sex non-parity of sex of the respondents. Regarding age, the results showed that majority of the students who participated in the study were 19 - 22 years than students who fell between 15 - 18 years, 23 - 26 years as well as those who were 27 years and above. In dealing with programme offered by respondents, the distribution of the respondents on programme offered revealed that one hundred and twenty-seven (127) representing 52.9% offered Bachelor of Education (Primary School) and one hundred and thirteen (113) also representing (47.1%) offered Bachelor of Education (Junior High School). The composition of the respondents showed that respondents who offered Bachelor of Education (Primary School) were more than Bachelor of Education (Junior High School).

4.3 Presentation of Findings

The study's findings are reported in this section and are organized in accordance with the research questions.

Research Question 1: What was the mean difference in achievement of students taught with GeoGebra and students without GeoGebra?

A pre-test with a total score of 10 marks was administered to both groups prior to the intervention to ascertain the effect of utilizing GeoGebra in teaching circle theorems (control group and experimental group). The goal of the pre-test was to ascertain whether students in both groups demonstrated comparable proficiency with circle theorems prior to the start of the treatments. A post-test with a total score of 10 was also administered following the intervention. The post-test was purposely designed to assess GeoGebra software effectiveness in teaching circle theorems. (See Appendix A).

Table 3: Descriptive statistics for both pre-test and post-test for the two groups.

Test	**Control**			**Experimental**		
	N	Mean	Std. Deviation	N	Mean	Std. Deviation
Pre -test	113	2.5575	1.37541	127	2.3228	1.18108
Post-test	113	4.5133	1.00103	127	7.6614	1.63891

Source: Field Work, 2022

From Table 3, the spread of marks from the mean marks of both groups were not much different in the pre-test. The results in Table 3 showed that the performance (pre-test marks) of the control group; was not statistically and significantly greater than the performance (pre-test marks) of experimental group. The implication of the findings of the pre-test was that students in both groups had identical level of circle theorem's ability before the intervention; therefore, any differences in the performance of students in circle theorems after the intervention was as a result of the intervention. However, in the post-test, the average performance between the control group and the experimental group was statistically significant.

In order to determine if the average performance between the control group and the experimental group was significant, paired sample t-test was conducted for the post-test results and the results is presented in Table 4.

Table 4: Paired sample t-test for post-test

	Mean	Std. Deviation	Std. Error Mean	95% CI Lower	95% CI Upper	T	Df	Sig (2-tailed)	Eta square
Post-test – Pre - test	5.3386	2.11653	.18781	4.96691	5.71026	28.425	126	.000	.87

Source: Field work, 2022

The hypothesis: there was statistically significant difference in academic achievement between students who were taught using GeoGebra and students not taught with GeoGebra was tested at 95% confidence interval. Results from Table 4 showed that there was a statistically significant difference in post-test scores of the experimental group and control group. This finding illustrated that the students in the experimental group who were taught circle theorems using GeoGebra performed better than the control group which used traditional or conventional method by 3.1481 out of a total of 10 marks. The Eta squared test statistic .87 (87%) as shown in Table 4 indicated large effect size which was caused by the intervention. Additionally, effect size of .87 (87%) explains high efficacy of the intervention on the experimental group.

Research Question 2: What are the usefulness of GeoGebra in teaching and learning of circle theorems.

To assess the usefulness of GeoGebra in teaching and learning of circle theorems, the respondents were asked to rate 5-point Likert scale item with 1 showing least rating and 5 showing strong rating (See appendix A). For analysis purposes, the mean and standard deviation of the responses given by the respondents were computed. The results of the respondents' responses on usefulness of GeoGebra were analysed with mean ranks. The responses from respondents on usefulness of GeoGebra was presented in Table 5.

Table 5: Respondents' opinion on usefulness of GeoGebra in teaching and learning circle theorems.

Variable	Mean	Stds.
GeoGebra helped me understand the inscribed angle theorems	4.433	.599
GeoGebra helped me visualise various positions of an angle in a semi-circle.	4.354	.650
With the use of GeoGebra I was able to draw various positions of a cyclic quadrilateral.	4.268	.684
The use of GeoGebra helped me understand the relationship of angles in the circle and the tangent.	4.221	.700
The visualization from GeoGebra helped me come out with the right theorems on my own.	4.181	.660
By using GeoGebra, I had fewer problems understanding the circle theorems.	4.173	.618
After using GeoGebra, I am able to solve problems using the right approach.	4.142	.721
I enjoyed the class lessons with the use of GeoGebra.	4.126	.735
I am happy the teacher used GeoGebra in teaching circle theorems.	4.110	.704
After the classroom lessons, I felt confident in my ability to solve questions in circle theorems.	4.095	.728
Grand mean	**4.210**	**.680**

Table 5 revealed that student's ability to solve problems in circle theorems using the right approach which indicated that respondents were in agreement to this sub-construct than students confident in solving in their ability to solve problems in circle theorems as well as students' ability to draw various positions of a cyclic quadrilateral. Students again agreed and came out that, they were able to come out with the right theorems on their own through visualisation than GeoGebra helping students to understand the relationship of angles in the circle and the tangent. Respondents' agreement to the variables: students feel happy if teacher uses GeoGebra in teaching circle theorems and students having fewer problems understanding circle theorems were almost the same. Students again agreed that they enjoyed the class lessons with the use of GeoGebra than students visualising various positions of an angle in a semi-circle than students understanding inscribed angle theorems in circle theorems.

In net shell, the results in Table 5 indicated that students agreed to all the components to be an embodiment of how usefulness GeoGebra is in the teaching and learning of circle theorems. The researcher could further say that students had agreed to the usefulness of GeoGebra in the teaching and learning of circle theorems with grand mean of 4.210 standard deviation of .680 and coefficient of variation 16.15 < 33%. This means that the views of the respondents on the usefulness of GeoGebra in the teaching and learning of circle theorems was homogeneous.

Research question 3: What are the ease-of-use of GeoGebra in learning circle theorems.

Again, to assess ease-of-use of GeoGebra software in the learning of circle theorems, the respondents were asked to rate 5-point Likert scale items with 1 showing least rating and 5 showing strong rating (See appendix A). For analysis purposes, the mean and standard deviation of the responses given by the respondents were computed. These responses were analysed with mean ranks. The results of the outcome of the analysis were presented in Table 6.

Table 6: Respondents' views on ease-of-use of GeoGebra in learning circle theorems.

Variable	Mean	Stds.
I found GeoGebra to be very flexible to interreact with in circle theorems.	4.480	.517
It was very easy for me to become skilful at using GeoGebra in circle theorems.	4.299	.660
Learning to find out the relationship between angles in circle theorems was easy.	4.134	.739
Using GeoGebra has made me understand circle theorems well.	4.047	.805
Using GeoGebra has improved my problem-solving skills in circle theorems.	4.223	.669
I found GeoGebra very useful in circle theorems	4.213	.741
Grand mean	**4.233**	**.689**

The information in Table 6 disclosed the ease-of-use of GeoGebra in teaching and learning circle theorems. Particularly, the findings showed that students found GeoGebra to be very flexible to interreact with in circle theorems as the most highly rated variable followed by it was very easy for respondents to become skilful at using GeoGebra in circle theorems Respondents again agreed and rated using GeoGebra in circle theorems has improved their problem-solving skills almost the same as students found GeoGebra useful in circle theorems than learning to find out the relationship between angles in a circle theorems was easy than GeoGebra has made me understand circle theorems well .

Generally, findings of the study revealed that students agreed to the ease-of-use of GeoGebra in circle theorems with the grand mean of and standard deviation of .689 and co-efficient of variation, of 16.277% < 33% which revealed that the opinions of the respondents was homogenous.

Research question 4: What are the students' perceived challenges of using GeoGebra in learning circle theorems?

Again, to find out students' perceived challenges of using GeoGebra software in the learning of circle theorems, the respondents were asked to rate 5-point Likert scale items with 1 showing least rating and 5 showing strong rating (See appendix A). For analysis purposes, the mean and standard deviation of the responses given by the respondents were computed. These responses were analysed with mean ranks. The results of the outcome of the analysis were presented in Table 7.

Table 7: Respondents' perceived challenges of using GeoGebra in learning circle theorems.

Variable	Mean	Stds.
Inability of teachers to practicalise the concept of circle theorems makes it difficult to understand.	4.394	.537
Teaching circle theorems without the use of application software (GeoGebra) makes it difficult to understand.	4.315	.639
Students do not try circle theorems questions in addition to what was given in class.	4.244	.639
Lack of teaching and learning aids by teachers make it difficult to grasp the concept of circle theorems	4.236	.610
Lack of teachers' motivation in learning circle theorems.	4.228	.645
The poor foundation of students in circle theorems at the senior high school level makes it difficult to learn circle theorems at the college level.	4.205	.682
Students without personal laptop or an android phone cannot practise on their own.	4.205	.671
Students perceived circle theorems has no application in real life.	4.197	.667
Student have a psychological fear for circle theorems.	4.189	.698
Grand mean	**4.246**	**.566**

The results as shown in the 7 indicated that inability of teachers to practicalise the concept of circle theorems make it difficult to understand is the most significant rating revealing homogeneity of responses expressed by the respondents that the items strongly measure perceived challenges of using GeoGebra in learning circle theorems.

Respondents agreed and rated teaching circle theorems without the use of application software (GeoGebra) made lessons difficult to understand as the second most rated variable than students not solving circle theorems questions in addition to what was given in school than lack of teaching and learning aids by teachers made lessons difficult to grasp the concept of circle theorems as well as lack of teachers' motivation in learning circle theorems.

Spontaneously, respondents agreed and shared the same opinion on the poor foundation of students in circle theorems at the senior high school level made it difficult to learn and understand circle theorems at the college level and students without personal laptop or an android phone cannot practice on their own. Last but one sub-construct that sought respondents' opinion on how students perceived circle theorems has no application in real life which indicated that respondents were in agreement.

The ninth and last sub-construct which measured students psychological fear for circle theorems was also agreed by respondents with.

Generally, findings of the study indicated that respondents agreed that there were perceived challenges of using GeoGebra in learning circle theorems with grand mean 4.246, standard deviation .566 and co-efficient of variation, of 13.33% < 33% which revealed homogeneity of the respondents' opinions.

Research Question 5: How mathematics self-efficacy influences students' achievement in circle theorems?

Qualitative research paradigm approach was used with semi-structured interviews to answer research question five. This was done with phenomenology study design, specifically to provide the lived experience of the participants using the GeoGebra software. The phenomenology provides insight or an in-depth understanding of the issue. The population for this phase was four purposively selected students from level 100 to share how mathematics self-efficacy influences students' achievement in circle theorems. The sampling techniques used in this study were purposive and concept or theory-based. Concept or theory-based procedure was used because the participants selected for the study had rich information and are experienced in learning circle theorems using GeoGebra. Purposive was also adopted because the researcher actually targeted the students who learnt using the GeoGebra.

The instrument used was a semi-structured interview developed by the researcher to collect data on how mathematics self-efficacy influences students' achievement in circle theorems. The semi-structured was designed taking into consideration the fifth objectives of the study. The semi-structure interview guide was pre-tested on 20 students in St. Ambrose College of Education, Dormaa – Akwamu in the Bono Region which was not part of the study. The pre-testing was done to identify how mathematics self-efficacy influences students' achievement in circle theorems for rectification before the main study. Fortunately, no challenges were found in the pre-testing, hence no revision was done. In addressing ethical issues, consent of the students was sought through personal contact, and an assurance was given that they could withdraw from the study at a time they felt uncomfortable to continue. Dates and specific times were agreed for the interviews based on the convenience and availability of each student. The researcher was open and honest with participants by disclosing the purpose of the research to participants. The researcher arranged an interview session with the students in their school through one-on-one basis.

To ensure the validity and reliability, semi-structured interview guide was given to experts in Mathematics and ICT Department for their assessment and comments on the items in the scope of clarity, ambiguity, relevance, and generality. The researcher further ensured ethical measures during the data collection process such as confidentiality, anonymity, and voluntary participation were all held in high esteem. The researcher ensured that the process of data collection was engaging in a genuine and natural environment. In the semi-structured, the participants were asked the same questions but in a different order. Data collection methods that were used to increase validity are a combination of participants' verbatim accounts and participant review. The researcher asked participants to review the researcher's synthesis of the interviews with participants for accuracy of representation. The participants were made to read the transcribed conversation to verify whether the data captured accurately reflected their actual position or ideas. During the interviews, audio recordings were done to serve as verbatim accounts of what transpires in the interview session material for reliability checks.

Immediately following the interview, the researcher manually transcribed the audio recordings by typing in text. This was done to remove all overlapping statements and assisted in obtaining verbatim accounts from the participants. The responses were coded, described, and categorized according to similar patterns based on the themes of the study. Also, connections and interrelations were found in the data which facilitated the narrative discussions which corroborate the themes of the study. The verbatim presentation was done in the sense that it captured the date, and informant identity or code to ensure anonymity.

The thematic analysis procedure was adopted to analyze the semi-structured interview data. The researcher then transcribed and grouped all the semi-structured interview responses into appropriate themes that match the objectives of the study. The responses were grouped and paraphrase by considering the patterns and relationships. Some responses were also written verbatim to support the themes.

The results of the study were presented and discussed according to the research questions posed in this study. The findings are also compared with existing literature through discussions. The abbreviations EGR and CGR followed by a number denotes a respondent in experimental group and control group respectively.

Looking at themes from the respondents on how circle theorems was taught using GeoGebra, the following themes were observed: lesson taught using GeoGebra, able to deduce theorems myself and came out with the theorems myself. The findings of the semi-structured interviews revealed that the respondents have grasped the concept of circle theorems using GeoGebra.

The following were respondents' responses on how the concept of circle was taught:

"Aaaaaammmmmh aaaammmmh it was cool, the teacher taught us circle theorems using GeoGebra. He first took us through the GeoGebra app and after that I was able to deduce circle theorems myself, from the day that we met you and you took us through some properties of the circle theorems and used that GeoGebra app to solve some problems under the circle theorems. We are able to use it hmmmmn the different ways to solve some of the GeoGebra questions." **(EGR 1, Interviewed Data, 2022)**

"Tools in the GeoGebra were made known to me by the teacher. He then guided me to deduce the theorems in circle theorems myself. With the use of GeoGebra, I was able to come out with the theorems myself. He taught us so many angles in the circle theorems so many theorems like for example a chord subtending an angle to the circumference have the same angles when they are many angles at the circumference." **(EGR 2, Interviewed Data, 2022)**

Again, on the theme that respondents feel excited, understood concept, lesson was practical, easy to relate angles, have self-confidence, unhappy if there is no laptop or android phone. The findings of the semi-structured interviews on how respondents feel after they have been taught circle theorems using GeoGebra revealed the admission of respondents that GeoGebra has them visualized various angles, motivated them and very effective in learning circle theorems. They added that they have the ability to solve circle theorems problems. In view of all these, the researcher can say that, the use of GeoGebra in teaching circle theorems has improved respondents understanding, problem-solving skills and ultimately improve their achievement in circle theorems.

When the respondents were again asked to comment on their feelings after being taught circle theorems by the use of GeoGebra, the following response emerged:

"I feel excited and have self-confidence because I was wondering how to get an app to solve most of the mathematics problems especially the GeoGebra when you eeeeiii sorry especially the circle theorems. So when you introduced such an app to us I feel excited that eeeey we have such a app to solve circle theorems questions, I feel excited, I understood the concept of circle theorems well, using the Geogebra was practical and easy to relate the angles. Hmmm students who don't have personal laptops or android phones can't practice. But in all, I recommend GeoGebra and any other app in teaching mathematics." **(EGR 1, Interviewed Data, 2022)**

"I see GeoGebra app to be the best app because it helped us to acquire more knowledge, understand circle theorems, GeoGebra is an activity-based and interactive when you are learning something about the circle theorems. I also feel very confident and happy." **(EGR 2, Interviewed Data, 2022)**

The following themes were recorded after the interview on how confident respondents were on the use of GeoGebra in circle theorems: motivated and high confidence in myself, help to acquire more knowledge, understand circle theorems, activity-based, interactive and feel very confident and happy. Considering these themes from respondents, researcher concluded that respondents are highly motivated, self-confidence and have ability to solve problems in circle theorems on their own.

When respondents in the experimental group were finally asked how confident are you in solving problems in circle theorems, below were the responses that respondents presented.

"Yeessss yeeesss yess, eeeemmmmmmm eeeeemmm on my own I practice GeoGebra because I am motivated and have high confidence in myself, I quite remember you did mention of eeeeerrrr a line tangential to circle yea and I tried it that one I think because of time you couldn't take us through so I tried it at home and I saw that it works it works." **(EGR 1, Interviewed Data, 2022)**

"With self-confidence, yea I can solve it with the help of GeoGebra app on my own. I have the ability. It has helped us a lot I have the will power to go close to circle theorems now I got more understanding. I will recommend the use of GeoGebra in teaching circle theorems, I would like if any other app will be used to teach other topics in mathematics." **(EGR 2, Interviewed Data, 2022)**

Similar interview was conducted for the respondents in control group to find out their views on teaching and learning of circle theorems using GeoGebra.

The themes: someone's theorem and must be learnt like that, marker-board (traditional) method of teaching, hard to understand, unfriendly. The findings of the semi-structured interview showed that respondents were not happy with the method employed in teaching circle theorems and did not understand the concept of circle theorems.

A question which reads, how was the concept of circle theorems taught? Respondents answered and below are the responses by respondents:

***"It was a teacher who introduced us to this concept and first of all made us understood that it was someone's theorems and must be learnt like that so when he was teaching us he went to the board and started drawing some on the board that this is a circle and here were some elements that he introduced us to that the segment of it, the chord of it, eeeer so he made us understood that when a chord subtends an angle on a circumference of a circle, is twice the angle at the centre*." (CGR 1, Interviewed Data, 2022)**

***"The teacher taught us circle theorems on the board (traditional method) teaching us how to solve examples. he gave us some small examples about circle theorems. It was so hard to understand. The way the teacher was teaching was unfriendly to us."* (CGR 2, Interviewed Data, 2022)**

On the themes: teaching of circle theorem was teacher centered, not interesting, very complicated, difficult to understand, felt confused, can't link relationship, cumbersome, abstract, respondents clearly declared their stand that circle theorems taught with traditional method was too abstract and impede the achievement of respondents in circle theorems.

The respondents' responses on how they feel after circle theorems lessons are as follows:

"Hmmmm, so it was like theory like eeeeerr teacher centered learning and I did not understand and got confused in the process of the teacher teaching us the theorems koraaaaaa. Frankly speaking not interesting, it was very complicated and hard for me especially because I am very very poor in memorizing theories so if you teach me base on only theorems that you use a verbal language to teach me without practicing it, it will go affe against me so it was very difficult for me to understand that concept." **(CGR 1, Interviewed Data, 2022)**

"I felt confused and couldn't link the relationship between the angles because the teaching was too abstract. Grasping the theorems was cumbersome." **(CGR 2, Interviewed Data, 2022)**

Researcher finally asked respondents how confident they are regarding the use of GeoGebra in teaching and learning circle theorems?

No self-confidence, understanding and motivation were not there, do recommend GeoGebra if it exist in teaching circle theorems were the theme that respondents expressed. The findings of the semi-structured interview revealed that respondents do not have the ability and no self-confidence to solve circle theorems problems.

The following responses were gathered from the respondents on the matter of how confident respondents are regarding the use of GeoGebra in teaching and learning circle theorems?

"Hmmmm somehow, somehow, I will not get the full mark because the understanding wasn't all that stable and I do not have the self-confidence. I will fully abreast a software

that can be used to learn circle theorems and yes yes if there are more sotware that can be used to teach mathematics, i recommend that." **(CGR 1, Interviewed Data, 2022)**

"hmmm somehow, I will try small de3, because the understanding and self-confidence were not too much. if there had been any software to be used to teaching circle theorems, i will welcome it paaaa. hahhahha because the last time the teacher taught on the board (traditional method) it was diffiulct but I think if there had been any app like that it will help me to learn. Mmmmmmm no, I don't have self confidence in me" **(CGR 2, Interviewed Data, 2022)**

4.4 Discussion of the Findings

To test the research hypothesis, the post-test descriptive statistics were run for both the control and the experimental group. Table 3 gives the details of the results obtained.

In the post-test, the average score (M = 7.66; SD = 1.64) of the experimental group, was higher than the control group average score (M = 4.51; SD = 1.00). To check if the difference between the two groups' circle theorems achievement were statistically significant, paired sample t-test was computed.

The results from Table 4 indicated that there was statistically significant difference between the mean achievement in circle theorems of students taught with GeoGebra and students who underwent the traditional method of teaching. The effect size statistic, Eta squared was .87 (87%) indicating high efficacy of the intervention on the experimental group. Therefore, null hypothesis: there was no statistically significant mean difference in academic achievement between students who were taught using GeoGebra and students not taught with GeoGebra was rejected. The researcher therefore concluded that the use of GeoGebra in teaching circle theorems has positive effect on students' achievement.

The result of this study was consistent with the earlier studies by Shadaan and Leong (2013) who investigated the effectiveness of using GeoGebra on students' understanding in learning circles by using quasi-experimental design. The result indicated that students in the experimental group not only outperformed those in the control group in academic achievement but also in levels of learning of transformation geometry. This study shows that when technology is effectively integrated into the teaching and learning of mathematics, students turn to think critically and therefore perform better. Tay and Wonkyi (2018) investigated the impact of GeoGebra on senior high school students' GeoGebra performance with an emphasis on circles. Tay and Wonkyi discovered that pupils who used the GeoGebra software to learn geometry showed improvement. Added that, GeoGebra-based instruction made lessons in the classroom more engaging and useful for pupils. Pavethira and Leong (2017) also did a study on students' performance in geometrical reflection using GeoGebra. The results indicated a significant difference between pre-test and post-test results. Similarly, the results also found a statistically significant difference in scores among the three student ability groups. In conclusion, the study implies using GeoGebra enhances students' achievement in geometrical studies.

Additionally, views from the respondents on the usefulness of GeoGebra in the teaching and learning of circle theorems was measured. Descriptive statistics: mean and standard deviation were computed to assess the respondents' views and their level of agreement pertaining to the usefulness of GeoGebra in the teaching and learning of circle theorems. In Table 5, the grand mean of 4.21 and standard deviation of .68 revealed that, the respondents agreed that GeoGebra is very useful in the teaching and learning of circle theorems.

The finding of the researcher is consistent with the earlier studies by Shadaan and Leong (2013) examined how GeoGebra affected students' comprehension of the teaching and learning of circles. 53 Year 9 students were divided into the treatment group and the control group for the study. To ascertain the impact of GeoGebra on the students' performance in learning circles, the study used a pre-test and post-test. The study's results revealed that pupils who used GeoGebra to learn about circles theorems outperformed their colleagues who did not utilize the program. Shadaan and Leong (2013) also revealed student perceptions of the use of GeoGebra were generally good, Chimuka (2017) also carried out research on how incorporating GeoGebra affected kids' mathematics skills. The results also indicated students taught with the GeoGebra perform significantly than their peers that did not use the software, Bhagat and Chang (2015) also examined the impact of using GeoGebra on 9th grade students' achievement in learning geometry and the finding revealed that teaching and learning Geometry with GeoGebra, helped students to improve their reasoning, visualization skills and representation of mathematical concepts in diverse ways.

Researcher further investigated Berekum College of Education students' views on the ease-of-use of GeoGebra in the teaching and learning of circle theorems using 6-item construct in relation to finding out the relationship between angles in a circle theorems, understanding circle theorems well with the use of GeoGebra, improving problem-solving skills, GeoGebra is flexible to interreact with in circle theorems, very easy to become skilful at the usage of GeoGebra and finding GeoGebra to be very useful in circle theorems.

Descriptive statistics: mean and standard deviation were computed to assess the respondents’ views and their level of agreement pertaining to the ease-of-use of GeoGebra in the teaching and learning of circle theorems.

The result from Table 6 revealed that the respondents were in agreement to the ease-of-use of GeoGebra in circle theorems with the grand mean of 4.233 and standard deviation of .689. Also, the coefficient of variation, of 18.20% which is less than 33% indicated that the respondents’ views on the ease-of-use of GeoGebra in circle theorems was homogenous. The result further indicated that, among the 6-item construct, GeoGebra is flexible to interreact with in circle theorems had a higher mean score of 4.48. This confirms Riley et al (2017) assertion that innovative teaching methods that provide positive mathematical learning experiences could help to enhance students’ achievement in mathematics if the learner is provided with the opportunity to explore their environment and provided an optimum learning environment then the learning becomes joyful and long lasting, Reis and Ozdemir (2010) used GeoGebra as a technological tool for instructional method for parabolas. The outcome from the data analysed shown that students can learn meaningfully with GeoGebra and Juniu (2011) observed that the introduction of word processors, databases, spreadsheets and other tools helped both teachers and learners to adopt easier way of teaching and learning without relying on the use mental capacity on trivial tasks. He added that technological tools are now integral parts of modern educational systems.

Again, students’ perceived challenges of using GeoGebra in learning circle theorems was measured. From Table 7, the grand mean of 4.246 and standard deviation of .566 with co-efficient of variation 13.33% < 33% revealed that, the respondents agreed that GeoGebra though is very useful in the teaching and learning of circle theorems, it has some challenges.

The finding of the researcher is consistent with the earlier studies by Adolphus (2011) also studied the problem of teaching and learning of geometry in secondary schools in Rivers State, Nigeria. The following were identified as some of the causes of students' challenges in learning geometry; poor foundations of most mathematics teachers in geometry, poor foundations of students in mathematics in general, poor teaching and learning environment of most schools, poor attitude of students towards learning of geometry, lack of commitment on the part of some teachers due to lack of motivation. Bosson-Amedenu (2017) also examined remedial students' perception of challenging concepts in senior high school core mathematics curriculum in Ghana. The most challenging topic identified by these students was circle theorems followed by plane geometry and the reasons assigned for these difficulties were; some mathematics teachers had difficulty with some topics in mathematics themselves, poor mathematics foundation by students, lack of resources for teaching mathematics by instructors, large class size in most senior high schools and lack of motivation in learning mathematics because of its abstractness. Mifetu et al. (2019) researched on geometry topics in mathematics perceived difficult to study by senior high school students in the Cape Coast Metropolis. The findings revealed that, four of the geometry concepts perceived challenges to learn by students are: circle theorems, perpendicularity of tangent and radius of a circle, angle between tangent and chord and tangent from an external point. The main reasons given for perceiving geometry concepts difficult include: unavailability of instructional materials to make the teaching of geometry real and lack of understanding of geometrical concepts by teachers.

Finally, the result from the quanlitative data revealed that, respondents in the experimental group on their own understood the concept of circle theorems very well, feel excited, well-motivated. They added how well interactive and visual GeoGebra is in circle theorems. Respondents again were having high self confidence in solving problems in circle theorems. However, respondents from the control group expressed that they could not understand the concept circle theorems let alone having interest and solving problems in circle theorems.

Generally, the researcher concluded that respondents who used GeoGebra in teaching and learning of circle theorems achieved better than respondents who learnt circle theorems using traditional or conventional method which confirms that mathematics self-efficacy influences students' achievement in circle theorems. The finding of the study is consistent with Phan (2012b) and Tariq and Durrani, (2012) asserted that students with higher levels of self-efficacy have higher levels of general achievement in mathematics, more easily overcome negative outcomes, display more positive attitudes towards mathematics, and possess a more comprehensive understanding of mathematics.

In conclusion, the findings from this research revealed that, teaching and learning circle theorems with GeoGebra helped students to improve their reasoning, visualization skills and representation of mathematical concepts in diverse ways.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

This chapter gives an overview of the purpose of the study, methodology and highlights findings of the study. Conclusions based on the findings of the study are offered. The chapter concludes with recommendations to draw attention to the effect of using GeoGebra in teaching circle theorems and provides suggestions for further research.

5.1 Summary of the Study

This study sought to find out the effect of using GeoGebra in teaching circle theorems on Berekum College of Education student's mathematics self-efficacy and achievement in the Bono Region of Ghana. This was done by answering four research questions and a hypothesis. The five research questions and the hypothesis were stated as:

Research Question 1: what is the effect of using GeoGebra in teaching circle theorems on students' achievement?

Research Question 2: What are the usefulness of GeoGebra in the teaching and learning of circle theorems?

Research Question 3: What are the ease-of-use of GeoGebra in teaching and learning circle theorems?

Research Question 4: What are the students' perceived challenges of using GeoGebra in learning circle theorems?

Research Question 5: How does mathematics self-efficacy influence students' achievement in circle theorems?

H_o: There was statistically significant difference in academic achievement between students who were taught using GeoGebra and students not taught with GeoGebra.

The philosophical underpinning the study was pragmatism. The research approach was mixed-method and among the four categories of mixed-method: convergent parallel approach, exploratory approach, embedded approach and explanatory approach, the researcher used embedded approach. This study was conducted using quasi-experimental research design. The research design involved non-random assignment of participants into two groups namely treatment (experimental) group and control group. Purposive sampling was used in determining the sample for the study. Two hundred and forty (240) participants from level 100 of Berekum College of Education were used for the study of which 127 participants offered Bachelor of Education (Primary School) option formed the experimental group and 113 participants also offering Bachelor of Education (Junior High School) option formed the control group. The study involved the administration of pre-test before treatment and post-test after treatment, administration of the research questionnaire and interview session. Analysis of data gathered from the pre-test and post-test was used to answer the first research question and hypothesis of the study. The second, third and fourth research questions were answered from the analysis of the data retrieved from the questionnaire administered. The fifth research question was answered qualitatively using thematic content analysis from the interview conducted. The analysis of data from the pre- and post-test was done using means, standard deviations and paired sample t-test. Also effect size statistic was used to measure the effect caused by the treatment on the experimental group. Again, means, standard deviation and co-efficient of variation were used to analyse the data from the questionnaire and qualitative analysis was carried out for the qualitative data.

5. 2 Major Findings

From the data analyses and theoretical point of view, the following findings were made.

1. When technology is effectively integrated into the teaching and learning of mathematics, students turn to understand concept, think critically and therefore perform better as indicated by the effect size (Eta squared) of 87%. Indicating high efficacy of the treatment or intervention on the experimental group.

2. Innovative teaching methods that provide positive mathematical learning experiences could help to enhance students' achievement in mathematics. If the learner is provided with the opportunity to explore their environment and provided an optimum learning environment then the learning becomes joyful and long lasting

3. When circle theorems is taught without practicalizing the concepts because of poor teaching and learning environment, it demotivates students and makes the concept of circle theorems too abstract and complex for students to grasp.

4. Students with higher levels of self-efficacy have higher levels of general achievement in mathematics, more easily overcome negative outcomes, display more positive attitudes towards mathematics, and possess a more comprehensive understanding of mathematics.

5.3 Conclusions

Three conclusions were drawn from the findings of this study. Firstly, it can be concluded that GeoGebra as a teaching and learning software helps improves academic performance in mathematics. This implies that when colleges of education students are taught mathematics with the use of GeoGebra, the students perform better as compared to being taught using the traditional or conventional method of teaching.

Secondly, with the use of GeoGebra in teaching and learning mathematics, students can improve their understanding of geometric concepts, high self-confidence and ability in solving problems. This implies that significant improvement in the performance of students is attributed to students' ability to establish the relationships that exist between concepts and theorems in mathematics.

Thirdly, the application of GeoGebra in the teaching and learning of circle theorems motivates students towards learning of circle theorems and mathematics in general. This means that using GeoGebra to teach Geometry as a blended instruction increases students' interest in the subject.

The findings of the study show that innovative methods (such as incorporating GeoGebra into teaching and learning) have proven to be more effective than traditional or conventional teaching method and as such, it can be integrated into classroom teaching as blended instruction. Previous studies are also favourably disposed to integrating GeoGebra into mathematics education. It is therefore concluded that GeoGebra helps to improves Berekum College of Education students' achievement in Geometry, particularly circle theorems.

5.4 Recommendations

From the findings of this study, the following recommendations are made for Ghana Tertiary Education Commission (GTEC) and for that matter Ministry of Education and other stakeholders for application.

1. Introducing tutors of colleges of education to GeoGebra as an innovative way to teach circle theorems and mathematics, in general, will be most helpful in raising the achievement of the college of education students. This introduction could be done through workshops and seminars organised by Ghana Tertiary Education Commission (GTEC) and/or other stakeholders in education. This will help enhance the teaching skills and strategy of tutors in mathematics.

2. Integrating the use of computers and interactive educational software like GeoGebra into the colleges of education mathematics curriculum by the various affiliate universities could also help tutors guide students to understand circle theorems concepts and improve students' ability in solving circle theorems problems significantly.

3. Interactive educational software and computers should be incorporated into teaching and learning activities. This will help make learning of circle theorems more interesting, motivating and visualize to students. This could be done by providing resource materials such as educational applet devices, computers and mathematical instruments to both tutors and students in the colleges of education.

5.5 Suggestions for Further Studies

The following are suggestions for further research:

1. Further studies with the use of GeoGebra could be done in other areas of mathematics where students have demonstrated poor performance such as trigonometry, probability and transformation (i.e Translation, Enlargement, Reflection and Rotation)

2. This study was limited to only level 100 students in the Berekum College of Education in the Bono Region of Ghana. Further studies could be replicated for a much larger sample for better generalisation.

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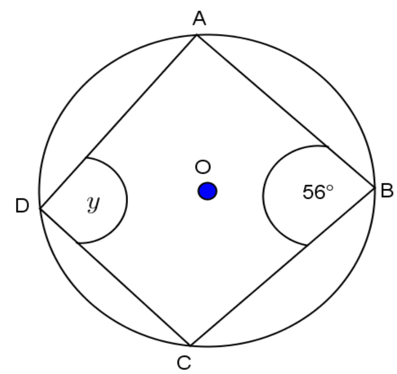
APPENDIX A

Sample pre-test questions administered to first year students

Find the values of the angles marked with letters in the following diagrams

1



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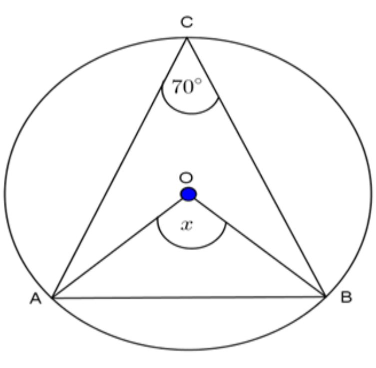

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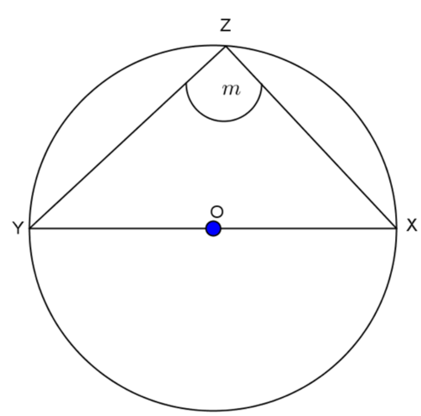

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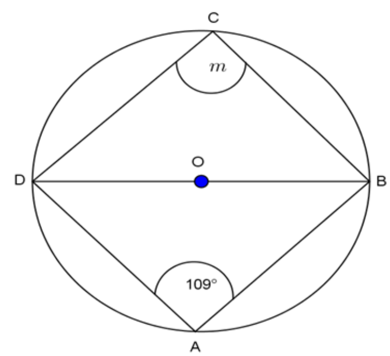

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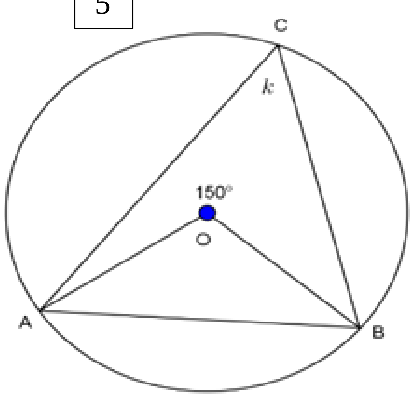


Marking scheme for sample pre-test questions administered to first year students.

Total marks = 10

1. $\angle ADC + \angle ABC = 180^0$
$y + 56^0 = 180^0$

$y = 180^0 - 56^0$

$y = 124^0$

2. $\angle AOB = 2\angle ACB$

$x = 2 \times 70^0$

$x = 140^0$

3. $\angle XYZ = 90^0$

$\Rightarrow m = 90^0$

4. $\angle BCD + \angle BAD = 180^0$

$\Rightarrow m + 109^0 = 180^0$

$m = 180^0 - 109^0$

$m = 71^0$

5. Angle at $O = 360^0$

$\Rightarrow \angle AOB = 360^0 - 150^0$

$\angle AOB = 210^0$

$$\angle ACB = \tfrac{1}{2}\angle AOB$$

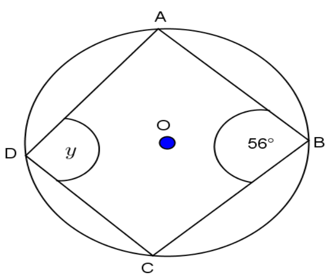
$$\Rightarrow k = \tfrac{1}{2}\left(210^0\right)$$

$$k = 105^0$$

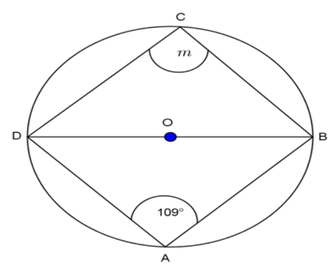
Sample post-test questions administered to first year students.

Answer all questions **Total marks = 10**

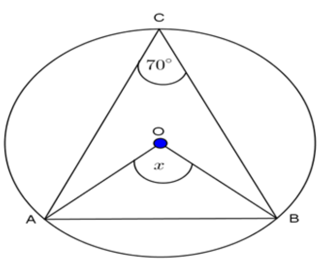
Find the values of the angles marked with letters in the following diagrams.



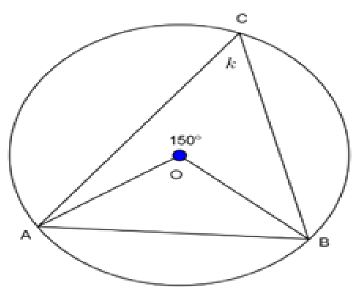
4.



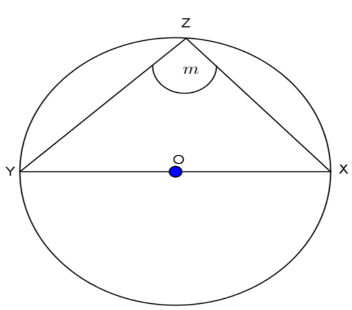
2.



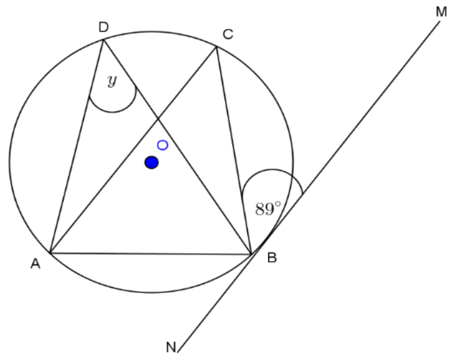
5.



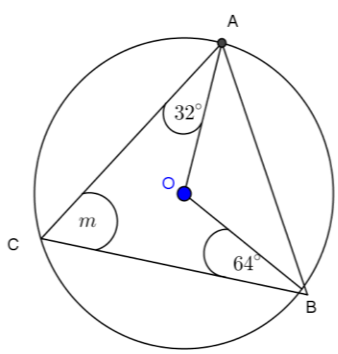
3.



6.

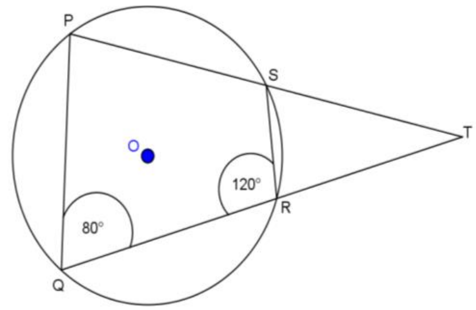


7.

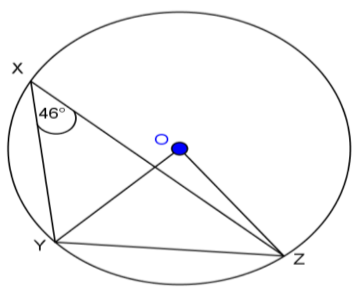


8. In the diagram, PQRS is a cyclic quadrilateral. PS and QR are produced to meet T.

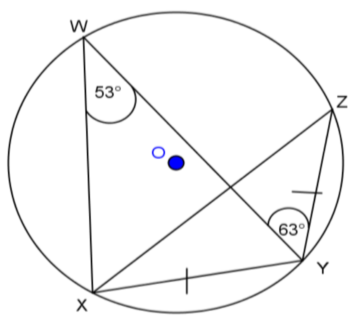
If <PQR = $80^0$and <QRS = 120^0, find <RTS.



9. In the diagram, X, Y and Z are points on the circle with centre O. If <YXZ = 46^0, find the reflex <YOZ.



10. In the diagram, |ZY| = |XY|, <WYZ = 65° and <XWY = 48°. Find <WYX.



Marking scheme for sample post-test questions administered to first year students.

Total marks = 10

1.

$\angle ADC + \angle ABC = 180^0$

$y + 56^0 = 180^0$

$y = 180^0 - 56^0$

$y = 124^0$

2. $\angle AOB = 2\angle ACB$

$x = 2 \times 70^0$

$x = 140^0$

3. $\angle XYZ = 90^0$

$\Rightarrow m = 90^0$

4. $\angle BCD + \angle BAD = 180^0$

$\Rightarrow m + 109^0 = 180^0$

$m = 180^0 - 109^0$

$m = 71^0$

5. Angle at $O = 360^0$

$\Rightarrow \angle AOB = 360^0 - 150^0$

$\angle AOB = 210^0$

$\angle ACB = \frac{1}{2}\angle AOB$

$\Rightarrow k = \frac{1}{2}\left(210^0\right)$

$k = 105^0$

6. $\angle ACB = \angle CBT$

$\Rightarrow \angle ACB = 89^0$

Also, $\angle ADB = \angle ACB$

$y = 89^0$

7. $\angle ABC = \angle CAO + \angle CBO$

$\angle ACB = 32^0 + 64^0$

$\angle ACB = 96^0$

$\therefore m = 96^0$

8. $\angle QPS = 180^0 - 120^0$

$\angle QPS = 60^0$

$\angle RTS + \angle QPS + \angle PQT = 180^0$

$\angle RTS + 60^0 + 80^0 = 180^0$

$\angle RTS + 140^0 = 180^0$

$\angle RTS = 180^0 - 140^0$

$\angle RTS = 40^0$

9. $\angle YOZ = 2\angle YXZ$

$\angle YOZ = 2(46^0)$

$\angle YOZ = 92^0$

But reflex $\angle YOZ = 360^0 - 92^0$

$\therefore \angle YOZ = 268^0$

10. $\angle XYZ = \angle XWY$

$\angle XZY = 53^0$

Also, $\angle ZXY = \angle XZY = 53^0$

$\angle XZY + \angle YXZ + \angle XYZ = 180^0$

$53^0 + 53^0 + \angle XYZ = 180^0$

$106^0 + \angle YXZ = 108^0$

$\angle XYZ = 180^0 - 106^0$

$\angle XYZ = 74^0$

But $\angle XYZ = \angle XYW + \angle WYZ$

$\Rightarrow 74^0 = \angle XYW + 63^0$

$$\angle XYW = 74^0 - 63^0$$

$$\angle XYW = 11^0$$

QUESTIONNAIRE ADMINISTERED TO FIRST YEAR STUDENTS

This questionnaire is supposed to provide data to be used in research. Please kindly answer the questions as frankly as possible. Your confidentiality is assured.

SECTION A: Respondent Information

Tick (√) the appropriate option.

1. Sex: Male [ ] Female [ ]

2. Age: 15 – 18 [ ] 19 – 22 [ ] 23 – 26 [ ] 27 – 30 [ ] 31 and above [ ]

3. Hall of Residence: Buchanan [ ] Stewart [ ] Nicholas [ ] Yiadom Boakye [ ]

4. Programme Offered: Primary Education [ ] JHS Education [ ]

SECTION B: Usefulness of GeoGebra in teaching and learning of circle theorems.

Tick (√) one option for each statement to indicate your degree of agreement with each statement.

	Statement	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1	GeoGebra helped me understand the inscribed angle theorems.					
2	GeoGebra helped me visualize various positions of an angle in a semicircle.					
3	With the use of GeoGebra I was able to draw various positions of a cyclic quadrilateral.					
4	The use of GeoGebra helped me understand the relationship of angles in the circle and the tangent.					
5	The visualisations from GeoGebra helped me come out with the right theorems on my own.					
6	By using GeoGebra I had fewer problems understanding the theorems.					
7	After using GeoGebra, I am able to solve problems by using the right approach.					
8	I enjoyed the class lessons with the use of GeoGebra.					

9	I am happy if the teacher uses GeoGebra in teaching circle theorems.					
10	After the class lessons I felt confident in my ability to solve questions in circle theorems.					

SECTION C: Ease-of-use of GeoGebra in learning circle theorems.

Tick (√) one option for each statement to indicate your degree of agreement with each statement

S/N	**Statement**	**Strongly Disagree**	**Disagree**	**Neutral**	**Agree**	**Strongly Agree**
1	Learning to find out the relationship between angles in a circle theorem was easy.					
2	Using GeoGebra has made me understand circle theorems well.					
3	Using GeoGebra improved my problem-solving skills in circle theorems.					
4	I found GeoGebra to be very flexible to interreact with in circle theorems.					
5	It was very easy for me to become skillful at using GeoGebra in circle theorems.					
6	I found GeoGebra very useful in circle theorems.					

SECTION D: Perceived challenges of using GeoGebra in learning circle theorems.

Tick (√) one option for each statement to indicate your degree of agreement with each statement.

S/N	Statement	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1	Lack of teaching and learning aids by teachers makes it difficult to grasp the concept of circle theorems.					
2	Lack of motivation in learning circle theorems.					
3	Students do not solve circle theorems questions in addition to what is given in school.					
4	Students have a psychological fear for circle theorems.					
5	Students perceived circle theorems has no application in real life.					
6	The poor foundations of students in circle theorems at the senior high school level makes it difficult to learn and understand circle theorems at the college level.					

7	Teaching circle theorems without the use of application software (GeoGebra) made it difficult to understand.					
8	Inability of teachers to practicalise the concept of circle theorems makes it difficult to understand.					
9.	Students without personal laptop or android phone cannot practice on their own.					

APPENDIX B

SAMPLE SIZE ESTIMATION FOR THE STUDY USING YAMANE'S SAMPLE SIZE FORMULA

Yamane's formula for sample size, $n = \frac{N}{1 + N(e)^2}$

Where $n =$ sample size of the study

$N =$ Population of the study

$e =$ the margin of error

Sample size of a study with population of 600 students and margin of error of 5% (0.005) is $n = \frac{600}{1 + 600(0.005)^2}$

$$n = \frac{600}{1 + 600(0.0025)}$$

$$n = \frac{600}{1 + 1.5}$$

$$n = \frac{600}{2.5}$$

$$n = 240$$

Therefore, sample size from population of 600 students at 5% (0.005) margin of error is 240 students.

DEMOGRAPHIC CHARACTERISTICS OF THE RESPONDENTS

Gender of respondents

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	MALE	138	57.5	57.5	57.5
	FEMALE	102	42.5	42.5	100.0
	Total	240	100.0	100.0	

Age of respondents

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	15 - 18 YEARS	7	2.9	2.9	2.9
	19 - 22 YEARS	168	70.0	70.0	72.9
	23 - 26 YEARS	56	23.3	23.3	96.3
	27 AND ABOVE	9	3.8	3.8	100.0
	Total	240	100.0	100.0	

Programme_offered by respondents

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	PRIMARY EDUCATION	127	52.9	52.9	52.9
	JHS EDUCATION	113	47.1	47.1	100.0
	Total	240	100.0	100.0	

Reliability test results from St. Ambrose College of Education

Case Processing Summary

		N	%
Cases	Valid	20	100.0
	Excluded[a]	0	.0
	Total	20	100.0

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics on usefulness of GeoGebra

in circle theorems

Cronbach's Alpha	N of Items
.811	10

Reliability Statistics on the ease-of-use of GeoGebra in circle theorems

Cronbach's Alpha	N of Items
.720	6

Reliability Statistics on perceived challenges on GeoGebra in circle theorems

Cronbach's Alpha	N of Items
.773	9

Results of paired Samples Statistics for control group

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	PRE_TEST_C1	2.5575	113	1.37541	.12939
	POST_TEST_C2	4.5133	113	1.00103	.09417

Results of paired Samples Statistics for experimental group

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	PRE_TEST_EXP1	2.3228	127	1.18108	.10480
	POST_TEST_EXP2	7.6614	127	1.63891	.14543

Results of the Paired Samples Test on the pre-test and post-test achievement of first year students

		Paired Differences							
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	POST_TEST_EXP2 - PRE_TEST_EXP1	5.33858	2.11653	.18781	4.96691	5.71026	28.425	126	.000

Descriptive statistics of respondents on the usefulness of GeoGebra in teaching circle theorems

	N	Minimum	Maximum	Mean	Std. Deviation
After using GeoGebra, I am able to solve problems by using the right approach	127	3.00	5.00	4.4331	.59882
After the class lessons I felt confident in my ability to solve questions in circle theorems.	127	2.00	5.00	4.3543	.64890
With the use of GeoGebra I was able to draw various positions of a cyclic quadrilateral.	127	2.00	5.00	4.2677	.68370
The visualisations from GeoGebra helped me come out with the right theorems on my own.	127	2.00	5.00	4.2205	.70049
The use of GeoGebra helped me understand the relationship of angles in the circle and the tangent.	127	3.00	5.00	4.1811	.65969

I am happy if the teacher uses GeoGebra in teaching circle theorems.	127	2.00	5.00	4.1732	.61843
By using GeoGebra I had fewer problems understanding the theorems.	127	2.00	5.00	4.1417	.72072
I enjoyed the class lessons with the use of GeoGebra.	127	2.00	5.00	4.1260	.73455
GeoGebra helped me visualize various positions of an angle in a semicircle.	127	2.00	5.00	4.1102	.70405
GeoGebra helped me understand the inscribed angle theorems.	127	2.00	5.00	4.0945	.72848
Valid N (listwise)	127				

Descriptive statistics of respondents on the ease-of-use of GeoGebra in teaching circle theorems

	N	Minimum	Maximum	Mean	Std. Deviation
I found GeoGebra to be very flexible to interreact with in circle theorems.	127	3.00	5.00	4.4803	.51717
It was very easy for me to become skillful at using GeoGebra in circle theorems.	127	3.00	5.00	4.2992	.59515
Using GeoGebra in cicle theorems has improved my problem-solving skills.	127	3.00	5.00	4.2283	.66891
I found GeoGebra useful in circle theorems.	127	2.00	5.00	4.2126	.74140
Learning to find out the relationship between angles in a circle theorem was easy.	127	2.00	5.00	4.1339	.73853
Using GeoGebra has made me understand circle theorems well.	127	2.00	5.00	4.0472	.80532
Valid N (listwise)	127				

Descriptive statistics of respondents on the ease-of-use of GeoGebra in teaching circle theorems

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Using GeoGebra has made me understand circle theorems well.	127	2.00	5.00	4.0472	.80532
Valid N (listwise)	127				

Descriptive statistics of respondents on perceived challenges of GeoGebra in teaching circle theorems

	N	Minimum	Maximum	Mean	Std. Deviation
Inability of teachers to practicalise the conept of circle theorems make it difficult to understand.	127	3.00	5.00	4.3937	.53686
Teaching circle theorems without the use of application software (GeoGebra) made it difficult to understand.	127	3.00	5.00	4.3150	.63871
Students do not solve circle theorems questions in addition to what is given in school.	127	2.00	5.00	4.2441	.63890
Lack of teaching and learning aids by teachers makes it difficult to grasp the concept of circle theorems.	127	2.00	5.00	4.2362	.61019
Lack of motivation in learning circle theorems.	127	3.00	5.00	4.2283	.64475
The poor foundations of students in circle theorems at the senior high school level makes it difficult to learn circle theorems at the college level.	127	2.00	5.00	4.2047	.68242
Students without personal laptop or android phone cannot practice on their own.	127	2.00	5.00	4.2047	.67069
students percieved circle theorems has no application in real life.	127	2.00	5.00	4.1969	.66714
Students have a psychological fear for circle theorems.	127	2.00	5.00	4.1890	.69844
Valid N (listwise)	127				

INTERVIEW OF RESPONDENTS AND THEIR RESPONSES

Experimental group: Respondents' responses and their respective codes

Interviewer	Respondent	Codes
How was the concept of circle theorems taught?	**Student One** Aaaaaammmmmh aaaammmmh it was cool, the teacher taught us circle theorems using GeoGebra. He first took us through the GeoGebra app and after that I was able to deduce circle theorems myself from the day that we meet you and you took us through some properties of the circle theorems and used that GeoGebra app to solve some problems	Lesson taught using GeoGebra, able to deduce theorems myself.

	under the circle theorems we are able to use it hmmmmn the different ways to solve some of the GeoGebra questions	
How do you feel after being taught circle theorems?	I feel excited and have self-confidence because I was wondering how to get an app to solve most of the mathematics problems especially the GeoGebra when you eeeeiii sorry especially the circle theorems. So when you introduced such an app to us I feel excited that eeeey we have such a app to	Feel excited, understood concept, lesson was practical, easy to relate angles, have self-confidence, unhappy if there is no laptop or

	solve circle theorems questions, I feel excited, I understood the concept of circle theorems well, using the Geogebra was practical and easy to relate the angles. Hmmm students who don't have personal laptops or android phones can't practice. But in all, I recommend GeoGebra and any other app in teaching mathematics.	android phone.
How confident are you in solving problems in	Yeessss yeeesss yess, eeeemmmmmmm eeeeemmm on my own I practice GeoGebra because I	Motivated and high confidence in myself

circle theorems?	am motivated and have high confidence in myself, I quite remember you did mention of eeeeerrrr a line tangential to circle yea and I tried it that one I think because of time you couldn't take us through so I tried it at home and I saw that it works it works. I therefore recommend that GeoGebra be used when it comes to the teaching of circle theorems	
Share with me how circle	**Student Two** Tools in the GeoGebra were made	Lesson was done using GeoGebra,

theorems was introduced to you?	known to me by the teacher. He then guided me to deduce the theorems in circle theorems myself. With the use of GeoGebra, I was able to come out with the theorems myself. He taught us so many angles in the circle theorems so many theorems like for example a chord subtending an angle to the circumference have the same angles when they are many angles at the circumference.	came out with the theorems myself.

How do you feel after being taught circle theorems?	I see GeoGebra app to be the best app because it helped us to acquire more knowledge, understand circle theorems, GeoGebra is an activity-based and interactive when you are learning something about the circle theorems. I also feel very confident and happy.	Help to acquire more knowledge, understand circle theorems, activity-based, interactive and feel very confident, happy
How confident are you in solving problems in	With self-confidence, yea I can solve it with the help of GeoGebra app on my own. I have the ability. It has helped us a lot I have the will power to	Have self-confidence and have the ability

circle theorems?	go close to circle theorems now I got more understanding. I will recommend the use of GeoGebra in teaching circle theorems, I would like if any other app will be used to teach other topics in mathematics.	

Control group: Respondents' responses and their respective codes

Interviewer	Respondent	Codes
How was the concept of circle theorems taught?	**Student One** It was a teacher who introduced us to this concept and first of all made us understood that it was someone's theorems	Someone's theorem and must be learnt like that, marker-baord (traditional)

	and must be learnt like that so when he was teaching us he went to the board and started drawing some on the board that this is a circle and here were some elements that he introduced us to that the segment of it, the chord of it, eeeer so he made us understood that when a chord subtends an angle on a circumference of a circle, is twice the angle at the centre .	method of teaching
How do you feel after being taught	Hmmmm, so it was like theory like eeeeerr teacher	Teacher centered, not interesting,

circle theorems?	centered learning and I did not understand and got confused in the process of the teacher teaching us the theorems koraaaaaa. Frankly speaking not interesting, it was very complicated and hard for me especially because I am very very poor in memorizing theories so if you teach me base on only theorems that you use a verbal language to teach me without practicing it, it will go affe against me so it was very difficult	very complicated, difficult to understand

	for me to understand that concept.	
How confident are you in solving problems in circle theorems?	Hmmmm somehow, somehow, I will not get the full mark because the understanding wasn't all that stable and I do not have the self-confidence. I will fully abreast a software that can be used to learn circle theorems and yes yes if there are more software that can be used to teach mathematics, i recommend that.	No self-confidence and do recommend GeoGebra if it exist in teaching circle theorems.
How was circle theorems	**Student Two** The teacher taught us circle theorems on	Hard to understand, unfriendly

introduced to you?	the board (traditional method) teaching us how to solve examples. he gave us some small examples about circle theorems. It was so hard to understand. The way the teacher was teaching was unfriendly to us.	
How is your feeling like after you were introduced to circle theorems?	I felt confused and couldn’t link the relationship between the angles because the teaching was too abstract. Grasping the theorems was cumbersome.	Felt confused, can’t link relationship, cumbersome, abstract
How confident are you in	hmmm somehow, I will try small de3, because the	Understanding and self-confidence

solving problems in circle theorems?	understanding and self-confidence were not too much. if there had been any software to be used to teaching circle theorems, i will welcome it paaaa. hahhahha because the last time the teacher taught on the board (traditional method) it was diffiulct but I think if there had been any app like that it will help me to learn. Mmmmmmm no, I don’t have self confidence in me when it comes to solving circle theorems problems.	were not there,