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Mathematical Model for Malaria Transmission with Optimal Control Strategies and Their Effects

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Authors' contributions

This work was carried out in collaboration between all authors. Author MARENO designed the study, performed the analysis and the numerical simulations, wrote the protocol and wrote the first draft of the manuscript. Author AE managed the literature searches and assisted in writing of the draft and reviewed of the final draft. Author IEK reviewed of the final draft. All authors read and approved the final manuscript.

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ABSTRACT

In this paper, we propose a SEIR-SEI optimal control model of malaria transmission with standard incidence rate. We present four control strategies to prevent the prevalence of infection in the society. In order to do this, we introduce an optimal control problem with an objective function, where the four control functions, prevention using Long-Lasting Insecticide Treated Net(LLITN) $u_1(t)$, the control effort on malaria treatment of infected individuals $u_2(t)$, the insecticide spray on the breeding grounds for the mosquito $u_3(t)$, the prevention using Indoor Residual Spraying $u_4(t)$, have been used as control measures for exposed and infected individuals. We show the existence of an optimal control pair for the optimal control problem and derive the optimality conditions. Our numerical simulation suggests that the two controls strategies $u_1(t)$ and $u_2(t)$ are more effective than the other control strategies in controlling (reducing) the number of exposed and infected individuals and also in increasing the number of recovered individuals.

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Keywords: Malaria transmission; optimal control; Pontryagin's principle; numerical simulation.

1 INTRODUCTION

Malaria is a serious parasitic disease in less developed countries, specifically, in Sub-Saharan Africa, causing high morbidity and mortality. It is estimated that nearly 300 to 400 million malaria cases occur worldwide, out of which 1.52 million die every year. Five species of Plasmodium can infect humans: *P. falciparum*, *P. vivax*, *P. ovale*, *P. malariae* and *P. knowlesi*. *P. falciparum* is the commonest and causes the most severe malarial infection. The female Anopheles mosquito is the primary vector of malarial parasite. The clinical manifestation of malaria disease varies over a spectrum, ranging from severe and complicated to mild and uncomplicated to asymptomatic [1, 2]. The mathematical computations which have been involved in the study of the dynamics of infectious diseases have brought a great surge of excitement in the hearts of modelers, governments, public health workers and all stakeholders as it has created awareness with regards to transmission dynamics and control measures of infectious diseases [3]. Mathematical models for transmission dynamics of malaria are useful in providing better insights into the behavior of the disease. The models have played great roles in influencing the decision making processes regarding intervention strategies for preventing and controlling the insurgence of malaria [4]. The study on malaria using mathematical modeling began in 1911 with Ronald Ross [5]. He introduced the first deterministic two-dimensional model with one variable representing human and the other representing mosquitoes where it was shown that reduction of mosquito population below a certain threshold was sufficient to eradicate malaria. In [6], the Ross model was modified by considering the latency period of the parasites in mosquitoes and their survival during that period. However, in this case, it was shown that reducing the number of mosquitoes is an inefficient control strategy that would have little effect on the epidemiology of malaria in areas of intense transmission. Further extension was described by Anderson and May [7], where the latency of infection in humans was introduced by making an additional exposed class in humans.

Jia Li [8] developed a simple SEIR malaria model with stage structured mosquitoes. He included metamorphic stages in the mosquito population and a simple stage mosquito population, where the mosquito population was divided into two classes namely; all three aquatic stages in one class and all adults in the other class. He concluded that the different dynamical behavior of the models in his study, compared to the behavior of most classical epidemiological models, and the possible occurrence of backward bifurcation make control of malaria more difficult. The new strategy for malaria prevention and control is emphasizing Integrated Vector Management (IVM). This approach reinforces linkages between health and environment, optimizing benefits to both. Integrated vector management is a dynamic and still-evolving field. IVM strategies are designed to achieve the greatest disease-control benefit in the most cost-effective manner, while minimizing negative impacts on ecosystems (e.g. depletion of biodiversity) and adverse side-effects on public health. A new WHO Global Strategic Framework for Integrated Vector Management defines IVM as a strategy to improve the efficacy, cost-effectiveness, ecological soundness and sustainability of disease vector control. IVM encourages a multi-disease control approach, integration with other disease control measures and systematic application of a range of interventions, often in combination and synergistically [9]. Malaria is entirely preventable and a treatable disease if the recommended interventions are properly applied. Individuals should take some aggressive measurements to decline malaria burden. Personal protection measures are the first line of defense against mosquito-borne diseases. Mosquito repellent is a method used for personal protection; and these are the substances used for exposed skin to prevent human-mosquito contact. Insecticide Treated Bed Nets (ITNs) are used for individuals against malaria to reduce the morbidity of childhood malaria (below five years of age) by 50% and global child mortality

by 20-30% [10, 11]. When used on a large scale, ITNs are suppose to represent efficient tools for malaria vector control but there is a limitation of resistance to insecticides used for a saturated net. The resistance of the most important African malaria *Anopheles gambiae* to protrude is already widespread in several West African countries [12, 13]. According to [14, 15] the World Health Organization (WHO) recommends insecticide-treated bed nets (ITNs), indoor residual spraying (IRS) and anti-malarial drug therapies, specifically, the treatment of clinical malaria with artemisinin based combination therapy (ACT), as the principal methods used to combat malaria. Catalyzed by the Roll Back Malaria Initiative around the United National Millennium Development Goals, a widespread scale-up of coverage of these control interventions successfully reduced and locally eliminated malaria in sub-Saharan Africa between 2000 and 2015, Plasmodium infection in endemic regions of Africa halved and the incidence of clinical disease fell by 40% [16]. Many people have applied optimal control methods to various epidemiological models especially on HIV and TB diseases dynamics but very little has been done in applying optimal control theory to study and analyze the dynamics of malaria. In the works of [17, 18, 19, 20], they studied the optimal chemotherapy treatment in controlling the virus reproduction in an HIV patient. Cesar [21] and Sethi and Staats [22], used optimal control to investigate the best strategy for educational campaign during the outbreak of an epidemic and at the same time minimizing the number of infective humans. Kar and Batabyal [23] have also used Optimal control to study a nonlinear mathematical SIR epidemic model with a vaccination program. In recent years, authors like Makinde and Okosun [24] have applied optimal control to study the impact of Chemo-therapy on malaria disease with infective immigrants while Blayneh et al. [25] studied the effects of prevention and treatment on malaria using an SEIR model. Rafikov et al [26] also used optimal control in a malaria model with genetically modified mosquitoes but without human population. In our work, we study the malaria transmission model with standard incidence rate together with four different control strategies; preventive using Long-lasting Insecticides Treated Net (LLITN), the control effort on malaria treatment of infected individuals, the insecticides spray on the breeding ground for the mosquito and the preventive using indoor Residual Spraying. Our goal is to present a mathematical model for malaria transmission and assess the effects of the optimal control strategies to minimize the number of exposed and infectious individuals. The rest of the paper is as follows: section 2 describes the mathematical model, section 3 presents analysis of optimal control and the numerical simulation. Conclusion to the study is presented in the final section.

2 MATHEMATICAL MODEL

2.1 Model Description

This subsection presents a malaria model with standard incidence rate. The malaria parasites are transmitted to the human host through a bite by an infected female anopheles mosquito. The human population at time t is categorized into four classes; susceptible human $S_h(t)$, exposed human $E_h(t)$, infectious human $I_h(t)$ and recovered human $R_h(t)$. The mosquito population is divided into three classes; susceptible mosquito $S_v(t)$, exposed mosquito $E_v(t)$ and infectious mosquito $I_v(t)$. The mosquito population does not include immune class as mosquitoes never recover from infection, that is, due to their relatively short life-cycle, their infective period ends with their death. According to [1], the recovered humans have some level of immunity to the disease, as such they do not get clinically ill, but they still harbor low levels of parasites in their blood streams which are transmitted to susceptible mosquitoes during bites. The total human and mosquito population at any time are respectively given by $N_h(t) = S_h(t)+E_h(t)+I_h(t)+R_h(t)$ and $N_v(t) = S_v(t)+E_v(t)+I_v(t)$. From the model it is assumed that people enter the susceptible class either through birth or immigration at a recruitment rate of Λ_h . After a susceptible human is bitten by an infectious mosquito, the probability that the malaria parasite is passed on to the human occurs at a rate of $b\frac{\beta_h I_v}{N_h}$, the person then moves to the exposed class. People from the exposed class enter the infectious class at a rate γ_h that

is the reciprocal of the duration of the latent period. When the infectious human recover, they move to the recovered class at a rate α_h. The disease induce a death rate of δ at the infectious class. It is assumed that recovered individuals have temporary immunity that can be lost and as a result, they become susceptible to reinfection at a rate of ρ. The natural death rate for all classes of the human population occurs at μ_h. In the case of the mosquito population, susceptible mosquitoes are recruited by birth at a constant rate Λ_v, this is independent of the actual number of adult mosquitoes. This assumption is true since only a fraction of a large reservoir of eggs and larvae matures to the adult stage, and this process does not depend directly on the size of the adult mosquito population. The natural death rate of the mosquito is given by μ_v. Susceptible mosquitoes become infectious after biting an infectious or recovered human. The susceptible mosquitoes which are now infected move to the exposed class at a rate of $\frac{b(\beta_v I_h + \beta_{vh} R_h)}{N_h}$. The exposed mosquitoes then move to the infected class at a rate of γ_v. From the above assumptions, the model for the dynamics of malaria in the human and mosquito populations is given by the following nonlinear system of differential equation

$$\begin{cases} \frac{dS_h}{dt} = \Lambda_h + \rho R_h - \lambda_h S_h - \mu_h S_h, \\ \frac{dE_h}{dt} = \lambda_h S_h - (\gamma_h + \mu_h) E_h, \\ \frac{dI_h}{dt} = \gamma_h E_h - (\alpha_h + \mu_h + \delta) I_h, \\ \frac{dR_h}{dt} = \alpha_h I_h - (\rho + \mu_h) R_h, \\ \frac{dS_v}{dt} = \Lambda_v - \lambda_v S_v - \mu_v S_V, \\ \frac{dE_v}{dt} = \lambda_v S_v - (\gamma_v + \mu_v) E_v, \\ \frac{dI_v}{dt} = \gamma_v E_v - u_v I_v, \end{cases} \tag{2.1}$$

With the initial conditions: $S_h(0) > 0$,$E_h(0) \geq 0$,$I_h(0) \geq 0$, $R_h(0) \geq 0$, $S_v(0) > 0$, $E_v(0) \geq 0$, $I_v(0) \geq 0$. Where $\lambda_h = \frac{b\beta_h I_v}{N_h}$ and $\lambda_v = b\frac{\beta_v I_h + \beta_{vh} R_h}{N_h}$.

3 ANALYSIS OF OPTIMAL CONTROL

In this section, the model (2.1) is modulated to estimate the impact of four control strategies: Preventive using Long-lasting Insecticide Treated Net(LLITN)$u_1(t)$, the control effort on malaria treatment of infected individuals $u_2(t)$. $u_3(t)$, represents the insecticide spray on the breeding grounds for the vector and the preventive using Indoor Residual Spraying (IRS) $u_4(t)$. The force of infection that is associated with the human population is reduced by a coefficient of $(1 - u_1(t))$, where $u_1(t)$ represent the use of LLTIN to prevent the direct contact and bite from infected mosquito. In the mosquito population the associate force of infection is reduced by a coefficient of $(1 - u_3(t))$, $u_3(t)$ represents an application of insecticide spray on breeding grounds under some conditions, for example (climate conditions). The control $u_2(t)$ measures the rate at which infected individuals are treated with the efficacy of treatments $c \in [0, 1]$, where $\delta_1 > 0$ is a constant rate. Also $d_1, d_2 \in [0, 1]$ are constants rate. The objective of this model is to minimize the number of exposed and infected individuals and the infected mosquito population and maximize the total number of recovered individuals through the optimal control strategies $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$ respectively.

We make use of Pontryagin's Principle in order to find the necessary conditions that establish the presence of optimal control of the malaria transmission model. We include time dependent controls into SEIR-SEI malaria model and attempt to explore the suitable optimal

control strategies for setting the malaria under the controls. We use four control variables, the controls $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$. The objective function used for the model is similar to [11, 12, 13]. With suitable initial conditions, we consider an optimal control problem to minimize our objective function which is given by:

$$J(u_1,u_2,u_3,u_4)=\int_0^{t_f}(A_1E_h+A_2I_h+A_3I_v+\frac{a_1}{2}u_1^2+\frac{a_2}{2}u_2^2+\frac{a_3}{2}u_3^2+\frac{a_4}{2}u_2^2)dt,. \tag{3.1}$$

where A_1, A_2, A_3 are the balancing cost factors due to scale and a_1, a_2, a_3 and a_4,denote the weighting constants for making use of prevention strategies using $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$, controls. Consequently, we attempt to expect an optimal control $u_1^*, u_2^*, u_3^*, u_4^*$ such that,

$$J(u_1^*,u_2^*,u_3^*,u_4^*)=minJ(u_1,u_2,u_3,u_4),\Delta=\{(u_1,u_2,u_3,u_4)|0\le u_i\le 1,i=1,2,3,4\}. \tag{3.2}$$

$$\begin{cases}\frac{dS_h}{dt}=\Lambda_h+\rho R_h-(1-u_1)\frac{b\beta_hI_v}{N_h}S_h-\mu_hS_h,\\ \frac{dE_h}{dt}=(1-u_1)\frac{b\beta_hI_v}{N_h}S_h-(\gamma_h+\mu_h)E_h,\\ \frac{dI_h}{dt}=\gamma_hE_h-(\alpha_h+\mu_h+\delta+cu_2)I_h,\\ \frac{dR_h}{dt}=(\alpha_h+cu_2)I_h-(\rho+\mu_h)R_h,\\ \frac{dS_v}{dt}=\Lambda_v-(1-u_3)b\frac{\beta_vI_h+\beta_{vh}R_h}{N_h}S_v-(\mu_v+d_1u_4+d_2u_1)S_v,\\ \frac{dE_v}{dt}=(1-u_3)b\frac{\beta_vI_h+\beta_{vh}R_h}{N_h}S_v-(\gamma_v+d_1u_4+d_2u_1)E_v,\\ \frac{dI_v}{dt}=\gamma_vE_v-(\mu_v+d_1u_4+d_2u_1)I_v,\end{cases} \tag{3.3}$$

The optimal control must conform to the necessary conditions that is emanated from the Pontryagin Maximum Principle [14]. This concept transpose the equations (3.2) and (3.3) into a type of problem charachterised with minimizing pointwise a Hamiltonian H, with respect to u_1 and u_2

$$\begin{aligned}&H=A_1E_h+A_2I_h+A_3I_v+\tfrac{a_1}{2}u_1^2+\tfrac{a_2}{2}u_2^2+\tfrac{a_3}{2}u_3^2+\tfrac{a_4}{2}u_2^2\\&+\lambda_1\left\{\Lambda_h+\rho R_h-(1-u_1)\tfrac{b\beta_hI_v}{N_h}S_h-\mu_hS_h\right\}\\&+\lambda_2\{(1-u_1)\tfrac{b\beta_hI_v}{N_h}S_h-(\gamma_h+\mu_h)E_h\}\\&+\lambda_3\{\gamma_hE_h-(\alpha_h+\mu_h+\delta+cu_2)I_h\}\\&+\lambda_4\{(\alpha_h+cu_2)I_h-(\rho+\mu_h)R_h\}\\&+\lambda_5\{\Lambda_v-(1-u_3)b\tfrac{\beta_vI_h+\beta_{vh}R_h}{N_h}S_v-(\mu_v+d_1u_4+d_2u_1)S_v\}\\&+\lambda_6\{(1-u_3)b\tfrac{\beta_vI_h+\beta_{vh}R_h}{N_h}S_v-(\gamma_v+d_1u_4+d_2u_1)E_v\}\\&+\lambda_7\{\gamma_vE_v-(\mu_v+d_1u_4+d_2u_1)I_v\}\end{aligned} \tag{3.4}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ and λ_7, represents the adjoint variables.
The system solution is attained by suitably taking partial derivatives of the Hamiltonian (3.4) with respect to the associated state variable.

Theorem 3.1. Given an optimal control $u_1^*, u_2^*, u_3^*, u_4^*$ and the solutions $S_h, E_h, I_h, R_h, S_v, E_v, I_v$ of the corresponding state system (2.1) and (3.3) that minimize $J(u_1, u_2, u_3, u_4)$ over Δ. Then there exists adjoint variables $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$, satisfying

$$\tfrac{-d\lambda_i}{dt}=\tfrac{\partial H}{\partial i} \tag{3.5}$$

Where $i = 1, 2, 3, 4, 5, 6, 7$ and with transversality conditions

$$\lambda_1(t_f)=\lambda_2(t_f)=\lambda_3(t_f)=\lambda_4(t_f)=\lambda_5(t_f)=\lambda_6(t_f)=\lambda_7(t_f)=0 \tag{3.6}$$

and

$$u_1^* = min\left\{1, max(0, \frac{1}{a_1}\left(\frac{b\beta_h I_v S_h}{N_h}(\lambda_2 - \lambda_1) + d_2\left(E_v\lambda_6 + I_v\lambda_7\lambda\right)\right)\right\} \tag{3.7}$$

$$u_2^* = min\left\{1, max(0, \frac{1}{a_2}\left(cI_h(\lambda_3 - \lambda_4) + c_1 I_v\lambda_7\right)\right\} \tag{3.8}$$

$$u_3^* = min\left\{1, max(0, \frac{1}{a_3}\left(b\frac{\beta_v I_h + \beta_{vh} R_h}{N_h}(\lambda_6 - \lambda_5)\right)\right\} \tag{3.9}$$

$$u_4^* = min\left\{1, max(0, \frac{1}{a_4}\left(b(E_v\lambda_6 + I_v\lambda_7)\right)\right\} \tag{3.10}$$

Proof. Theorem 4.1 and Corollary 4.1 of [27] gives the conditions of possible existence of an optimal control based on the convexity of the integrand of $J(u_1, u_2, u_3, u_4)$ with respect to u_1, u_2, u_3 and u_4 a priori boundedness of the state solutions, and the resulting Lipschitz characteristics of the state system of the ODE's with the state variables. The Hamiltonian function determines at the optimal control level leads to the adjoint variables. Thus, the adjoint equations can be rearranged as

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= (1-u_1)b\frac{\beta_h I_v}{N_h}(\lambda_1 - \lambda_2) + (1-u_1)b\frac{\beta_h I_v}{N_h^2}(\lambda_2 - \lambda_1) + \mu_h\lambda_1 + (1-u_3)b\frac{\beta_v I_h + \beta_{vh} R_h}{N_h^2}S_h(\lambda_6 - \lambda_5)\\
\frac{d\lambda_2}{dt} &= -A_1 + (1-u_1)b\frac{\beta_h I_v S_h}{N_h^2}(\lambda_2 - \lambda_1) + (\gamma_h + \mu_h)\lambda_h - \gamma_h\lambda_3 + (1-u_3)b\frac{[\beta_v I_h + \beta_{vh} R_h]S_v}{N_h^2}(\lambda_6 - \lambda_5)\\
\frac{d\lambda_3}{dt} &= -A_2 + (1-u_1)b\frac{\beta_h I_v S_h}{N_h^2}(\lambda_2 - \lambda_1) + (\alpha_h + \mu_h + \delta_h + cu_2)\lambda_3 - (c\mu_2 + \alpha_h)\lambda_4 +\\
&\quad (1-u_3)b\frac{\beta_v S_v}{N_h}(\lambda_5 - \lambda_6) + (1-u_3)b\frac{[\beta_v I_h + \beta_{vh} R_h]S_v}{N_h^2}(\lambda_6 - \lambda_5)\\
\frac{d\lambda_4}{dt} &= (1-u_1)b\frac{\beta_h I_v S_h}{N_h^2}(\lambda_2 - \lambda_1) + (\rho + \mu_h)\lambda_4 - \rho\lambda_1 + (1-u_3)b\frac{\beta_{vh} S_v}{N_h}(\lambda_5 - \lambda_6)\\
&\quad +(1-u_3)b\frac{[\beta_v I_v + \beta_{vh} R_h]S_v}{N_h^2}(\lambda_6 - \lambda_5)\\
\frac{d\lambda_5}{dt} &= (1-u_3)b\frac{[\beta_v I_v + \beta_{vh} R_h]S_v}{N_h^2}(\lambda_5 - \lambda_6) + \mu_v\lambda_h\\
\frac{d\lambda_6}{dt} &= (\gamma_v + \mu_v + d_1u_4 + d_2u_1)\lambda_6 - \gamma_v\lambda_7\\
\frac{d\lambda_7}{dt} &= -A_3 + (1-u_1)b\frac{\beta_h S_h}{N_h}(\lambda_1 - \lambda_2) + (\mu_v + d_1u_4 + d_2u_1 + c_1u_2)\lambda_7
\end{aligned}$$

□

3.1 Numerical Simulations of Optimal Control

In this section, we discuss the numerical outcomes of our various optimal control strategies on the spread of malaria.

3.1.1 Prevention of malaria through the use of LLITN (u_1) and treatment (u_2) only

With this strategy, long-lasting Insecticide Treated Net (u_1) and treatment (u_2) are employed to optimize the objective function J, while the control on insecticide spray (u_3) and Indoor Residual spraying (u_4) are set to zero. In Figure 1(a) there is a significant difference when the controls are used. The number of exposed mosquitoes decreases when this strategy is used compared to without the control. Figure 1(b) also shows that the use of treated net and treatment reduce the number of infected mosquitoes substantially. From Figure 1(c), the exposed human fall very low, until about 11 days before it starts rising again, when the controls (u_1) and (u_2) are used. However, Figure 1(d) indicates that the controls (u_1) and (u_2) do very little to bring down the number of the infected mosquitoes. The Figure, 1(e) indicates that there is a significant difference between the number of recovered humans with the control compared to without the controls (u_1) and (u_2). With the control, the recovered humans increases tremendously.

Table 1: Parameter values for model (3.3)

Parameter	Description	Value	Source
Λ_h	Recruitment rate of human	2.5	Assumed
Λ_v	Recruitment rate of mosquito	500	Assumed
b	mosquito biting rate	0.39	[28]
μ_v	Natural death rate of mosquito	0.0714	[28]
γ_h	Progression rate from E_h to I_h compartment	0.08333	[29]
γ_v	Progression rate from E_v to I_h compartment	0.1	[30]
μ_h	Natural death rate of human	0.00004	[31]
β_h	Transmission probability from I_v to S_h	0.9	Assumed
β_v	Transmission probability from I_h to S_v	0.8	Assumed
$\beta_v h$	Transmission probability from R_h to S_v	0.009	Assumed
α	Infectious human recovery rate	0.003704	Assumed
δ	Disease induced death rate	0.00354	Assumed

3.1.2 Prevention of malaria through the use of LLITN (u_1) and insecticide spray (u_3) only

For this strategy, prevention effort at making effective use of long-lasting Insecticide Treated Net (u_1) and Insecticide spray (u_3) are employed to optimize the objective function while treatment (u_2) and Indoor Residual Spraying (u_4) are set to zero. From Figure 2(a), the number of exposed mosquitoes with the control decreases sharply until day 10 before it starts to increase again. However, the number without the control initially increases slightly after day 4, reaching a peak on day 6 before it gently decreases. After day 14, the control is no longer effective since the number of exposed mosquito with the control is higher than the number without the control. Figure 2(b) indicates that the control strategy decreases the number of Infected mosquito significantly. We see from Figure 2(c) that the number of exposed human with the control decreases until day 13 before it starts rising again. It is however efficient in reducing the number of exposed human. From Figures 2(d) and 2(e) , the graphs indicate that the strategy of using Long-lasting Insecticides Treated Net and insecticides spray have no effect on the number of infected and recovered human.

3.1.3 Prevention of malaria through the use of LLITN (u_1) and insecticide spray (u_4) only

Here the control on Long-Lasting Insecticide Treated Net (u_1) and Indoor Residual spraying (u_4) are both employed to optimize the objective function. We observed in Figure 3(a) that the control strategy reduces the number of exposed mosquitoes until after day 13 when it begins to increase again. Moreover it can be seen that, the strategy is not effective after day 17. From Figure 3(b), the number of infected mosquitoes with the control reduces significantly until after day 16, then it starts to rise again. This strategy effectively reduces the Infected mosquito. The graph from Figure 3(c) indicates that the control strategy reduces exposed human significantly till about day 10 before it starts increasing. The Figures 3(d) and 3(e) show that the control strategies of Long-lasting Insecticide Treated Net (u_1) and Indoor Residual spraying (u_4) do not show any difference in the numbers of infected human and recovered human when the strategy is used.

3.1.4 Prevention of malaria through the use of treatment (u_2) and insecticides spray (u_3) only

With this strategy, the control (u_2) on treatment and (u_3) on insecticides spray are used to optimize the objective function J. Figure 4(a) shows that the strategy is effective until after day 15. However there is a sharp decrease in the number of exposed mosquitoes when using the strategy until after day 11, it then begins to rise again. From Figure 4 (b), there is a little difference between the strategy with control and without control. The Figure 4(d) shows that the strategy reduces the number of infected human slightly. Clearly, from Figure 4 (e), the strategy of using treatment and insecticide spray increases the number of recovered humans. This shows that (u_2) and (u_3) are very effective in increasing the recovered humans.

3.1.5 Prevention of malaria through the use of treatment (u_2) and IRS u_4 only

Here, the control on treatment (u_2) and Indoor Residual spraying (u_4) are used to optimize the objective function while setting the control on Long-lasting Insecticide spray (u_3) to zero. Figure 5(a) indicates that the strategy decreases the number exposed mosquitoes significantly. However, the number showing the control begins to rise after day 18. We can see from Figure 5(b) that the strategy effectively reduce the Infected mosquito as the days go by. The graph in Figure 5(c) , shows that the strategy of using Indoor Residual spraying and treatment has no effect on the number of exposed human. According to the graph on Figure 5(d), employing this strategy will slightly reduce the number of Infected human after day 14. Figure 5 (e) shows that the use of (u_2) and (u_4) increases the number of recovered human. This means that the strategy of using (u_2) and (u_4) is very effective.

3.1.6 Prevention of malaria through the use of insecticide spray (u_3) and IRS (u_4) only

In this strategy, the control on insecticide spray (u_3) and Indoor Residual spraying (u_4) are employed to maximize the objective function while setting long-Lasting Insecticide Treated Net (u_1) and treatment (u_2) to zero. From Figure 6(a), the control strategy leads to a decrease in number of exposed mosquitoes until after day 10 where it starts to rise again. The control strategy of using (u_3) and (u_4) is no longer effective after day 14 since the number with the control is higher than the number without control. In Figure 6(b), the control strategy leads to a decrease in the number of infected mosquitoes as compare to without control. Figures 6(c), 6(d) and 6(e) show that the strategy do not affect the number of exposed humans, infected humans and recovered human.

3.1.7 Prevention of malaria through the use of the LLITN (u_1), treatment (u_2) and insecticide spray (u_3) only

With this strategy, the control on long-lasting Insecticide Treated Net (u_1), treatment (u_2) and Insecticide spray (u_3) are used to optimize the objective function J while setting Indoor Residual spraying (u_4) to zero. The Figure 7(a) indicates that, the strategy decreases the number of exposed mosquitoes until day 10 before it starts increasing again. However, the strategy is not effective after day 14. This is because the number with the control is higher than the number of Infected mosquito after day 14. We see clearly from Figure 7(b) that the strategy effectively reduces the number of infected mosquitoes. Figure 7(c) shows that the strategy reduces the number of exposed human until day 13 before it starts increasing again. From Figure 7(d), we can see that the strategy decreases the number of infected humans slightly after day 12 onwards. But before day 12, there is no difference in the number of infected human with control and the number without control. Figure 7(e), shows that there is a significant increase in the number of recovered human when this strategy is employed. This strategy is good in decreasing the infected mosquitoes as well as increasing the recovered humans.

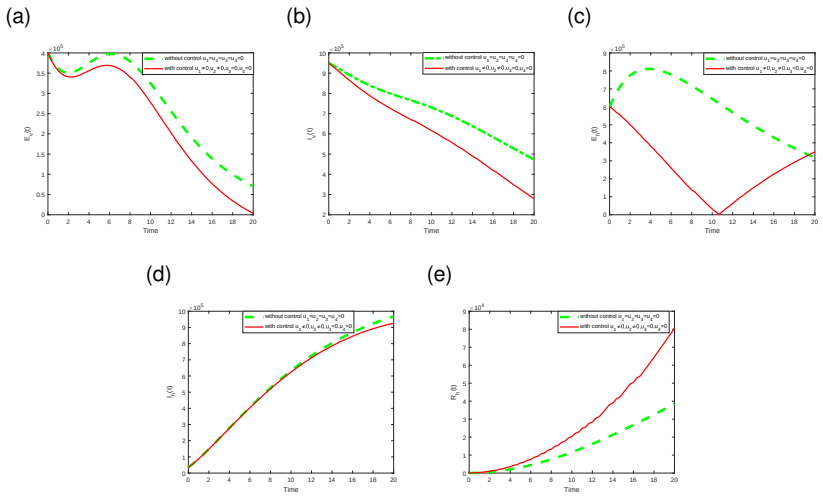

Figure 1: Simulations of the model showing the effect of malaria prevention through the use of 'LLITN' and treatment only on transmission. Fig 1 (a-e) represents the behavior of exposed and infected mosquitoes, exposed, infected and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 \neq 0, u_2 \neq 0, u_3 = 0, u_4 = 0.$)

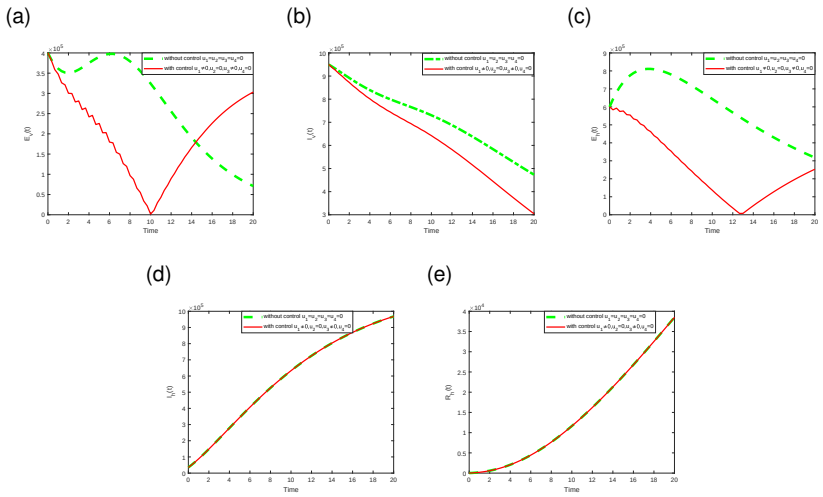

Figure 2: Simulations of the model showing the effect of malaria prevention through use of 'LLITN' and insecticide spray only on transmission. Fig 2 (a-e) represents the behavior of exposed and infected mosquitoes, exposed, infected and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 \neq 0, u_2 = 0, u_3 \neq 0, u_4 = 0.$)

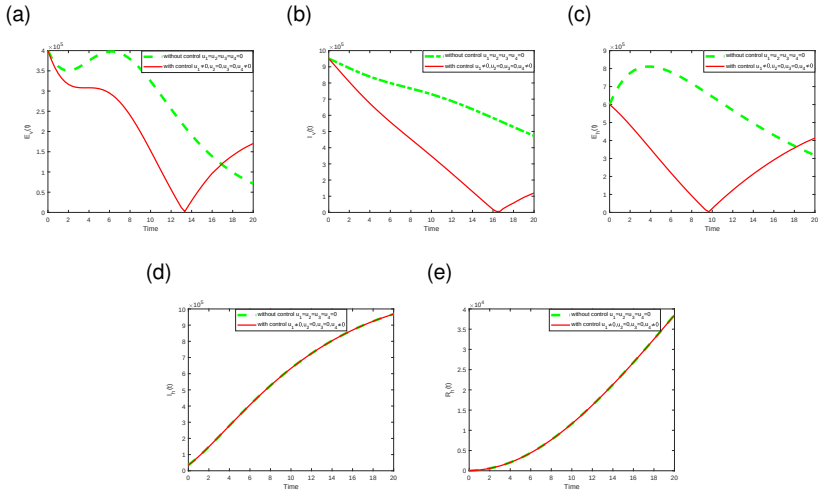

Figure 3: Simulations of the model showing the effect of malaria prevention through the use of 'LLITN' and IRS only on transmission. Fig 3 (a-e) represents the behavior of exposed and infected mosquitoes, exposed, infected and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 \neq 0, u_2 = 0, u_3 = 0, u_4 \neq 0.$)

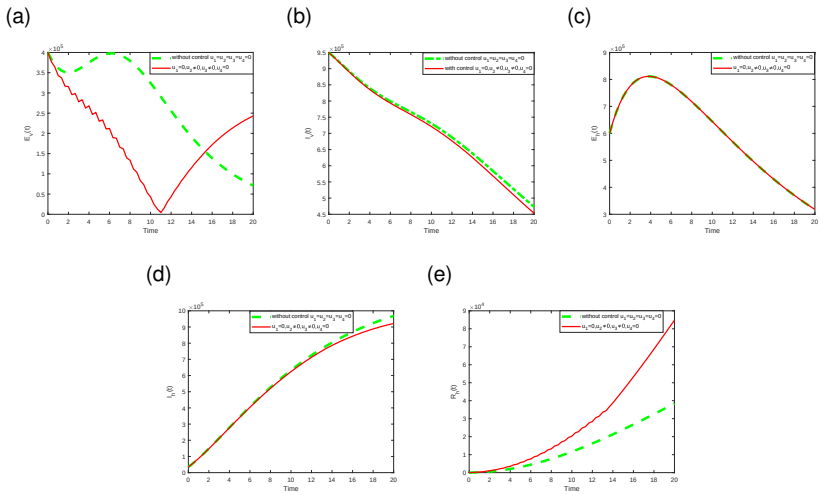

Figure 4: Simulations of the model showing the effect of malaria prevention through the use of treatment and insecticide spray only on transmission. Fig 4 (a-e) represents behavior of exposed and infected mosquitoes, exposed, infected and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 = 0, u_2 \neq 0, u_3 \neq 0, u_4 = 0.$)

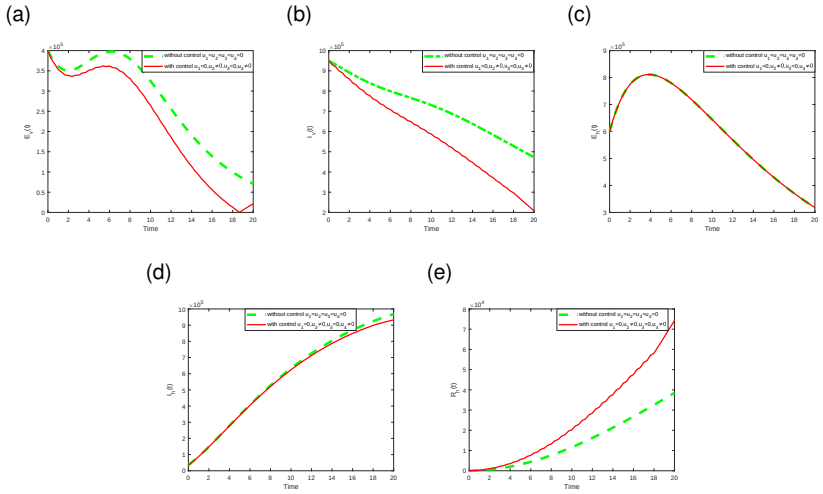

Figure 5: Simulations of the model showing the effect of malaria prevention through use of treatment and IRS only on transmission. Fig 5 (a-e) represents behavior of exposed and infected mosquitoes, exposed, infected and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 = 0, u_2 \neq 0, u_3 = 0, u_4 \neq 0$.)

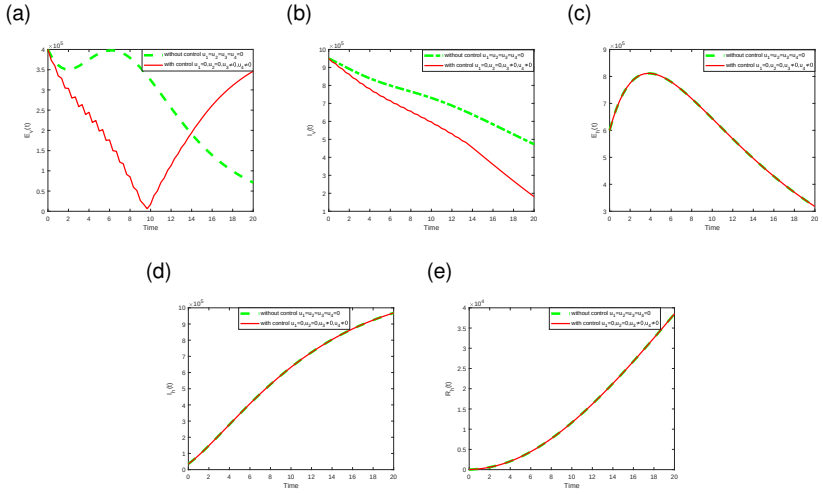

Figure 6: Simulations of the model showing the effect of malaria prevention through use of insecticide spray and IRS only on transmission. Fig 6 (a-e) represents behavior of exposed, infected mosquitoes exposed, infected humans and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 = 0, u_2 = 0, u_3 \neq 0, u_4 \neq 0$.)

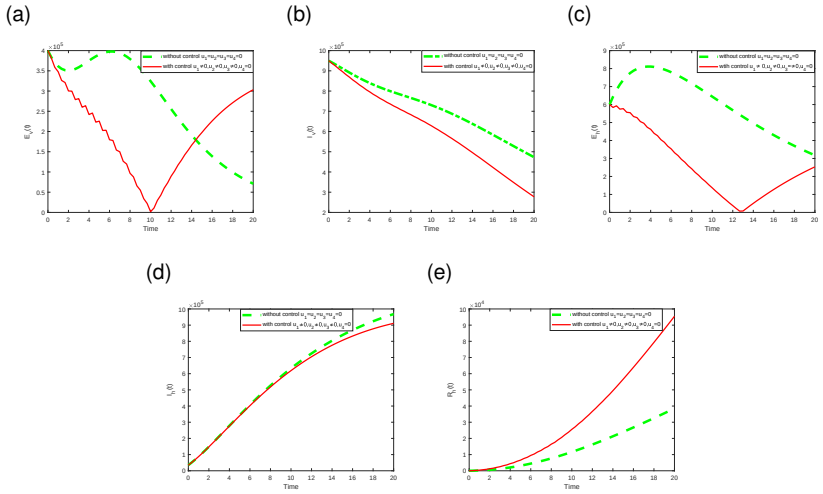

Figure 7: Simulations of the model showing the effect of malaria prevention through the use of 'LLITN', treatment and insecticide spary only on transmission. Fig 7 (a-e) represents behavior of exposed and infected mosquitoes, exposed, infected and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 \neq 0, u_2 \neq 0, u_3 \neq 0, u_4 = 0$.)

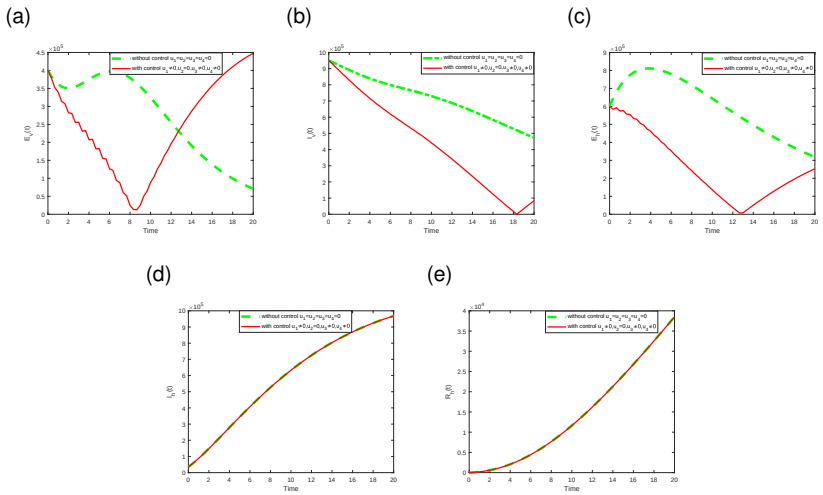

Figure 8: Simulations of the model showing the effect of malaria prevention through the use of LLITN, insecticide spray and IRS only on transmission. Fig 8 (a-e) represents behavior of exposed and infected mosquitoes, exposed, infected and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 \neq 0, u_2 = 0, u_3 \neq 0, u_4 \neq 0$.)

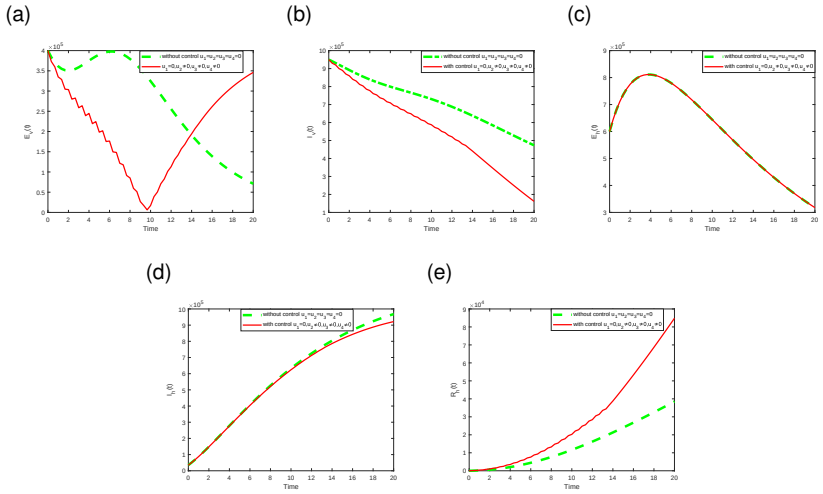

Figure 9: Simulations of the model showing the effect of malaria prevention through the use treatment, insecticide spray and IRS only on transmission. Fig 9 (a-e) represents behavior of exposed and infected mosquitoes, exposed, infected and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 = 0, u_2 \neq 0, u_3 \neq 0, u_4 \neq 0.$)

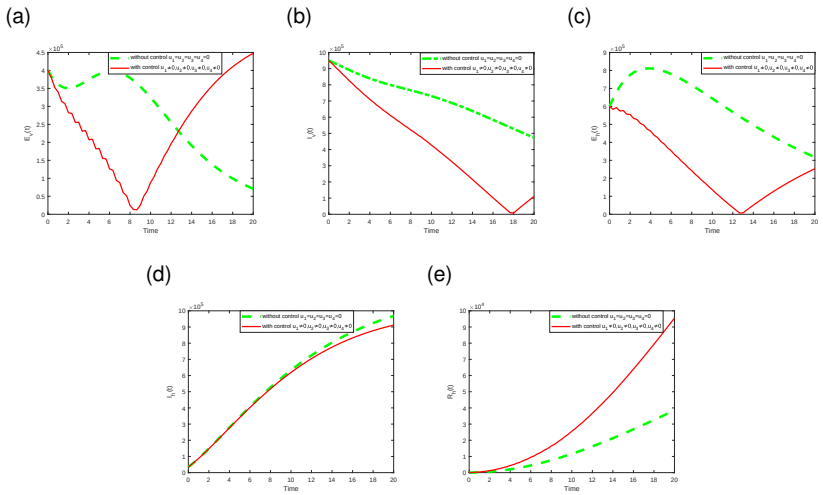

Figure 10: Simulations of the model showing the effect of malaria prevention 'LLITN',treatment, insecticide spary and IRS only on transmission. Fig 10 (a-e) represents behavior of exposed and infected mosquitoes, exposed, infected and recovered humans respectively. Dashed line represents system without control ($u_1 = u_2 = u_3 = u_4 = 0$) and solid line shows the system with control ($u_1 \neq 0, u_2 \neq 0, u_3 \neq 0, u_4 \neq 0.$)

3.1.8 Prevention of malaria through the use of the LLITN (u_1), Insecticide spray (u_3) and IRS (u_4) only

For this strategy, the control on Long-lasting Insecticide Treated Net (u_1), Insecticide spray (u_3) and Indoor Residual spraying (u_4) are employed to optimize the objective function J while treatment (u_2) is set to zero. Figure 8(a), shows that the strategy sharply decreases the number with the control until day 9 and then it starts increasing again. This strategy is not effective after day 13. We see from Figure 8(b) that the control strategy decreases the infected mosquito greatly compared to without the control. Figure 8(c) indicates that the control strategy decreases the number of exposed human until around day 13 before the number start increasing. From Figures 8(d) and 8(e), we see that the control strategies have no effect on the number of the infected and recovered humans.

3.1.9 Prevention of malaria through the use of treatment (u_2), Insecticide spray (u_3) and IRS (u_4) only

With this strategy, the control on treatment (u_2), insecticide spray (u_3) and Indoor Residual spraying (u_4) are used to optimize the objective function while setting the control on Long-lasting Insecticide Treated to zero. From Figure 9(a), the number with the control decreases until around day 10 before it starts increasing. The control is however not effective after day 14. Figure 9(b), clearly shows that the control decreases the number of infected mosquitoes significantly. However from Figure 9(c), the control does not affect the number of exposed human. Moreover, from Figure 9(d) the control decreases the number of infected human slightly after day 14 onwards. Specifically, we observed from Figure 9(e) that the control strategy leads to a sharp increase in the number of recovered human while the uncontrolled case resulted in a decrease of the recovered humans.

3.1.10 Prevention of malaria through the use of the LLITN (u_1), treatment (u_2), Insecticide spray (u_3) and IRS (u_4) only

For this strategy, we employ all the four controls u_1, u_2, u_3 and u_4 in order to optimize the objective function. From Figure 10(a), the number of the exposed mosquitoes reduces with the control until day 9 before it starts rising again. The strategy is not effective after day 13 since the number of exposed mosquitoes with the control is higher than the number without the control. The activation of all the control is effective in minimizing the number of Infected mosquitoes this is clearly seen in Figure 10(b). Similarly, the number of exposed human is drastically reduced when this strategy is employed until after day 13 where the number with the control begins to rise again. This is clearly seen from figure 10(c). However from Figure 10(d), the strategy shows no difference until after day 10 where the number of infected human decreases slightly. It is clearly seen from Figure 10(e) that the presence of all the controls increases the number of recovered human greatly.

4 CONCLUSION

In this paper, an optimal control model of malaria transmission with four control strategies and standard incidence rate was proposed to study the transmission of the malaria and control the malaria at minimum cost. Four control strategies were introduced to estimate and measure empirically the effectiveness of the use of Long-Lasting Insecticide Treated Net, the control effort on malaria treatment of infected individuals, the insecticide spray on the breeding grounds for the mosquito and the preventive using indoor residual spraying. Our numerical simulations suggest that the use of two controls (Long-Lasting Insecticide Treated Net $u_1(t)$ and the control effort on malaria treatment $u_2(t)$) together are more effective than the other control strategies in reducing the number of exposed and infected individuals and also increasing the number of recovered individuals.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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